Review Problems
for
Introductory Physics 1

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Contents

Chapter 1

Preface

The problems in this review guide are provided as is ***without any guarantee of being correct!*** That's not to suggest that they are all broken – on the contrary, most of them are well-tested and have been used as homework, quiz and exam problems for decades if not centuries. It is to suggest that they have typos in them, errors of other sorts, bad figures, and one or two of them are really too difficult for this course but haven't been sorted out or altered to make them doable.

Leaving these in just adds to the fun. Physics problems are *not* all cut and dried; physics itself isn't. One thing you should be building up as you work is an appreciation for what is easy, what is difficult, what is correct and what is incorrect. If you find an error and bring it to my attention, I'll do my best to correct it, of course, but in the meantime, be warned!

A *few* of the problems have rather detailed solutions (due to Prof. Ronen Plesser and myself), provided as examples of how a really good solution might develop, with considerable annotation. However, most problems do not have included solutions and *never will have.* I am actually philosophically opposed to providing students with solutions that they are then immediately tempted to memorize. This guide is provided so that you can learn to solve problems and work sufficiently carefully that they can trust the solutions.

Students invariably then ask: But how are we to know if we've solved the problems correctly?"

The answer is simple. The same way you would *in the real world!* Work on them in groups and check your algebra, your approach, and your answers against one another's. Build a consensus. Solve them with mentoring (course TAs, professors, former students, tutors all are *happy* to help you). Find answers through research on the web or in the literature.

To be honest, almost any of the ways that involve *hard work on your part* are good ways to learn to solve physics problems. The only *bad* way to (try to) learn is to have the material all laid out, cut and dried, so that you don't have to *struggle* to learn, so that you don't have to *work hard* and thereby *permanently* imprint the knowledge on your brain as you go. Physics requires engagement and investment of time and energy like no subject you have ever taken, if only because it is one of the most difficult subjects you've ever tried to learn (at the same time it is remarkably simple, paradoxically enough).

In any event, to use this guide most effectively, first skim through the whole thing to see what is there, then start in at the beginning and work through it, again and again, reviewing repeatedly all of the problems and material you've covered so far as you go on to what you are working on currently in class and on the homework and for the upcoming exam(s). Don't be afraid to solve problems more than once, or even more than three or four times.

And *work in groups!* Seriously! With pizza and beer...

Chapter 2

Short Math Review Problems

The problems below are a diagnostic for what you are likely to need in order to work physics problems. There aren't really enough of them to constitute practice", but if you have difficulty with *any* of them, you should probably find a math review (there is usually one in almost any introductory physics text and there are a number available online) and work through it.

Weakness in geometry, trigonometry, algebra, calculus, solving simultaneous equations, or general visualization and graphing will all negatively impact your physics performance and, if uncorrected, your grade.

Problem 1. problems-1/math-sa-binomial-expansion.tex

Write down the **binomial expansion** for the following expressions, given the conditions indicated. FYI, the binomial expansion is:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

where x can be positive or negative and where n is any real number and only converges if $|x| < 1$. Write at least the first *three non-zero* terms in the expansion:

a) For $x > a$:
$$\frac{1}{(x+a)^2}$$

b) For $x > a$:
$$\frac{1}{(x+a)^{3/2}}$$

c) For $x > a$:
$$(x+a)^{1/2}$$

d) For $x > a$:
$$\frac{1}{(x+a)^{1/2}} - \frac{1}{(x-a)^{1/2}}$$

e) For $r > a$:
$$\frac{1}{(r^2 + a^2 - 2ar\cos(\theta))^1/2}$$

Problem 2. problems-1/math-sa-differentiate-expressions.tex

Evaluate the following expressions, where $\frac{d}{dt}$ means "differentiate with respect to t":

a) $\frac{d}{dt}\sin(\omega t) =$ $\boxed{}$

b) $\frac{d}{dt}\cos(\omega t) =$ $\boxed{}$

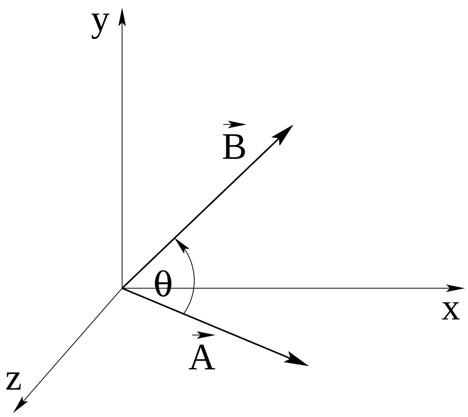
c) $\frac{d}{dt}\ln(at) =$ $\boxed{}$

d) $\frac{d}{dt}\left(at^5 + bt^2 + c\right) =$ $\boxed{}$

e) $\frac{d}{dt}e^{\lambda t} =$ $\boxed{}$

f) $\frac{d}{dt}\left(1 + at^2\right)^3 =$ $\boxed{}$

Problem 3. problems-1/math-sa-evaluate-vector-products.tex



a) Express the dot product in terms of its Cartesian components e.g. $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}$:

$\vec{A} \cdot \vec{B} =$ $\boxed{}$

b) Express the dot product in terms of the magnitudes A, B and θ:

$\vec{A} \cdot \vec{B} =$ $\boxed{}$

c) Express the *magnitude* of cross product in terms of the magnitudes A, B and θ:

$\left|\vec{A} \times \vec{B}\right| =$ $\boxed{}$

d) Express the cross product in terms of its Cartesian components e.g. $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}$ (this has a lot of terms):

$\vec{A} \times \vec{B} =$ $\boxed{}$

Problem 4. problems-1/math-sa-general-arithmetic.tex

Solve the following short problems:

a)

$$\frac{10 * 2^4}{5} \quad = \quad \boxed{}$$

b)

$$7.1 * 3. \times 10^3 + 12. \quad = \quad \boxed{}$$

c)

$$\frac{\sqrt{3}}{2} * \frac{1}{\sqrt{6}} \quad = \quad \boxed{}$$

d)

$$\cos(\pi/6)/9 \quad = \quad \boxed{}$$

e)

$$\sin(30^\circ) * 3.14 \quad = \quad \boxed{}$$

Problem 5. problems-1/math-sa-integrate-expressions.tex

Evaluate the following indefinite and definite integrals:

a) $\displaystyle\int_0^{\pi/2} \sin(\theta)\ d\theta = \boxed{}$

b) $\displaystyle\int \cos(\omega t)\ dt = \boxed{}$

c) $\displaystyle\int x^n\ dx = \boxed{}$

d) $\displaystyle\int_a^b \frac{1}{x}\ dx = \boxed{}$

e) $\displaystyle\int (-gt + v_0)\ dt = \boxed{}$

Problem 6. problems-1/math-sa-simple-series.tex

Evaluate the first *three nonzero terms* for the **Taylor Series** for the following expressions. Recall that the radius of convergence for the binomial expansion (another name for the first taylor series in the list below) is $|x| < 1$ – this gives you two ways to consider the expansions of the form $(x + a)^n$.

a) Expand about $x = 0$:

$(1 + x)^{-2} \approx$

b) Expand about $x = 0$:

$e^x \approx$

c) For $x > a$ (expand about x or use the binomial expansion after factoring):

$(x + a)^{-2} \approx$

d) Estimate $0.9^{1/4}$ to within 1% without a calculator, if you can. Explain your reasoning.

Problem 7. problems-1/math-sa-solve-simple-equations.tex

Solve for t. Your answer should be an equation, although you may give a number answer for the last one as ***well*** as the algebraic answer if you have a calculator handy. You may find $\ln(2) \approx 0.693$ to be a useful thing to know if not.

a) $v_0 t - x_0 = 0$ $\qquad t = \boxed{}$

b) $-\frac{1}{2}gt^2 + v_0 t = 0$ $\qquad t = \boxed{}$

c) $-\frac{1}{2}gt^2 + v_0 t + x_0 = 0$ $\qquad t = \boxed{}$

d) $A/2 = Ae^{-t}$ $\qquad t = \boxed{}$

(for $A = 5$).

Problem 8. problems-1/math-sa-solve-simultaneous-equations.tex

Solve the following system of simultaneous equations for a and T. ***Show your work*** and give ***algebraic*** answers in terms of m_1, m_2, θ and g:

$$m_1 g \sin(\theta) - T = m_1 a$$
$$T - m_2 g = m_2 a$$

$a = \boxed{}$

$T = \boxed{}$

Problem 9. problems-1/math-sa-sum-two-vectors-1.tex

Suppose vector $\vec{\boldsymbol{A}} = -4\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$ and vector $\vec{\boldsymbol{B}} = 9\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$. Then the vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$:

a) is in the first quadrant (x+,y+) and has magnitude 17.

b) is in the fourth quadrant (x+,y-) and has magnitude 12.

c) is in the first quadrant (x+,y+) and has magnitude 13.

d) is in the second quadrant (x-,y+) and has magnitude 17.

e) is in the third quadrant (x-,y-) and has magnitude 13.

Problem 10. problems-1/math-sa-sum-two-vectors.tex

a) Suppose vector $\vec{\boldsymbol{A}} = 3\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$ and bector $\vec{\boldsymbol{B}} = -7\hat{\boldsymbol{x}} - 3\hat{\boldsymbol{y}}$. Then the vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$:

A) is in the first quadrant (x+,y+) and has magnitude 7.

B) is in the second quadrant (x-,y+) and has magnitude 7.

C) is in the second quadrant (x-,y+) and has magnitude 5.

D) is in the fourth quadrant (x+,y-) and has magnitude 5.

E) is in the third quadrant (x-,y-) and has magnitude 6.

b) Suppose vector $\vec{\boldsymbol{A}} = -4\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$ and vector $\vec{\boldsymbol{B}} = 9\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$. Then the vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$:

A) is in the x-direction and has magnitude 17.

B) is in the y-direction and has magnitude 13.

C) is in the $-y$-direction and has magnitude 12.

D) is in the x-direction and has magnitude 5.

E) is in the $-x$-direction and has magnitude 13.

Problem 11. problems-1/math-sa-taylor-series.tex

Evaluate the first *three nonzero terms* for the **Taylor series** for the following expressions. Expand about the indicated point:

a) Expand about $x = 0$:
$$(1+x)^n \approx$$

b) Expand about $x = 0$:
$$\sin(x) \approx$$

c) Expand about $x = 0$:
$$\cos(x) \approx$$

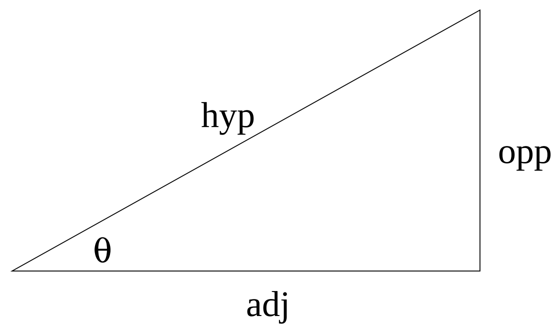
d) Expand about $x = 0$:
$$e^x \approx$$

e) Expand about $x = 0$ (note: $i^2 = -1$):
$$e^{ix} \approx$$

Verify that the expansions of both sides of the following expression match:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

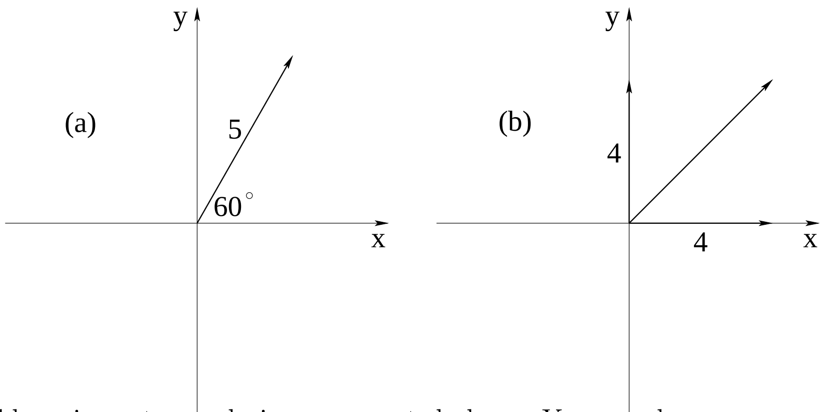
Problem 12. problems-1/math-sa-trig-basic.tex



Fill in the following in terms of the marked sides. For example, one of the answers below might be (but probably isn't) $\frac{\text{hyp}}{\text{opp}}$:

$\sin(\theta) =$ $\boxed{}$ $\cos(\theta) =$ $\boxed{}$ $\tan(\theta) =$ $\boxed{}$

Problem 13. problems-1/math-sa-vector-components.tex



Two simple problems in vector analysis are presented above. You may leave your answers in terms of radical fractions (e.g. $\sqrt{7}/13$) where appropriate. You may not use calculators!

a) Find the **cartesian** coordinate components (X, Y) of the vector given.

$X \quad =$

$Y \quad =$

b) Find the **polar** coordinate components (V, θ) of the vector given.

$|V| \quad =$

$\theta \quad =$

2.0.1 Long Problems

Problem 14. problems-1/math-pr-elliptical-trajectory.tex

The position of a particle as a function of time is given by:

$$\vec{x}(t) = x_0 \cos(\omega t)\hat{x} + y_0 \sin(\omega t)\hat{y}$$

where $x_0 > y_0$.

a) What is $\vec{v}(t)$ for this particle?

b) What is $\vec{a}(t)$ for this particle?

c) Draw a generic plot of the trajectory function for the particle. What kind of shape is this? In what direction/sense is the particle moving (indicate with arrow on trajectory)?

d) Draw separate plots of $x(t)$ and $y(t)$ on the same axes.

Chapter 3

Essential Laws, Theorems, and Principles

The questions below guide you through basic physical laws and concepts. They are the stuff that one way or another you should know" going into any exam or quiz following the lecture in which they are covered. Note that there aren't really all that many of them, and a lot of them are actually easily derived from the most important ones.

There is absolutely no point in memorizing solutions to all of the problems in this guide. In fact, for all but truly prodigious memories, memorizing them all would be impossible (presuming that one could work out all of the solutions into an even *larger* book to memorize!). However, *every student* should memorize, internalize, learn, *know* the principles, laws, and and theorems covered in this section (and perhaps a few that haven't yet been added). These are things upon which all the rest of the solutions are based.

Short Problem 1.

problems-1/true-facts-angular-momentum-conservation.tex

When is the angular momentum of a system conserved?

Short Problem 2.

problems-1/true-facts-archimedes-principle.tex

What is Archimedes' Principle? (Equation with associated diagram or *clear and correct* statement in words.)

Short Problem 3.

problems-1/true-facts-bernoullis-equation.tex

What is Bernoulli's equation? What does it describe? Draw a small picture to illustrate.

Short Problem 4.

problems-1/true-facts-coefficient-of-performance.tex

How is the coefficient of performance of a refrigerator defined? Draw a small diagram that schematically indicates the flow of heat and work between reservoirs.

Short Problem 5.

problems-1/true-facts-conditions-static-equilibrium.tex

What are the two conditions for a rigid object to be in static equilibrium?

Condition 1:

Condition 1:

Short Problem 6.

problems-1/true-facts-coriolis-force.tex

What "force" makes hurricanes spin counterclockwise in the northern hemisphere and clockwise in the southern hemisphere?

Short Problem 7.

problems-1/true-facts-definition-of-decibel.tex

One measures sound intensity in decibels. What is a decibel? (Equation, please, and define and give value of all constants.)

Short Problem 8.

problems-1/true-facts-doppler-shift-moving-source.tex

What is the equation for the Doppler shift, specifically for the frequency f' heard by a stationary observer when a source emitting waves with speed u at frequence f_0 is approaching at speed u_s?

Short Problem 9.

problems-1/true-facts-equipartition-theorem.tex

What is the Equipartition Theorem?

Short Problem 10.

problems-1/true-facts-four-forces-of-nature.tex

Name the four fundamental forces of nature as we know them now.

a)

b)

c)

d)

Short Problem 11.

problems-1/true-facts-generalized-work-energy.tex

What is the *Generalized* Work-Mechanical-Energy Theorem? (Equation only. This is the one that differentiates between conservative and non-conservative forces.)

Short Problem 12.

problems-1/true-facts-heat-capacity-monoatomic-gas.tex

What is the heat capacity at constant volume C_V of N molecules of an ideal monoatomic gas? What is its heat capacity at constant pressure C_P?

Short Problem 13.

problems-1/true-facts-heat-engine-efficiency.tex

What is the algebraic definition of the efficiency of a heat engine? Draw a small diagram that schematically indicates the flow of heat and work between reservoirs.

Short Problem 14.

problems-1/true-facts-inelastic-collision-conservation.tex

What is conserved (and what *isn't*) in an inelastic collision?

Short Problem 15.

problems-1/true-facts-integral-definition-moment-of-inertia.tex

Write the *integral* definition of the moment of inertia of an object about a particular axis of rotation. Draw a picture illustrating what "dm" is within the object relative to the axis of rotation.

Short Problem 16.

problems-1/true-facts-kepler1.tex

What is Kepler's First Law?

Short Problem 17.

problems-1/true-facts-kepler2.tex

What is Kepler's Second Law and what physical principle does it correspond to?

Short Problem 18.

problems-1/true-facts-kepler3.tex

What is Kepler's Third Law?

Short Problem 19.

problems-1/true-facts-momentum-conservation.tex

Under what condition(s) is the linear momentum of a system conserved?

Short Problem 20.

problems-1/true-facts-n1.tex

What is Newton's First Law?

Short Problem 21.

problems-1/true-facts-n2.tex

What is Newton's Second Law?

Short Problem 22.

problems-1/true-facts-n3.tex

What is Newton's Third Law?

Short Problem 23.

problems-1/true-facts-newtons-law-gravitation.tex

What is Newton's Law for Gravitation? Draw a picture showing the coordinates used (for two pointlike masses at arbitrary positions), and indicate the value of G in SI units.

Short Problem 24.

problems-1/true-facts-parallel-axis-theorem.tex

Write the parallel axis theorem for the moment of inertia of an object around an axis parallel to one through its center of mass. Draw a picture to go with it, if it helps.

Short Problem 25.

problems-1/true-facts-pascals-principle.tex

What is Pascal's principle? A small picture would help.

Short Problem 26.

problems-1/true-facts-perpendicular-axis-theorem.tex

Write the perpendicular axis theorem for a mass distributed in the $x - y$ plane. Draw a picture to go with it, if it helps.

Short Problem 27.

problems-1/true-facts-toricellis-law.tex

What is Toricelli's Law (for fluid flow) and what is the condition required for it to be approximately true?

Short Problem 28.

problems-1/true-facts-venturi-effect.tex

What is the Venturi Effect?

Short Problem 29.

problems-1/true-facts-wave-equation-string.tex

Write the wave equation (the differential equation) for waves on a string with tension T and mass density μ. Identify all parts.

Short Problem 30.

problems-1/true-facts-work-energy.tex

What is the Work-Kinetic Energy Theorem?

Short Problem 31.

problems-1/true-facts-youngs-modulus.tex

What is the definition of Young's modulus Y? Draw a picture illustrating the physical situation it describes and define all terms used in terms of the picture.

Chapter 4

Problem Solving

The following problems are, at last, the meat of the matter: serious, moderately to extremely difficult physics problems. An A" student would be able to construct *beautiful solutions*, or almost all, of these problems.

Note well the phrase beautiful solutions". In no case is the answer" to these problems an equation, or a number (or set of equations or numbers). It is a *process*. Skillful physics involves a systematic progression that involves:

- Visualization and conceptualization. What's going on? What will happen?
- Drawing figures and graphs and pictures to help with the process of determining what physics principles to use and how to use them. The paper should be an extension of your brain, helping you associate coordinates and quantities with the problem and working out a solution strategy. For example: drawing a free body diagram" in a problem where there are various forces acting on various bodies in various directions will usually help you break a large, complex problem into much smaller and more manageable pieces.
- Identifying (on the basis of these first two steps) the *physical principles* to use in solving the problem. These are almost invariably things from the Laws, Theorems and Principles chapter above, and with practice, you will get to where you can easily identify a Newtons Second Law" problem (or part of a problem) or an Energy Conservation" (part of) a problem.
- Once these principles are identified (and identifying them *by name* is a good practice, especially at first!) one can proceed to *formulate* the solution. Often this involves translating your figures into equations using the laws and principles, for example creating a free body diagram and translating it into Newton's Second Law for each mass and coordinate direction separately.
- At this point, believe it or not, the hard part is usually done (and most of the *credit* for the problem is *already secured*). What's left is using *algebra* and other mathematical techniques (e.g. trigonometry, differentiation, integration, solution of simultaneous equations that combine the results from different laws or principles into a single answer) to obtain a *completely algebraic* (symbolic) expression or set of expressions that answer the question(s).
- At this point you should *check your units!* One of several good reasons to solve the problem algebraically is that all the symbols one uses carry *implicit* units, so usually it is a simple matter to check whether or not your answer has the right ones. If it does, that's good! It means you

probably didn't make any trivial algebra mistakes like dividing instead of multiplying, as that sort of thing would have led to the wrong units.

Remember, an answer with the right units may be wrong, but it's not crazy and will probably get lots of credit if the reasoning process is clear. On the other hand, an answer with the *wrong* units isn't just mistaken, its *crazy* mistaken, impossible, silly. Even if you can't see your error, if you check your answer and get the wrong units *say so*; your instructor can then give you a few points for being diligent and checking and knowing that you are wrong, and can usually quickly help you find your mistake and permanently correct it.

- Finally, *at the very end*, substitute any numbers given for the algebraic symbols, do the arithmetic, and determine the final numerical answers.

Most of the problems below won't have any numbers in them at all to emphasize how unimportant this last step is *in learning physics!* Sure, you should learn to be careful in your doing of arithmetic, but anybody can (with practice) learn to punch numbers into a calculator or enter them into a computer that will do all of the arithmetic flawlessly no matter what. It is the process of determining *how* to punch those numbers or program the computer to evaluate a *correct formula* that is what physics is all about. Indeed, with skill and practice (especially practice at estimation and conceptual problem solving) you will usually be able to at least *approximate* an answer and fully understand what is going on and what will happen even without doing any arithmetic at all, or doing only arithmetic you can do in your head.

As with all things, practice makes perfect, wax on, wax off, and the more *fun* you have while doing, the more you will learn. Work in groups, with friends, over pizza and beer. Learning physics should not be punishment, it should be a pleasure. And the ultimate reward is seeing the entire world around you with different eyes...

Chapter 5

Newton's Laws

The following problems include both kinematics" problems that can be solved using nothing but mathematics with units (no real physical principle but the notion of acceleration, for example) and true ***dynamics*** problems – ones based on Newton's Second Law plus various force laws, used to find the acceleration(s) of various masses and, the differential equation(s) of motion of those masses, and from those differential equations (by integration or later in the course, by more complicated methods of solution) the velocity and the position.

Some problems are very short and can be answered on the basis of physics concepts with little or no computation. Others must be solved algebraically with some real effort, usually using multiple physics concepts and several mathematical skills synthesized together. A few problems are presented and then completely solved to illustrate good solution methodology.

The main thing to remember is – ***be sure to connect the basic physical principles involved*** to ***each*** problem as you solve them. Write them down separately, in English as well as equation form (if there is one). Well over half of the difficulty of solving physics problems on exams comes from being clueless concerning what the problem is all about. Typically, anywhere from 1/3 to 1/2 the credit for a problem can be obtained by simply ***getting the basic physics right*** even if you screw up the algebra afterwards or have no idea how to proceed from the concept to the solution.

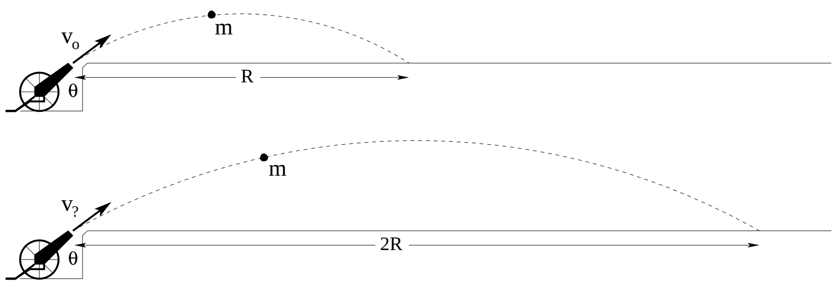
But you will find, especially with practice, that once you *do* learn to identify the right physics to go with a problem you can usually make it through the algebra and other math to get a creditable solution, presuming only that you honestly have met the mathematical prerequisites for this course (or have beefed up your math skills along the way).

Good luck!

5.1 Kinematics

5.1.1 Multiple Choice

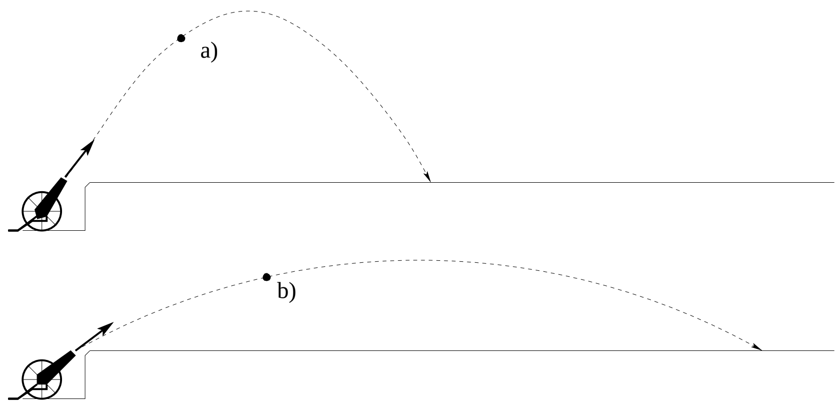
Problem 15. problems-1/kinematics-mc-range-of-cannon-on-plain.tex



A cannon sits on a horizontal plain. When it fires a cannonball of mass m at speed v_0 at an angle θ relative to the ground it has a range R (neglecting friction and drag). Suppose one wishes to fire at a target a distance $2R$ away without altering the the elevation angle θ. The initial speed of the cannonball as it leaves the cannon in terms of v_0 must then be (circle the correct answer):

a) $\sqrt{2}v_0$ b) $2v_0$ c) $3v_0$ d) $4v_0$ e) We cannot tell because the answer depends upon θ.

Problem 16. problems-1/kinematics-mc-cannonball-timed-trajectories-icp.tex



Two cannons fire projectiles into the air along the trajectories shown. Neglect the drag force of the air.

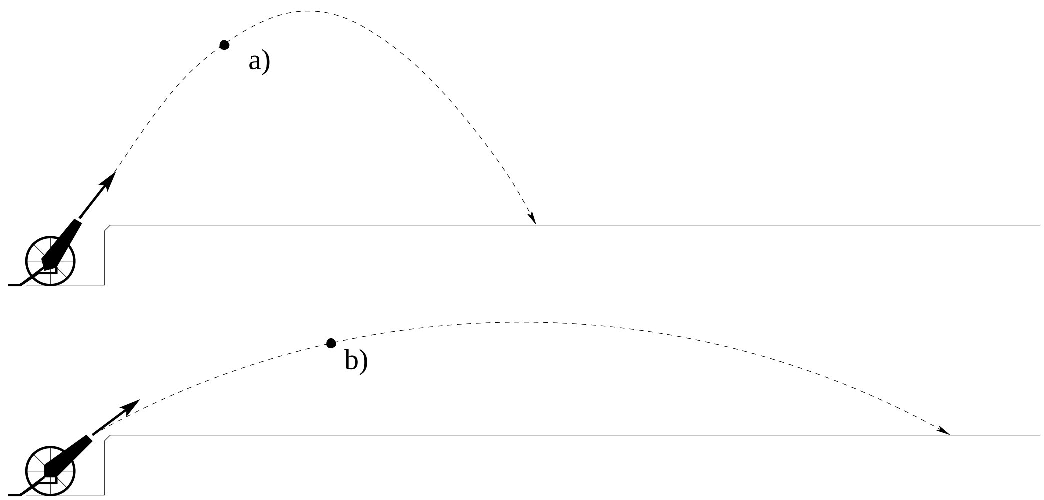
a) Cannonball $\boldsymbol{a}$ is in the air longer.

b) Cannonball $\boldsymbol{b}$ is in the air longer.

c) Cannonballs $\boldsymbol{a}$ and $\boldsymbol{b}$ are in the air the same amount of time.

d) We cannot tell which is in the air longer without more information than is given in the picture.

Problem 17. problems-1/kinematics-mc-cannonball-timed-trajectories-icp-soln.tex



The motion in x and y are ***independent*** for 2D trajectory problems. You can therefore ***ignore the x motion altogether*** when you assess the answer to this question. So, considering ***only how high each cannonball goes***, which one is in the air longer?

Problem 18. problems-1/kinematics-mc-shooting-the-monkey.tex

A zookeeper wants to shoot a monkey sitting on a branch a height H above the gun muzzle in a tree a horizontal distance R away with a tranquilizer gun. Where must the zookeeper aim the gun in order to hit the monkey if the monkey falls asleep in the tree after being shot (that is, does ***not*** drop from the tree at the instant he fires)? (Neglect air resistance, justify your answer with a sketch or some work.)

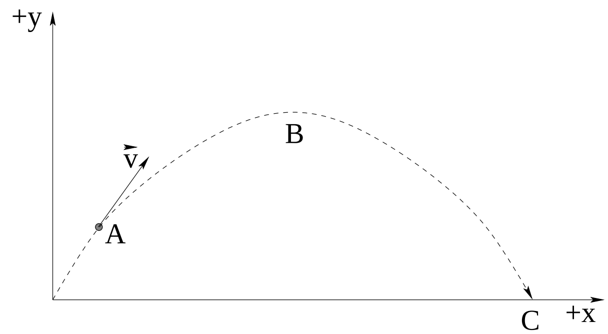
a) Straight at the monkey.

b) Slightly below the monkey.

c) Slightly above the monkey.

d) Cannot tell without knowing the mass of the monkey and the dart.

Problem 19. problems-1/kinematics-mc-velocity-acceleration-2D-1.tex



A small ball of mass m is thrown so that it follows the parabolic trajectory shown. Neglect drag forces. Circle the true statement(s) (there can be more than one) below:

a) The minimum speed occurs at point A.

b) The maximum speed occurs at point C.

c) The acceleration is larger at point B than it is at point C.

d) The acceleration is the same at points A and C.

e) The speed is the same at points A and C.

Problem 20. problems-1/kinematics-mc-falling-time-mars.tex

Surface gravity on Mars is roughly 1/3 that of the Earth. Suppose you drop a rock (initially at rest) from a height H_m on Mars and it takes a time t_g to hit the ground. From what height H_e do you need to drop the mass on Earth so that it hits the ground in the same amount of time?

a) $H_e = \sqrt{3}H_m$

b) $H_e = 3H_m$

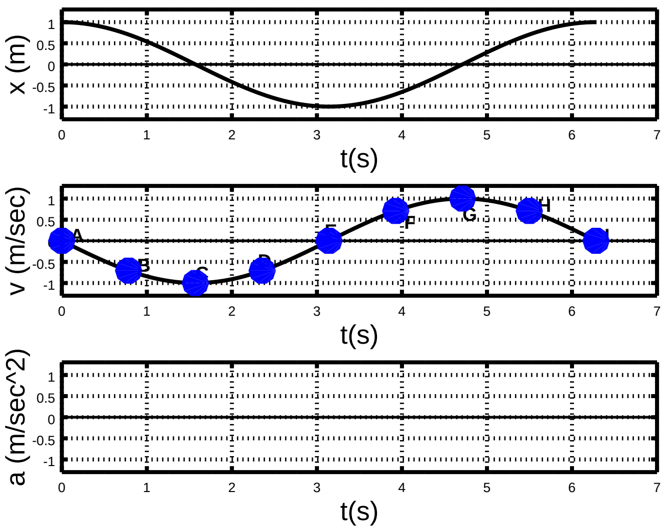
c) $H_e = 9H_m$

d) $H_e = H_m/\sqrt{3}$

e) $H_e = H_m/3$

5.1.2 Short Answer

Problem 21. problems-1/kinematics-sa-a2v2x-2.tex



A particle undergoes ***one-dimensional motion*** such that its position $x(t) = cos(t)$. Its velocity is therefore given by $v(t) = dx/dt = -\sin(t)$. Both of these are plotted in the graph on the right.

a) Sketch the acceleration in the x-direction, $a(t)$, in the lowest graph box provided.

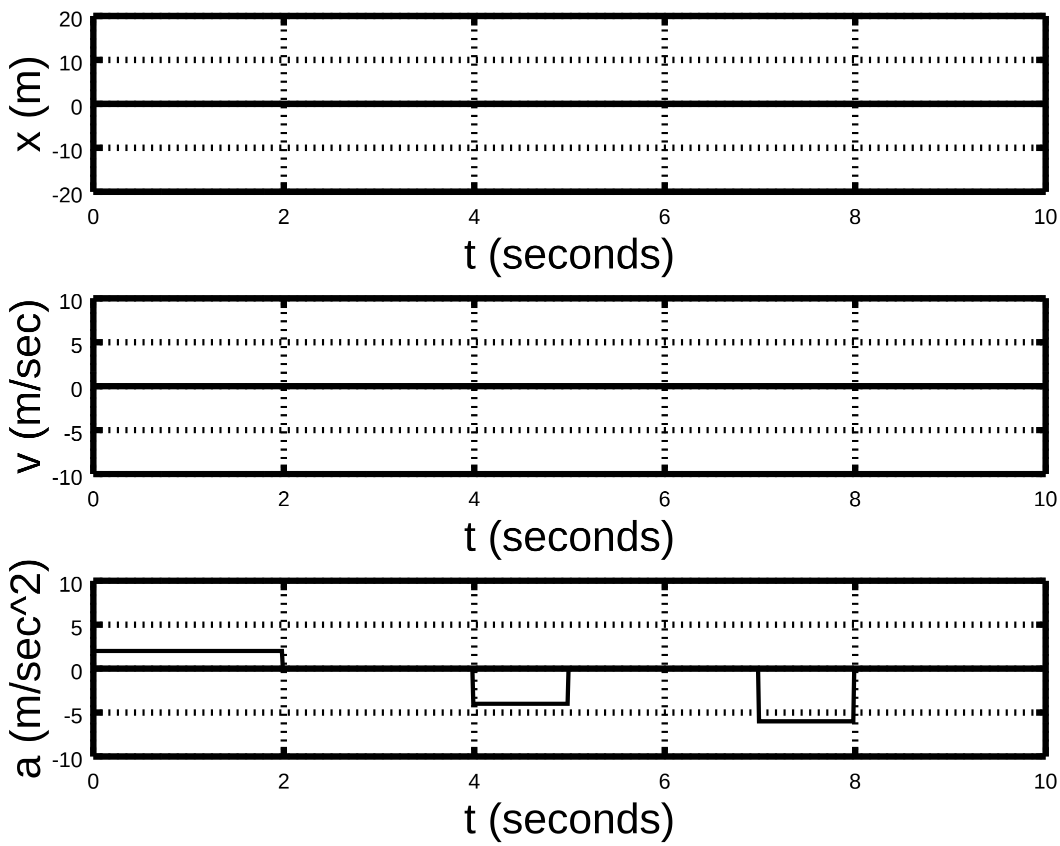
b) For which letter(s) A,B,...,I in the middle panel does the particle have nonzero velocity and zero acceleration?

c) For which letter(s) A,B,...,I in the middle panel does the particle have a zero velocity and nonzero acceleration?

d) For which letter(s) A,B,...,I in the middle panel is the particle speeding up while its velocity is negative?

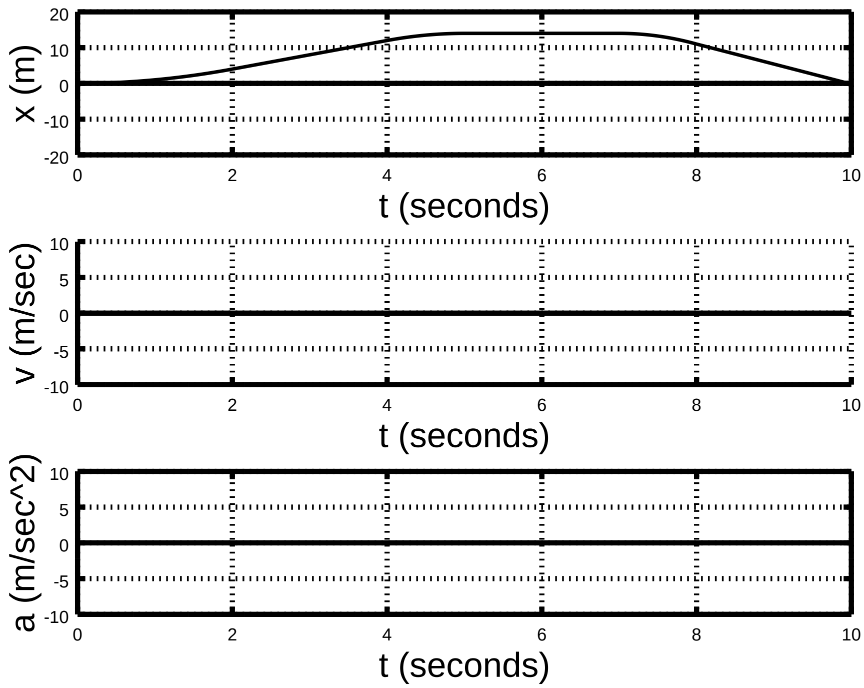
e) Discussion: In physics, functions that have a power series expansion (like sine and cosine and the exponential function) ***cannot*** have physical dimensions, nor can their arguments. Yet, in this problem, you are given $x(t) = cos(t)$ where x has units of *meters*, and t has units of *seconds*. Find an explanation for this that makes sense. Are there hidden constants in this expression? This is one of many reasons that algebraic expressions are superior to ones with (often hidden) dimensioned numbers!

Problem 22. problems-1/kinematics-sa-a2v2x.tex



On the figure above the acceleration $a(t)$ for one-dimensional motion is plotted in the bottom panel. In the first and second panel sketch in *approximate* curves that represent $x(t)$ and $v(t)$ respectively. Your curves should at least *qualitatively* agree with the bottom figure. Assume that the particle in question starts *from rest* and is *at the origin* at time $t = 0$.

Problem 23. problems-1/kinematics-sa-x-to-a-graphs.tex



On the figure above a trajectory $x(t)$ for one-dimensional motion is plotted in the top panel. In the second and third panel sketch in *approximate* curves that represent $v(t)$ and $a(t)$ respectively. Your curves should at least *qualitatively* agree with the top figure and correctly identify ranges of positive, negative and zero velocity and acceleration.

Problem 24. problems-1/kinematics-sa-scaling-fall-times-mars.tex

Suppose that a mass dropped dropped from rest on Mars 2 meters above the surface takes 1 second to reach the ground. How high do you have to drop it from for it to take 4 seconds to reach the ground? (Neglect air resistance and show work to justify your answer.)

Problem 25. problems-1/kinematics-sa-scaling-fall-times-icp.tex

A mass m is used to perform some experiments close to the surface of the planet *Mongo* (which will figure prominently in a number of problems this semester). The near-surface gravitational force on Mongo, like that of Earth, is given by:

$$F_y = -mg_m$$

where the gravitational "constant" of Mongo is represented by the (unknown, so far) constant g_m. You observe that when it is dropped from 4 meters above the ground, it takes 1 second to reach the surface.

a) How high do you have to drop it from for it to take 3 seconds to reach the ground? (Neglect all forces but gravity and show work to justify your answer.)

b) Just for fun, what is the gravitational acceleration g_m on Mongo relative to that of Earth?

$$\frac{g_m}{g_e} = \boxed{}$$

Problem 26. problems-1/kinematics-sa-scaling-fall-times-icp-soln.tex

It's easiest to do this ***algebraically*** and take a ratio. It eliminates most of the arithmetic. We follow the ritual demonstrated in lecture. For either mass:

- $F_y = -mg_m = ma_y$ (N2)
- $a_y = -g_m$ (solve for acceleration)
- $v_y(t) = -\int g_m\,dt = -g_m t$ (Initial Condition: $v_y(0) = 0$)
- $y(t) = \int v_y(t)\,dt = -\int g_m t\,dt = H - \frac{1}{2}g_m t^2$ (Initial Condition: $y(0) = H$)

Note well that we ***do not write units*** next to symbols with known/implicit units. In this case, y and H are obviously lengths, t is a time, g_m is an (unknown) acceleration. We don't even need to specify a *system* of units for these equations to be correct!

After the first week or two, you will not need to do the integrals explicitly and will just remember the constant acceleration results *without* ever tring to "memorize" them, but for now you should! Then (writing each solution and rearranging):

$$H_1 = \frac{1}{2}g_m t_1^2$$

$$H_2 = \frac{1}{2}g_m t_2^2$$

We can take the ***ratio*** of these two equations, which eliminates the unknown g_m:

$$\frac{H_2}{H_1} = \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{3^2}{1^2}\right) = 9$$

or

This is an example of ***scaling***. Since the distance fallen *scales* with time squared, it falls 9 times as far in 3 times the time. A physicist might not even write down the equations to arrive at this conclusion! Remember, the time it takes to answer a question *matters* on a quiz or exam. One can solve this many ways, some of them purely arithmetical (for example, find g_m from the first equation, substitute it into the second equation), but most of them are a lot more work and take more *time*!

Our final answers, of course, should be explicitly written and have units:

a)

$$H_2 = 9H_1 = 9 \times 4 = 36 \text{ meters.}$$

b)

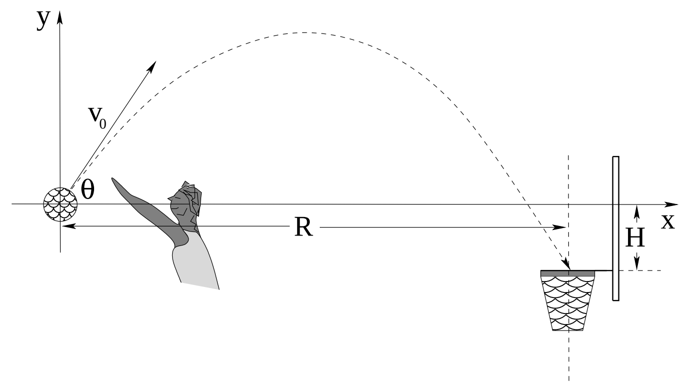
$$\frac{g_m}{g_e} = \boxed{0.8}$$

Problem 27. problems-1/kinematics-sa-falling-time-earth-moon.tex

Gravity on the surface of the moon is weaker than it is on the surface of the earth: $g_{moon} \approx g_{earth}/6$. If a mass dropped from 5 meters above the earth takes 1 second to reach the surface of the earth, how long does it take a mass dropped from the same height above the moon take to reach the surface of the moon? (You can express the answer as a rational fraction, no need for calculators.)

5.1.3 Long Problems

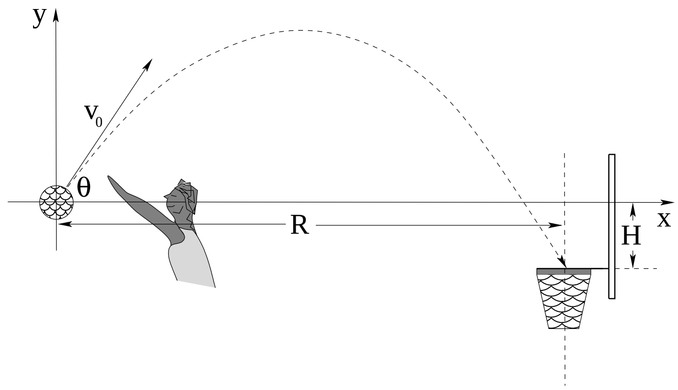
Problem 28. problems-1/kinematics-pr-2D-basketball-trajectory.tex



A basketball player shoots a jump hook at a (horizontal) distance R from the basket, releasing the ball at a height H above the rim as shown. To shoot over his opponent's outstretched arm, he releases the basketball at an angle θ with respect to the horizontal.

Find v_0, the ***speed*** he must release the basketball with (in terms of H, R, g and θ) for the ball to go through the hoop "perfectly" as shown. Assume that his release is on line and undeflected, at initial speed v_0 and that the acceleration of the basketball is $\vec{a} = -g\hat{\jmath}$, ignoring drag.

Problem 29. problems-1/kinematics-pr-2D-basketball-trajectory-soln.tex



First, note that

$$a_x = 0, v_{0x} = v_0 \cos(\theta), x_0 = 0$$

and

$$a_y = -g, v_{0y} = v_0 \sin(\theta), y_0 = 0$$

define the initial conditions of two ***independent*** 1D constant acceleration problems.

Integrate $a_x = 0$ twice to get:

$$x(t) = v_0 \cos(\theta) t$$

Integrate $a_y = -g$ twice to get:

$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin(\theta) t$$

Next, find the ***time*** t_b that the basketball reaches the horizontal position of the hoop:

$$R = v_0 \cos(\theta) t_b \Rightarrow t_b = R/(v_0 \cos(\theta))$$

This must also be the time that the ball has exactly the height of the hoop:

$$-H = -\frac{1}{2} g t_b^2 + v_0 \sin(\theta) t_b$$

$$-H = -\frac{gR^2}{2v_0^2 \cos^2(\theta)} + R \tan(\theta)$$

$$\frac{gR^2}{2v_0^2 \cos^2(\theta)} = R \tan(\theta) + H$$

And finally, we solve for v_0:

$$\boxed{v_0 = \sqrt{\frac{gR^2}{2(R \sin(\theta) \cos(\theta) + H \cos^2(\theta))}}}$$

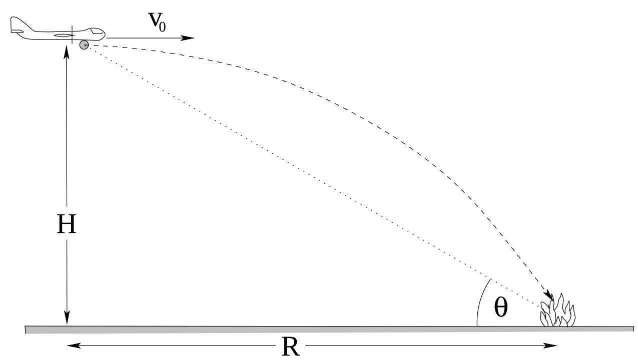
After doing the algebra, check the dimensions. Are they OK?

Check "common sense" – does the solution vary the way you expect? Well, if g goes up, he must shoot the ball faster to overcome gravity on (say) Jupiter. Makes sense. If H goes up, must shoot faster even here on Earth to reach the hoop. Makes sense.

Note that solution doesn't tell us whether a shot at the ***given*** angle will hit the rim, but if θ points directly at center of hoop ($\tan(\theta) = -H/R$) then v_0 has to become "infinite" for ball to travel in a straight line to the target. There are ***no solutions*** for angles less than this as we can tell because the solution speed becomes imaginary! This too makes "sense".

Note Well: There is a second, much more painful solution that involves finding the time that the basketball reaches the right height H ***first***, then substituting it into equation for R. **This solution can work**, but it is not easy. The trick is to isolate the radical on one side of the equals sign, square both sides (to make the radical go away), and then solve for v_0. Done perfectly, it will give you precisely the same answer obtained above.

Problem 30. problems-1/kinematics-pr-bombing-run-trajectory.tex



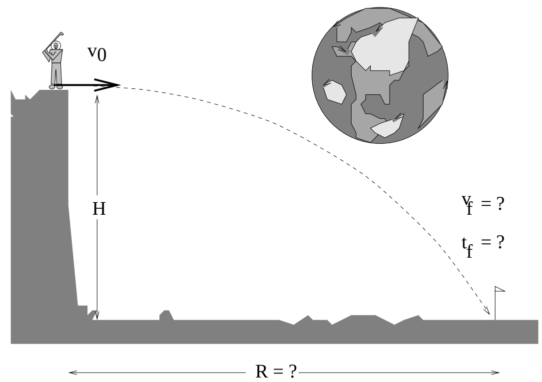
A forest fire is burning. A forest service plane flying ***horizontally*** at a height H above the ground is about to release a water bomb with ***initial horizontal velocity*** $\boldsymbol{v_0\hat{x}}$ at just the right instant so that it will freely fall with the right trajectory to hit the fire and put it out. Neglect drag forces and answer the following questions in terms of the givens H and v_0 and physical constants you know such as g.

a) How long will it take for the bomb to reach the ground?

b) What should the angle θ of the line that leads ***directly from the fire to the plane*** as shown be ***at the instant of release*** if the bomber wishes to hit the fire? (Note that this line is ***not*** the trajectory of the water bomb, which is the curved line above it.)

c) What is the ***speed*** of the the bomb when it hits the fire?

Problem 31. problems-1/kinematics-pr-golf-on-moon-icp.tex



An astronaut on the moon hits a golf ball of mass m horizontally from a tee H meters above the plane as shown. The initial speed of the ball is v_0 in the x-direction only. The gravitational force law for the moon is:

$$\vec{\boldsymbol{F}}_m = -m\frac{g}{6}\hat{\boldsymbol{j}}$$

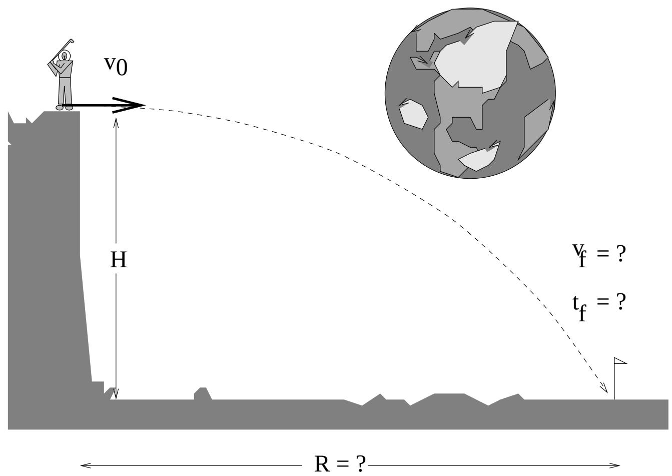
Note that there are no drag forces as the moon is in a vacuum, and that the lunar plane is flat on the scale of this picture. Use Newton's second law to answer the following questions:

a) How long does it take the ball to reach the ground?

b) How far from the base of the cliff where the tee is located does the ball strike?

c) How fast is the ball going when it hits the ground?

Problem 32. problems-1/kinematics-pr-golf-on-moon-icp-soln.tex



a) $t_g = \sqrt{\frac{2H}{g'}} = \sqrt{\frac{12H}{g}}$

b) $R = x(t_g) = v_0 t_g = v_0 \sqrt{\frac{2H}{g'}}$

c) This one is a bit tricky (and gets much easier with energy conservation later):

$v_x(t_g) = v_0$

$v_y(t_g) = \sqrt{\frac{2Hg}{6}}$

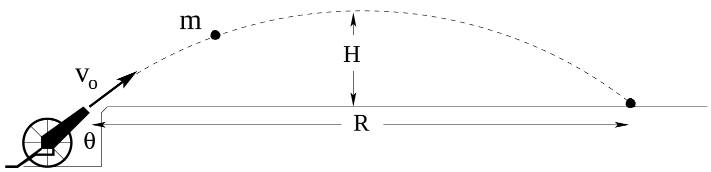
$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \frac{2Hg}{6}}$

Problem 33. problems-1/kinematics-pr-green-laser-falls.tex

The Green Lantern's daughter, Green Laserbeam, steps off of a tall building to follow her dad to the ground. She falls freely (from rest) to the ground, falling the last **third** of the total distance in a time t_2. Find the following ***ratio*** of the ***total time it takes for her to reach the ground to*** $\boldsymbol{t_2}$. Your answer should be a number.

$$\frac{t_{\rm tot}}{t_2} = \boxed{}$$

Problem 34. problems-1/kinematics-pr-range-of-cannon-on-plain.tex



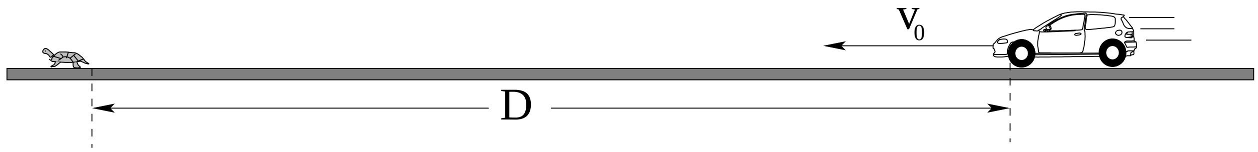
A cannon sits on a horizontal plain. It fires a cannonball of mass m at speed v_0 at an angle θ relative to the ground. Find:

a) The maximum height H of the cannonball's trajectory.

b) The time t_a the cannonball is in the air.

c) The range R of the cannonball.

Problem 35. problems-1/kinematics-pr-stopping-before-a-turtle.tex



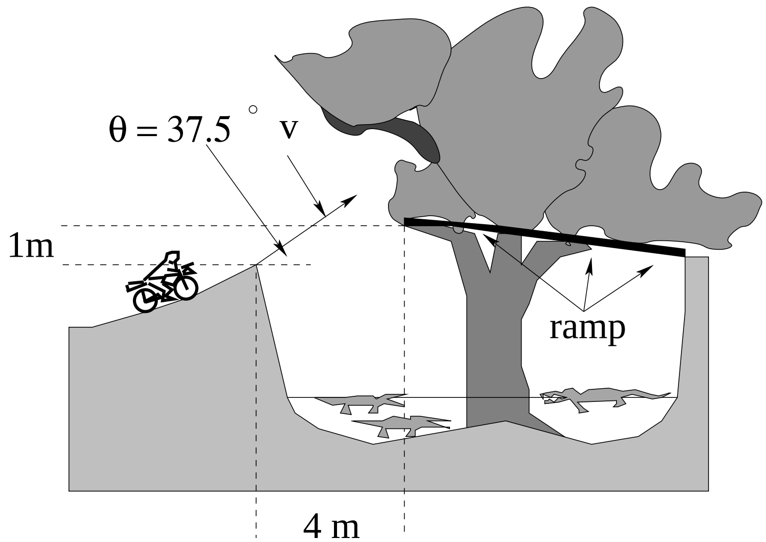
A distance of D meters ahead of your car you see a box turtle sitting on the road. Your car is traveling at a speed of v_0 meters per second straight at the turtle (along the straight road).

a) What is the (algebraic) magnitude of the *minimum* acceleration your car must have in order to stop before hitting the turtle? What is its direction?

b) How long does it take to stop your car at this acceleration?

c) Evaluate your algebraic answers for $D = 50$ m, $v_0 = 20$ m/sec (about 45 mph). If your car's maximum braking acceleration magnitude is $a = 5$ m/sec^2, do you hit the turtle?

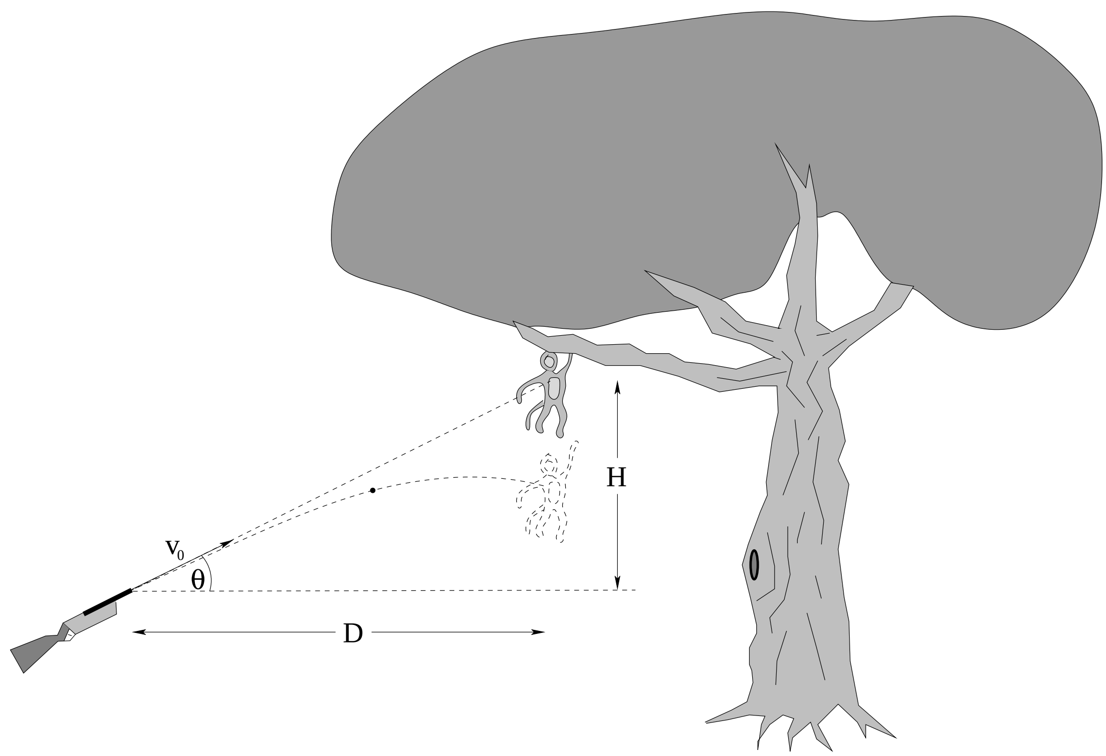
Problem 36. problems-1/kinematics-pr-evie-kniebel-trajectory.tex



Evie Kniebel, a stunt woman for a movie, is trying to jump a motorcycle across a crocodile-filled ditch and land on a special ramp that is built into the branch of a convenient tree. The horizontal gap she must leap to reach the ramp is **4 m**. The ramp is vertically **1 m** above the lip of the takeoff ramp. The angle of the takeoff ramp is fixed at 37.5°(which just happens to be the angle of a 3-4-5 right triangle). If Evie jumps even a bit too high, she will wreck on an overhanging branch. If she jumps too low, she bounces back. Either way the crocs that have already dined on Evie's stunt-cousins Evel, Weevel, and Abel Kneibel will eat well again.

With what speed v_0 must she take off to complete the jump ***just right*** (and live to get paid)? Answer the question algebraically FIRST and only then worry about numbers.

Problem 37. problems-1/kinematics-pr-monkey-gun.tex



A hunter aims his gun directly at a monkey in a distant tree. Just as she fires, the monkey lets go and drops in free fall towards the ground. Show that the bullet hits the monkey.

Problem 38. problems-1/kinematics-pr-stopping-before-a-bicycle-icp.tex

Sally is driving on a straight country road at night at a speed of $v_s = 20$ meters/second when she sees a bicyle without any lights loom ahead of her. The bike is travelling in the same direction as Sally at $v_b = 5$ meters/second. She slams on her brakes when she is a distance $D = 50$ meters from the bike, and her car brakes with a constant acceleration of $a = -5$ meters/second^2.

a) Write an ***algebraic*** condition for *car hitting the bike*. Do not use any of the numbers given above in the algebra; use symbols.

b) Draw a graph representing the trajectories of the car and the bike. Again, this doesn't need to be for particular values of the givens; you may want to draw several graphs representing different cases where the bike is or isn't hit.

c) The biker is deaf and doesn't hear her brakes or alter his speed in any way. Does Sally's car hit the bike for the specific values given?

d) Suppose that the bike is travelling *towards* her (on the wrong side of the road). Does she hit the bike (before she stops moving forward) then for the specific values given?

Problem 39. problems-1/kinematics-pr-stopping-before-a-bicycle-icp-soln.tex

First, ignore the numbers.

a)
$$x_{\rm car}(t_c) = x_{\rm bike}(t_c)$$

or in the coordinate system indicated:

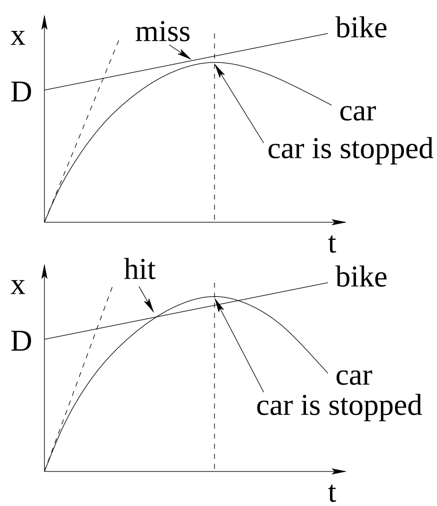
$$v_s t_c - \frac{1}{2}at_c^2 = D + v_b t_c$$

This is just a quadratic equation:

$$\frac{1}{2}at_c^2 + (v_b - v_s)t_c + D = 0$$

If one can find a real time t_c that solves this equation, the car hits the bike. If the time(s) that satisfy the quadratic are ***imaginary***, then the car stops before hitting the bicycle.

b) Here are two graphs. In the first one the car barely misses the bike. In the second one, it hits the bike (but only once – why?).

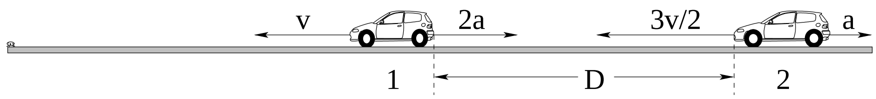


c) No bikers were injured in this problem. If the quadratic bothers you (it shouldn't) note that it takes 3 seconcds for Sally to slow to the speed of the biker. If her position at that time is still behind the bike's position at that time, she will never reach it. The bike is at $x_b = 65$ meters, far beyond the 37.5 meters she reaches at this time.

d) It now takes her 4 seconds to stop completely. She stops at $x_s = 40$ meters. But at that same time, $x_b = 30$ meters! So she hits it.

This is why bikers should *always* ride *with* the traffic! They do ***not*** obey the same rule as a pedestrian who walks facing the traffic! The pedestrian can just step off of the road, but a biker often cannot because of a curb or other obstruction!

Problem 40. problems-1/kinematics-pr-two-braking-cars.tex



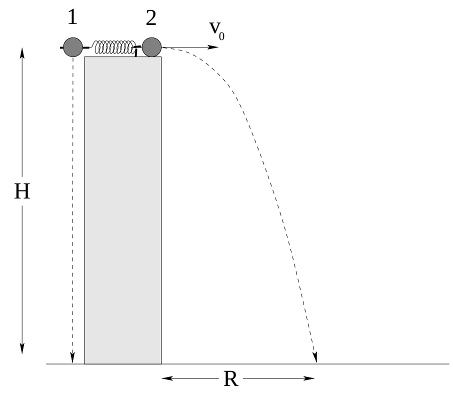
Two cars are driving down a straight country road when the driver of the first car (travelling at speed $v_1 = v$) sees a turtle crossing the road in front of her. She quickly applies the brake, causing her car to slow down with a (negative) acceleration $a_1 = 2a$. The second car is a distance D behind her and is travelling at an initial speed $v_2 = 3v/2$. Its driver immediately applies his brakes as well – assume at the same time as the driver of the first car – but his car is heavier and his tires are not so good and his car only slows down with a (negative) acceleration $a_2 = a$.

a) Find the minimum value of D such that the cars do not collide.

b) ***Qualitatively*** graph $x_1(t)$ and $x_2(t)$ as functions of time on a reasonable scale.

c) ***Qualitatively*** graph $v_1(t)$ and $v_2(t)$ as functions of time on a reasonable scale with the same time axis.

Problem 41. problems-1/kinematics-pr-two-falling-balls.tex



When a trigger is pulled at time $t = 0$, a compressed spring simultaneously drops ball 1 and hits identical ball 2 so that it is shot out to the right as initial speed v_0 as shown. The two balls then independently fall a height H. Answer the following questions, assuming that the balls fall only under the influence of gravity. (Neglect drag forces, and express all answers in terms of the givens, in this case H and v_0 and (assumed) gravitational acceleration g.).

a) Which ball strikes the ground first (or do they strike at the same time)? Prove your answer by finding the time that each ball hits the ground.

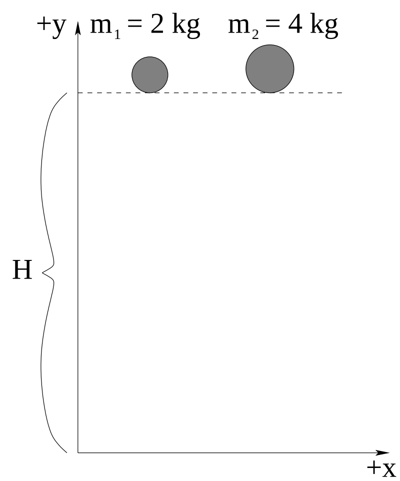
b) Which ball is travelling faster when it hits the ground (or do they hit at the same speed)? Prove your answer by finding an expression for the speed each ball has when it hits the ground.

c) Find an expression for R, the horizontal distance ball 2 travels before hitting the ground.

5.2 Dynamics

5.2.1 Multiple Choice

Problem 42. problems-1/force-mc-acceleration-two-masses-1.tex



A mass $m_1 = 2$ kg and a mass of $m_2 = 4$ kg are both dropped from rest from the same height at the same time. Mark the true statements with an "X" below (there can be more than one). **Neglect drag forces.**

☐ While the two masses are falling, the force acting on m_1 and the force acting on m_2 are equal in magnitude.

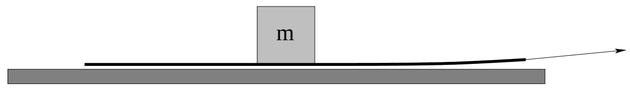
☐ While the two masses are falling, the acceleration of m_1 and the acceleration of m_2 are equal in magnitude.

☐ Mass m_2 will strike the ground first.

☐ Mass m_1 will strike the ground first.

☐ The two masses will strike the ground at the same time.

Problem 43. problems-1/force-mc-block-on-paper-icp.tex

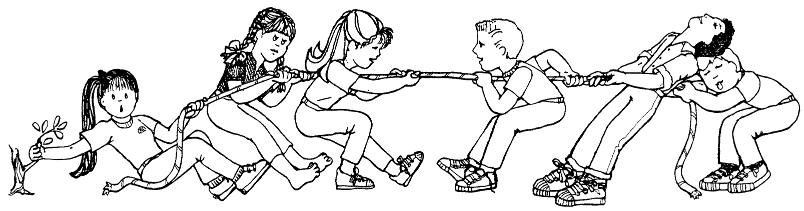


A block of mass m is resting on a long piece of smooth paper. The block has coefficient of static and kinetic friction μ_s, μ_k with the paper, respectively. You jerk the paper horizontally so it ***slides*** out from under the block quickly in the direction indicated by the arrow without sticking. Which of the following statements about the force acting on and acceleration of the ***block*** are true?

a) $F = \mu_s mg$, $a = \mu_s g$, both to the right.
b) $F = \mu_k mg$ to the right, $a = \mu_k g$ to the left.
c) $F = \mu_k mg$, $a = \mu_k g$ both to the left.
d) $F = \mu_k mg$ to the left, $a = \mu_k g$ to the right.
e) $F = \mu_k mg$, $a = \mu_k g$ both to the right.
f) None of the above.

Problem 44. problems-1/force-mc-N3-pick-list-1-icp.tex

The following sentences each describe two specific forces exerted by objects in a physical situation. Circle the letter of the sentences where those two forces form a ***Newton's Third Law*** force pair. ***More than one sentence or no sentences at all*** in the list may describe a Newton's Third Law force pair.



a) In an evenly matched tug of war (where the rope does not move); team one pulls the rope to the left with some force and team two pulls the rope to the right with an equal magnitude force in the opposite direction.

b) Gravity pulls me down; the normal force exerted by a scale I'm standing on pushes me up with an equal magnitude force in the opposite direction.

c) The air surrounding a helium balloon pushes it up with a buoyant force; the balloon pushes the air down with an equal magnitude force in the opposite direction.

Problem 45. problems-1/force-mc-N3-pick-list.tex

Identify the Newton's Third Law pairs from the following list of forces (more than one could be right):

a) The Earth pulls me down with gravity, the normal force exerted by the ground pushes me up.

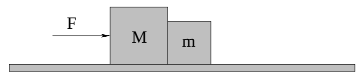
b) I pull hard on a rope in a game of tug-of-war, the rope pulls back hard on me.

c) A table exerts a force upwards on a book sitting on it; the book exerts a force downward on the table.

d) I pull up on a fishing rod trying to land a big fish; the fish pulls down on the fishing rod to get away.

e) My car is stuck in the mud. I push hard on the car to free it, but the car pushes back on me hard enough that I slip and fall on my face in the mire.

Problem 46. problems-1/force-mc-pushing-two-blocks.tex



The figure shows two blocks of mass M and m that are being pushed along a horizontal frictionless surface by a force of magnitude F as shown. What is the magnitude of the force that the block of mass M exerts on the block of mass m?

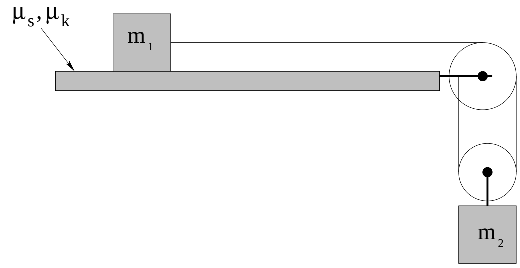
a) F

b) $m\frac{F}{M}$

c) $m\frac{F}{(M+m)}$

d) $M\frac{F}{(M+m)}$

Problem 47. problems-1/force-mc-sliding-block-and-tackle.tex



A block of mass m_1 sits on a rough table. The coefficient of static and kinetic friction between the mass and the table are μ_s and μ_k, respectively. Another mass m_2 is suspended as indicated in the figure above (where the pulleys are massless and the string is massless and unstretchable). What is the maximum mass m_2 for which the blocks remain at rest?

a) $m_2 = 2m_1\mu_k$

b) $m_2 = m_1\mu_k/2$

c) $m_2 = m_1/\mu_s$

d) $m_2 = 2m_1\mu_s$

e) $m_2 = m_1\mu_s/2$

Problem 48. problems-1/force-mc-terminal-velocity-vsq-2bs.tex

Two spherical objects, both with mass m, are falling freely under the influence of gravity through air. The air exerts a drag force on the two spheres in the opposite direction to their motion with *magnitude* $F_1 = b_1 v_1^2$ and $F_2 = b_2 v_2^2$ respectively, with $\boldsymbol{b_2 = 2b_1}$.

Suppose the terminal speed for object 1 is v_t. Then the terminal speed of object 2 is:

a) $2v_t$

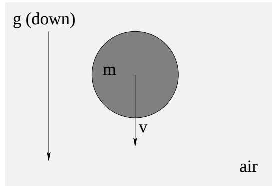
b) $\sqrt{2}v_t$

c) v_t

d) $\frac{\sqrt{2}}{2}v_t$

e) $v_t/2$

Problem 49. problems-1/force-mc-terminal-velocity-vsq.tex



In the figure above, a spherical mass m is falling freely under the influence of gravity through air. The air exerts a drag force on the sphere in the opposite direction to its motion of *magnitude* $F_d = bv^2$ (where the *drag coefficient* b is determined by the shape of the object and its interaction with the air). After a (long) time, the falling mass approaches a constant *terminal speed* v_t, where:

a) $v_t = \frac{F_d}{b}$

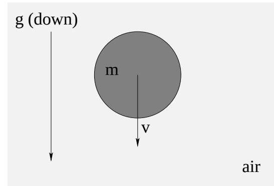
b) $v_t = \frac{mg}{b}$

c) $v_t = \left(\frac{mg}{b}\right)^2$

d) $v_t = \sqrt{\frac{mg}{b}}$

e) $v_t = \left(\frac{F_d}{m}\right) t$

Problem 50. problems-1/force-mc-terminal-velocity-v.tex



In the figure above, a spherical mass m is falling freely under the influence of gravity through air. The air exerts a drag force on the sphere in the opposite direction to its motion of *magnitude* $F_d = bv$ (where the *drag coefficient* b is determined by the shape of the object and its interaction with the air). After a (long) time, the falling mass approaches a constant *terminal speed* v_t, where:

a) $v_t = \frac{F_d}{b}$

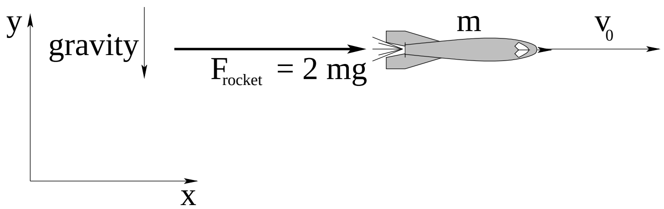
b) $v_t = \frac{mg}{b}$

c) $v_t = \left(\frac{mg}{b}\right)^2$

d) $v_t = \sqrt{\frac{mg}{b}}$

e) $v_t = \left(\frac{F_d}{m}\right) t$

Problem 51. problems-1/force-mc-two-constant-forces.tex



In the figure above, a rocket engine exerts a ***constant*** force $\vec{\boldsymbol{F}} = 2mg\,\hat{\boldsymbol{x}}$ to the right on a ***freely falling mass*** near the surface of the earth. The object is ***initially moving at velocity*** $\boldsymbol{v_0}$ ***to the right*** ($+x$ direction). No drag or frictional forces are present – consider only the two forces of gravity and the rocket engine. The object:

a) Moves in a straight line with an acceleration of magnitude $3g$.

b) Moves in a straight line with an acceleration of magnitude $\sqrt{5}g$.

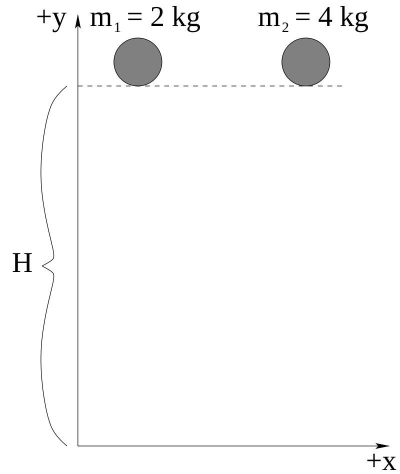
c) Moves in a parabolic trajectory with an acceleration of magnitude $3g$.

d) Moves in a parabolic trajectory with an acceleration of magnitude $\sqrt{5}g$.

e) We cannot determine the trajectory and/or the magnitude of the acceleration from the information given.

Sketch your best guess for the trajectory of the particle in on the figure above as a dashed line with an arrow.

Problem 52. problems-1/force-mc-two-masses-falling-drag-icp.tex



A mass $m_1 = 2$ kg and a mass of $m_2 = 4$ kg have identical size, shape, and surface characteristics, and are both dropped from rest from the same height $H \approx 50$ meters at the same time. ***Air resistance (drag force) is present!*** Place a T/F in each box below as required:

☐ Initially, the acceleration of both masses is the same.

☐ The 2 kg mass hits the ground first.

☐ The 4 kg mass hits the ground first.

☐ Both masses hit the ground at the same time.

☐ Just before they hit, the acceleration of the heavier mass is greater.

Problem 53. problems-1/force-mc-inertial-reference-frames.tex

Newton's Second Law states that $\vec{F}_{\rm tot} = m\vec{a}$ where $\vec{F}_{\rm tot}$ is the total force exerted on a given mass m by *actual forces* of nature or force rules that idealize actual forces of nature (such as Hooke's Law, normal forces, tension in a string), but only if one defines $\vec{a}$ in an **inertial reference frame**.

Select the ***best*** (most complete and accurate) explanation for the inertial reference frame requirement below:

a) It is too difficult to solve for the acceleration of a mass in a non-inertial reference frame.

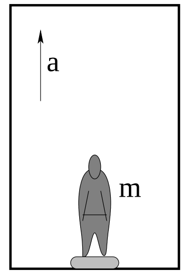
b) In non-inertial reference frames, the sum of the actual forces acting on a mass is no longer equal to the mass times its acceleration in the frame.

c) The Earth's surface is "the" reference inertial reference frame; we use it as the basis for physics in all other frames moving at constant velocity relative to the Earth.

d) Inertial reference frames allow one to use pseudoforces when forces alone are not enough.

e) Because the inertia/mass of an object cannot be measured in a non-inertial reference frame, Newton's Second Law doesn't hold there.

Problem 54. problems-1/force-mc-weight-in-elevator-icp.tex



In the figure above, a person of mass m is standing on a scale in an elevator (near the Earth's surface) that is accelerating upwards with acceleration a. What does the scale read?

☐ mg. ☐ ma. ☐ $m(g + a)$. ☐ $m(g - a)$.

☐ We cannot tell what the scale would show without more information.

Problem 55. problems-1/force-mc-coriolis-dropped-mass-at-equator.tex

A dense mass m is dropped "from rest" from a high tower built at the equator. As the mass falls, it to a person standing on the ground appears to be deflected as it falls to the:

a) East.

b) West.

c) North.

d) South.

e) Cannot tell from the information given.

Problem 56. problems-1/force-mc-coriolis.tex

The Earth is a rotating sphere, and hence is not really an inertial reference frame. Select the true answers from the following list for the ***apparent*** behavior of e.g. naval projectiles or freely falling objects:

a) A naval projectile fired due North in the northern hemisphere will be (apparently) deflected East (spinward).

b) A naval projectile fired due South in the northern hemisphere will be (apparently) deflected East (spinward).

c) A bomb dropped from a helicopter hovering over a fixed point on the surface in the northern hemisphere will be (apparently) deflected West (antispinward).

d) A bomb dropped from a helicopter hovering over a fixed point on the surface in the northern hemisphere will be (apparently) deflected East (spinward).

e) An object placed at (apparent) "rest" on the surface of the Earth in the Northern hemisphere experiences an (apparent) force to the North.

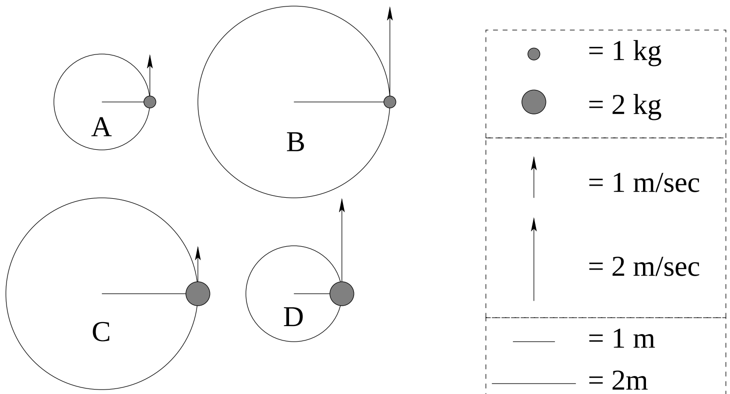
f) An object placed at (apparent) "rest" on the surface of the Earth in the Northern hemisphere experiences an (apparent) force to the South.

g) The true weight of an object measured with a spring balance in a laboratory on the equator is a bit larger than the measured weight.

h) The true weight of an object measured with a spring balance in a laboratory on the equator is a bit smaller than the measured weight.

5.2.2 Ranking/Scaling

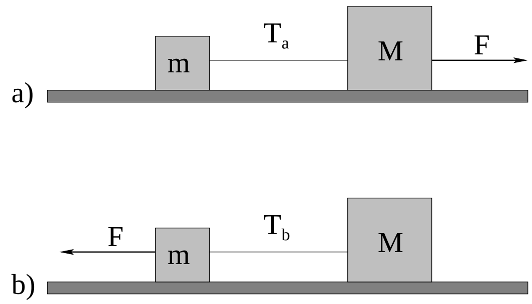
Problem 57. problems-1/force-ra-circular-motion-tension.tex



In the four figures above, you are looking down on a mass sitting on a frictionless table being whirled on the end of a string. The mass, length of string, and speed of the mass in each figure are indicated in the key on the right.

Rank the tension in the string in each of the four figures above, from ***lowest to highest***. Equality is a possibility. An example of a possible answer is thus: $A < B = C < D$.

Problem 58. problems-1/force-ra-tension-between-two-unequal-blocks-friction.tex



In the figure above a block of mass m is connected to a block of mass $M > m$ by a string. Both blocks sit on a smooth surface with a coefficient of kinetic friction μ_k between either block and the surface. In figure **a)**, a force of magnitude F (large enough to cause both blocks to slide) is exerted on block M to pull the system to the **right**. In figure **b)**, a force of (the same) magnitude F is exerted on block m to pull the system to the **left**.

Circle the true statement:

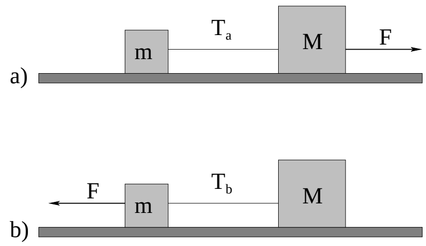
a) The tension $T_a > T_b$.

b) The tension $T_a < T_b$.

c) The tension $T_a = T_b$.

d) There is not enough information to determine the relative tension in the two cases.

Problem 59. problems-1/force-ra-tension-between-two-unequal-blocks-icp.tex



In the figure above a block of mass m is connected to a block of mass $M > m$ by a string. Both blocks sit on a frictionless floor. In a), a force of magnitude F is exerted on block M to pull the system to the right. In b), a force of (the same) magnitude F is exerted on block m to pull the system to the left. Circle the true statement:

a) The tension $T_a > T_b$.

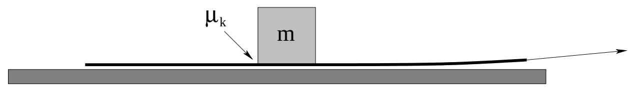
b) The tension $T_a < T_b$.

c) The tension $T_a = T_b$.

d) There is not enough information to determine the relative tension in the two cases.

5.2.3 Short Answer

Problem 60. problems-1/force-sa-block-on-paper.tex

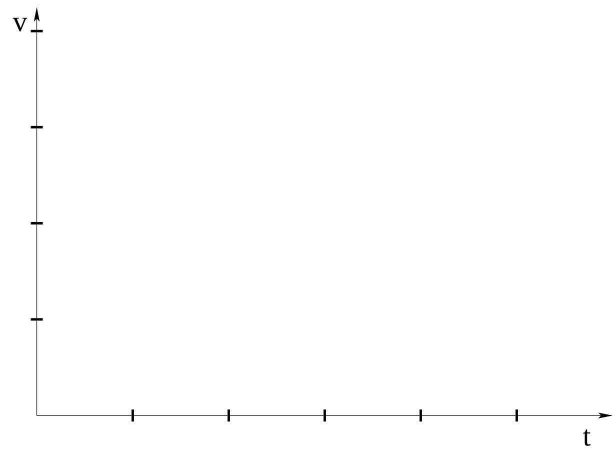


A block of mass m is resting on a long piece of smooth paper. The block has a coefficient of kinetic friction μ_k with the paper. You pull the paper horizontally out from under the block quickly in the direction indicated by the arrow.

a) Draw the ***direction*** of the frictional force acting on the ***block***.

b) What is the ***magnitude and direction*** of the acceleration of the block?

Problem 61. problems-1/force-sa-free-fall-cliff-icp.tex

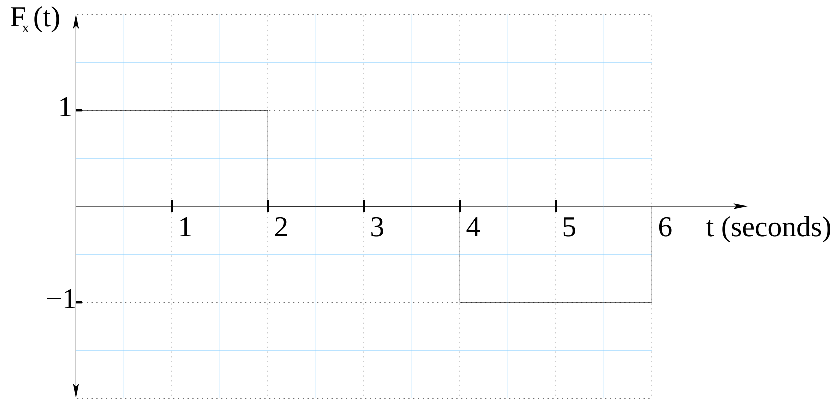


A ball of mass m is dropped from rest over the edge of a very tall (kilometer high) cliff. It experiences a drag force opposite to its velocity of $F_d = -bv^2$.

a) On the axes above, *qualitatively* plot its downward ***speed*** as function of time.

b) What is its approximate speed when it hits after falling a long time/distance?

Problem 62. problems-1/force-sa-kinematic-graphs-1-icp.tex



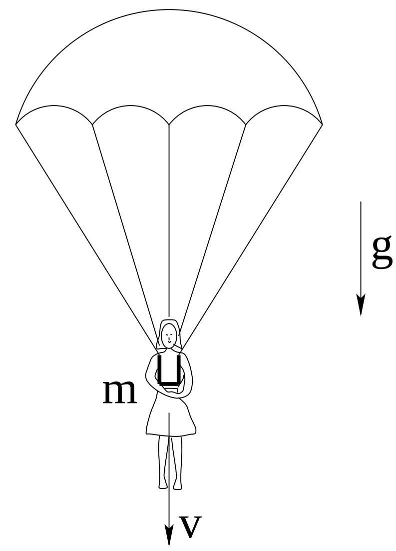
The graph above represents the force in the positive x direction $F_x(t)$ applied to a mass $m = 1$ kg as a function of time in seconds. The mass begins at rest at $x = 0$. The force F is given in Newtons, the position x is given in meters.

a) What is the acceleration of the mass during the time interval from $t = 0$ to $t = 6$ seconds (sketch a curve)?

b) How fast is the mass going at the end of 6 seconds?

c) How far has the mass travelled at the end of 6 seconds?

Problem 63. problems-1/force-sa-parachutist.tex



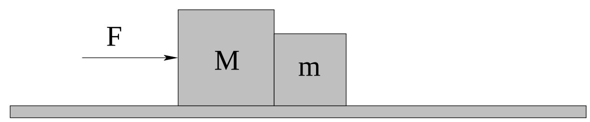
A skydiver of mass m jumps from a helicopter and immediately opens her parachute (so that her initial downward speed with the parachute open is basically zero.) The parachute exerts a ***quadratic*** drag force (proportional to v^2) with a drag coefficient b.

a) Draw a free body diagram showing the forces acting on the skydiver a short time later when her downward speed is v. Write down an expression for the *magnitude* of her acceleration at this speed.

b) Find her terminal (asymptotic) speed as she falls over a very long distance.

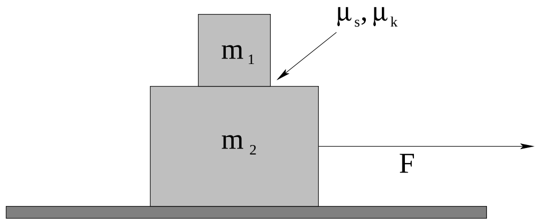
c) If her terminal speed needs to be reduced by a factor of 2 for her to land safely, by what factor must b be increased?

Problem 64. problems-1/force-sa-pushing-two-blocks.tex



The figure shows two blocks of mass M and m that are being pushed along a horizontal frictionless surface by a force of magnitude F as shown. What is the magnitude of the (contact/normal) force that the block of mass M exerts on the block of mass m?

Problem 65. problems-1/force-sa-two-blocks-friction.tex



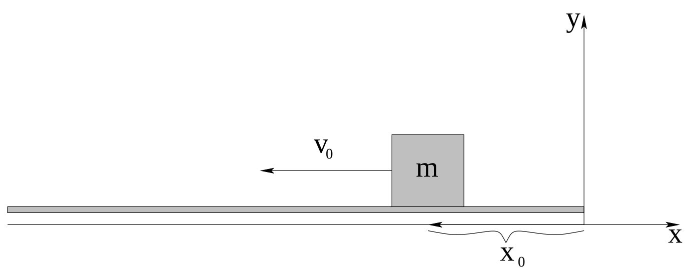
A block of mass m_1 is placed on a larger block of mass $m_2 > m_1$, where there is a coefficient of static friction $\mu_s = 0.25$ and a coefficient of kinetic friction $\mu_k = 0.2$ for the surface in contact between the blocks. Both blocks are on a frictionless table. A force of magnitude $F = 3(m_1 + m_2)g$ is applied to the ***bottom*** block only.

a) Is the ***magnitude*** of the acceleration of the lower block greater than, less than, or equal to the magnitude of the accleration of the upper block?

b) Find the acceleration of the ***top block only*** (magnitude and direction).

5.2.4 Long Problems

Problem 66. problems-1/force-pr-a-constant-simplest-icp.tex



A block of mass m sits on a horizontal frictionless table as shown. A constant force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{x}}$ in the $+x$-direction (to the right) is applied to it. The mass is initially moving to the left with speed v_0, and starts a the position x_0 as shown.

a) Draw a ***force diagram*** for the mass m onto the figure above. This should include ***all*** the forces, including those that cancel.

b) Write down an expression for the acceleration $\vec{\boldsymbol{a}}$ of the mass.

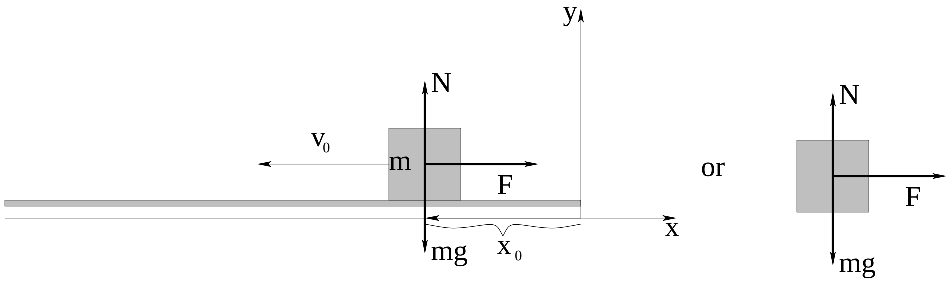
c) ***Integrate*** the acceleration one time to find $\vec{\boldsymbol{v}}(t)$.

d) ***Integrate*** the velocity one time to find $\vec{\boldsymbol{x}}(t)$.

e) How long will it take to bring the particle to rest (where infinity is a possible answer)?

f) Where will it be when it comes to rest?

Problem 67. problems-1/force-pr-a-constant-simplest-soln.tex



A block of mass m sits on a horizontal frictionless table as shown. A constant force $\vec{F} = F\hat{x}$ in the $+x$-direction (to the right) is applied to it. The mass is initially moving to the left with speed v_0, and starts a the position x_0 as shown.

a) Draw a ***force diagram*** for the mass m onto the figure above. This should include ***all*** the forces, including those that cancel.

b) Write down an expression for the acceleration $\vec{a}$ of the mass.

c) ***Integrate*** the acceleration one time to find $\vec{v}(t)$.

d) ***Integrate*** the velocity one time to find $\vec{x}(t)$.

e) How long will it take to bring the particle to rest (where infinity is a possible answer)?

f) Where will it be when it comes to rest?

For a), see above.

For b) in the y-direction:

$$\sum_y F_y = N - mg = ma_y = m0 = 0$$

$$a_y = 0$$

and nothing interesting happens in the y-direction. The block doesn't jump into the air or fall through the solid table! In the x-direction:

$$\sum_x F_x = F = ma_x$$

$$a_x = F/m$$

Acceleration is a ***vector*** so we must specify its ***magnitude and direction*** in some way or some coordinate frame. Any of the following are acceptable ways:

$$\vec{a} = F/m\hat{x} = (F/m, 0) = (|\vec{a}| = F/m, \theta_{\vec{a}} = 0)(\text{polar}) = F/m \text{ ``to the right''}$$

or just $a_x = F/m, a_y = 0$ as obtained above.

For part c), $a_y = 0 = dv_y/dt$ so $v_y =$ a constant. But its initial y-velocity is zero, so $v_y = 0 = dy/dt$. Thus y is a constant. But the initial value for y is 0 in a reasonable coordinate system and in any event it does not change, so we'll choose coordinates where boring old $y = 0$ throughout.

x is more interesting:

$$a_x = \frac{F}{m} = \frac{dv_x}{dt}$$
$$dv_x = \left(\frac{F}{m}\right) dt$$
$$v_x = \int dv_y = \left(\frac{F}{m}\right) \int dt = \left(\frac{F}{m}\right) t + v_{0x}$$

hence (using the ***given*** fact that $v_x(0) = -v_0$):

$$v_x = \left(\frac{F}{m}\right) t - v_0 = \frac{dx}{dt}$$

For part d) we now repeat this process for dx:

$$x = \int dx = \int \left\{ \left(\frac{F}{m}\right) t - v_0 \right\} dt = \frac{1}{2}\left(\frac{F}{m}\right) t^2 - v_0 t - x_0$$

where we used the ***given*** fact that $x(0) = -x_0$ in the coordinate system shown.

Again, we could express both of these ***vector*** answers in any acceptable way. I'll use cartesian coordinates:

$$\vec{\boldsymbol{v}}(t) = \left\{ \left(\frac{F}{m}\right) t - v_0] \right\} \hat{\boldsymbol{x}}$$
$$x(t) = \left\{ \frac{1}{2}\left(\frac{F}{m}\right) t^2 - v_0 t - x_0 \right\} \hat{\boldsymbol{x}}$$

For part e) we try to solve:

$$v_x(t_s) = \left(\frac{F}{m}\right) t_s - v_0 = 0$$

for the particular time t_s that the block's velocity will equal zero. We get:

$$t_s = \frac{mv_0}{F}$$

At that time its y-coordinate will remain zero (of course) but its x-coordinate will be:

$$\begin{aligned} x(t_s) &= \frac{1}{2}\left(\frac{F}{m}\right) t_s^2 - v_0 t_s - x_0 \\ &= \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{mv_0}{F}\right)^2 - v_0\left(\frac{mv_0}{F}\right) - x_0 \\ &= -\frac{1}{2}\left(\frac{mv_0^2}{F}\right) - x_0 \end{aligned}$$

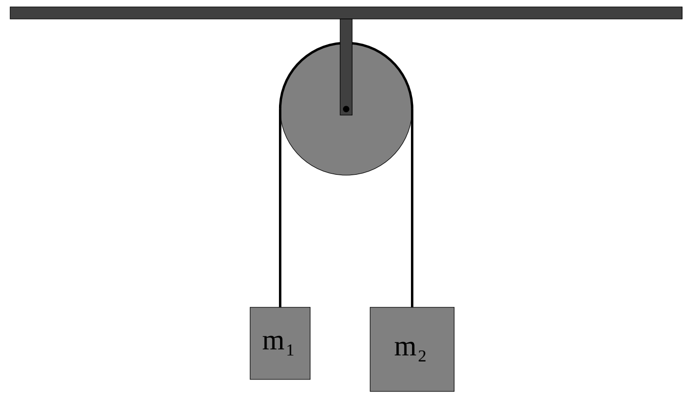
Note well! This last result can be made familiar to us by noting that the acceleration is constant, so that $v_f^2 - v_0^2 = 2a_x\Delta x$, with $a_x = F/m$. Hence:

$$x_f - (-x_0) = \Delta x = -v_0^2/(2F/m) = -\frac{1}{2}\left(\frac{mv_0^2}{F}\right)$$

and $x_f = x(t_s)$ as expected. There is yet another way to do it using work and energy.

This example solution has been worked in *more* detail than would usually be required on a problem, but I would still recommend that you start out working homework and additional examples from this guide at exactly this level. After a bit some of the steps will be so obvious and easy and boring (like the discussion of the *nothing interesting* that happens in y above) that you can safely omit them or cover them with a phrase like "$a_y = 0$ so y is unchanged", but wait for that to happen and pay attention to the stated requirements of your class's particular grader(s).

Problem 68. problems-1/force-pr-atwoods-machine-icp.tex



In the figure above Atwood's machine is drawn – two masses m_1 and m_2 hanging over a massless frictionless pulley, connected by a massless unstretchable string.

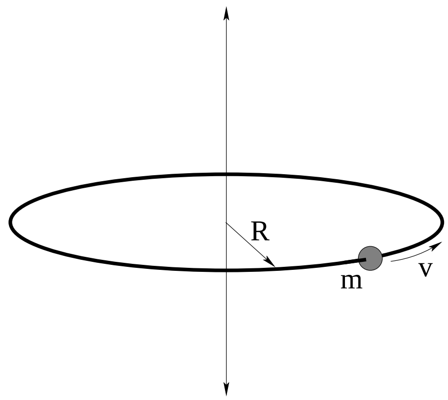
a) Draw free body diagrams (isolated diagrams for each object showing just the forces acting on that object) for the two masses in the figure above.

b) Convert each free body diagram into a statement of Newton's Second Law for that object.

c) Find the acceleration of the system and the tensions in the string on *both* sides of the pulley in terms of m_1, m_2, and g.

d) Suppose mass $m_2 > m_1$ and the system is released from rest with the masses at equal heights. When mass m_2 has descended a distance H, find the speed of the masses.

Problem 69. problems-1/force-pr-bead-on-hoop.tex



A bead of mass m is threaded on a metal hoop of radius R. There is a coefficient of kinetic friction μ_k between the bead and the hoop. It is given a push to start it sliding around the hoop with initial speed v_0. The hoop is located on the space station, so you can ignore gravity.

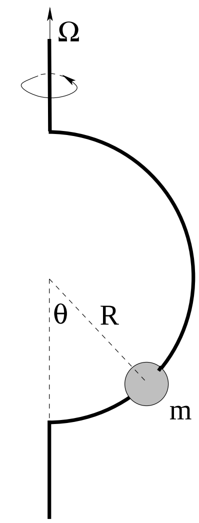
a) Find the normal force exerted by the hoop on the bead as a function of its speed.

b) Find the dynamical frictional force exerted by the hoop on the bead as a function of its speed.

c) Find its speed as a function of time. This involves using the frictional force on the bead in Newton's second law, finding its *tangential* acceleration on the hoop (which is the time rate of change of its speed) and solving the equation of motion.

All answers should be given in terms of m, μ_k, R, v (where requested) and v_0.

Problem 70. problems-1/force-pr-bead-on-semicircular-hoop.tex

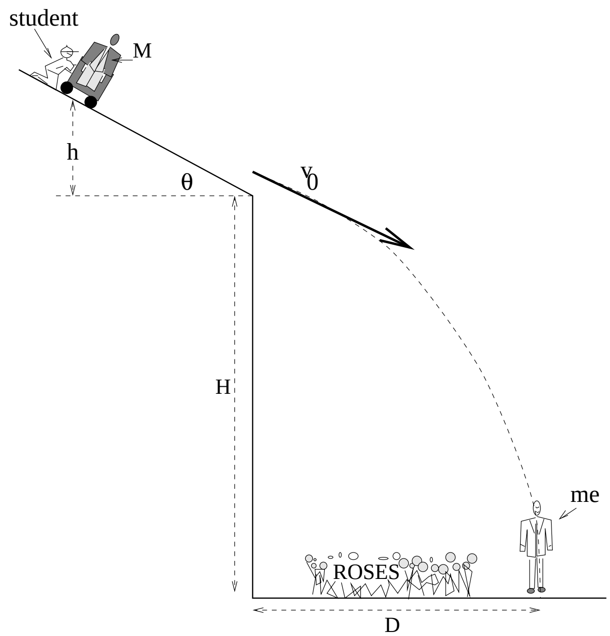


A small frictionless bead is threaded on a semicircular wire hoop with radius R, which is then spun on its vertical axis as shown above at angular velocity Ω.

a) Find the angle θ where the bead will remain stationary relative to the rotating wire as a function of R, g, and Ω.

b) From your answer to the previous part, it should be apparent that there is a ***minimum*** angular velocity $\Omega_{\rm min}$ that the hoop must have before the bead moves up from the bottom at all. What is it? (Hint: Think about where the previous answer has solutions.)

Problem 71. problems-1/force-pr-dropping-washington-duke-trajectory.tex



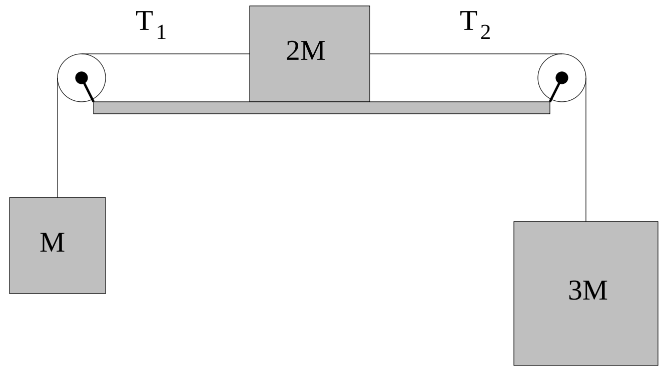
A physics student irritated by the personal mannerisms of their physics professor decides to rid the world of him. The student plans to drop a large, massive object (the statue of Washington Duke, actually, recently stolen by pranksters from his fraternity), mounted on nearly frictionless casters, from a tall building of height H with a smooth roof sloped at the angle θ as shown. However, the student (being a thoughtful sociopath) wants to make sure that the mass M will make it *over the roses* to the path a distance D from the base of the building and needs to know how far to let the statue roll down the roof to get the right speed.

Unfortunately, the student isn't very good at physics and comes to *you* for help. Since they don't want to tell you *which* building or *which* path they want to use (you might be able to testify against them!) they want you to find (in **two steps**, each counting as a separate problem) a *general formula* for the requisite distance.

a) Help them out. Start by finding v_0 in terms of H, M, D, θ and g (the gravitational constant) that will drop M on RGB assuming *no friction or drag forces.* (That way I'm still pretty safe).

b) Now that you know the speed (or rather, assuming that you know the speed, as the case may be) find h (the vertical distance the statue must roll down, released from rest, to come off with the right speed). Explicitly show that your overall answer (in which v_0 should NOT appear) has the right units. If you were clueless in problem 4) you may leave v_0 in your answer but should still try to find SOME combination of the letters H, M, D, θ and g that has the right units and varies the way you expect the answer to (more height H means *smaller* h, for example, so it probably belongs on the bottom).

Problem 72. problems-1/force-pr-flat-plane-three-blocks.tex



Three blocks of mass M, $2M$ and $3M$ are drawn above. The middle block ($2M$) sits on a frictionless table. The other two blocks are connected to it by massless unstretchable strings that run over massless frictionless pulleys. At time $t = 0$ the system is released from rest. Find:

a) The acceleration of the middle block sitting on the table.

b) The tensions T_1 and T_2 in the strings as indicated.

Problem 73. problems-1/force-pr-flat-plane-two-blocks.tex

A mass m_1 is attached to a second mass m_2 by an Acme (massless, unstretchable) string. m_1 sits on a frictionless table; m_2 is hanging over the ends of a table, suspended by the taut string from an Acme (frictionless, massless) pulley. At time $t = 0$ both masses are released.

a) Draw the force/free body diagram for this problem.

b) Find the acceleration of the two masses.

c) How fast are the two blocks moving when mass m_2 has fallen a height H (assuming that m_1 hasn't yet hit the pulley)?

Problem 74. problems-1/force-pr-flat-plane-two-blocks-friction.tex

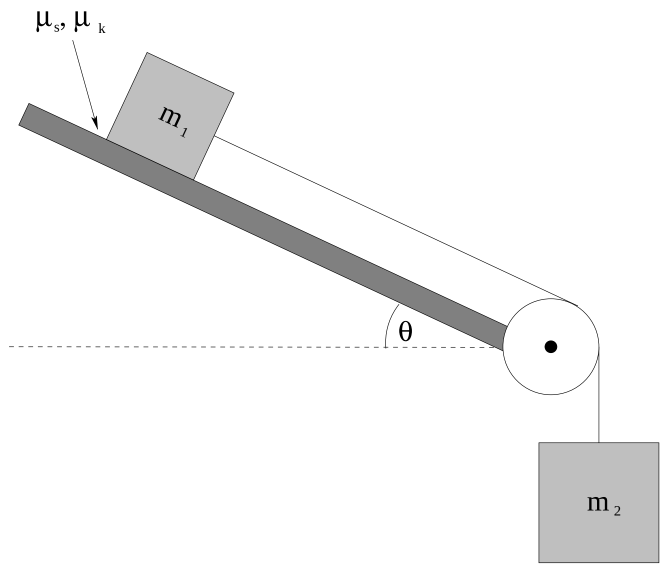
A mass m_1 is attached to a second mass m_2 by a massless, unstretchable string. m_1 sits on a rough table with coefficients of static and kinetic friction μ_s and μ_k respectively. m_2 is hanging over the end of the table, suspended by the taut string from a frictionless, massless pulley.

At time $t = 0$ both masses are released from rest. Answer the following two questions:

a) What is the minimum mass $m_{2,\mathrm{min}}$ such that the two masses begin to move?

b) Suppose $m_2 > m_{2,\mathrm{min}}$. Determine how fast the two blocks are moving when mass m_2 has fallen a height H (assuming that m_1 hasn't yet hit the pulley)?

Problem 75. problems-1/force-pr-inclined-atwoods-machine-downhill-friction-icp.tex



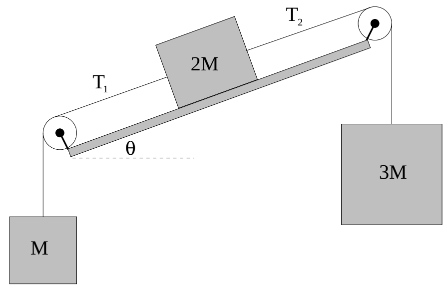
A block of mass m_1 sits on a plane inclined at the angle θ as shown. It is connected with a massless, unstretchable string running over a massless, frictionless pulley to m_2, which is hanging over a drop to the ground. The two masses are released initially from rest. The inclined plane has coefficients of static and kinetic friction with m_1 of μ_s and μ_k respectively.

a) Draw separate free-body diagrams for each mass m_1 and m_2, and select (indicate on your figure) an appropriate coordinate system for each diagram;

b) Find the minimum mass $m_{2,\mathrm{min}}$ such that the two masses begin to move;

c) If $m_2 > m_{2,\mathrm{min}}$ (so that the block definitely slides), determine the magnitude of the acceleration of the blocks.

Problem 76. problems-1/force-pr-inclined-plane-three-blocks.tex



Three blocks of mass M, $2M$ and $3M$ are drawn above. The middle block $(2M)$ sits on a frictionless table tipped at an angle θ with the horizontal as shown. The other two blocks are connected to it by massless unstretchable strings that run over massless frictionless pulleys. At time $t = 0$ the system is released from rest. Find:

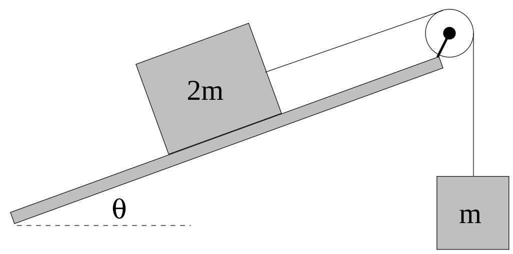
a) The magnitude of the acceleration of the middle block sitting on the table.

b) The tensions T_1 and T_2 in the strings as indicated.

c) Suppose $\theta = 30^\circ$. Which way will the system of blocks accelerate?

☐ Down on the right ☐ Down on the left ☐ They won't move.

Problem 77. problems-1/force-pr-inclined-plane-two-blocks-30deg-icp.tex



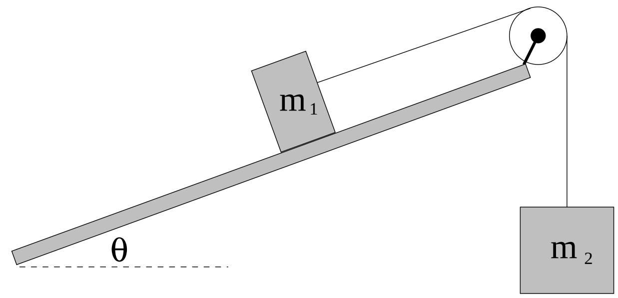
A block of mass $2m$ sits on a frictionless incline held at an angle θ relative to the horizontal as shown in the figure above. It is connected by a massless, unstretchable string that runs over a frictionless, massless pulley to a block m hanging over a drop. The two blocks are initially ***at rest***.

a) For what angle θ_0 will this system will be in ***force balance*** (and hence remain stationary).

b) If the incline is lifted from this angle to a new (given) angle $\theta > \theta_0$, what is the subsequent direction of motion for both blocks? Indicate the direction on the figure above for each block.

c) At this angle θ, find the magnitude of the acceleration a and the magnitude of the tension T in the string.

Problem 78. problems-1/force-pr-inclined-plane-two-blocks.tex



Two blocks of mass m_1 and $m_2 > m_1$ are drawn above. The block m_1 sits on a frictionless inclined plane tipped at an angle θ with the horizontal as shown. Block m_2 is connected to m_1 by a massless unstretchable string that runs over a massless, frictionless pulley to hang over a considerable drop. At time $t = 0$ the system is ***released from rest***.

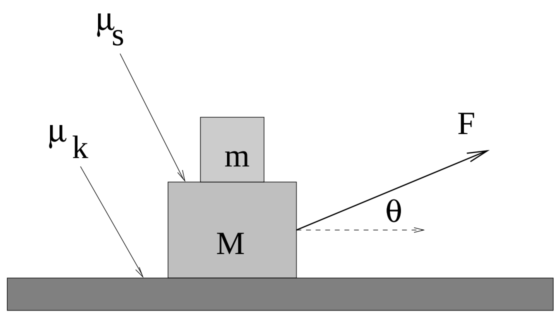
a) Draw a force/free body diagram for the two masses.

b) Find the magnitude of the acceleration of two masses.

c) Find the tension T in the string.

d) When mass m_2 has fallen a height H, how fast are the two masses moving?

Problem 79. problems-1/force-pr-pulling-rough-blocks-on-rough-table.tex



A rope at an angle θ with the horizontal is pulled with a force vF. It pulls, in turn, two blocks, the bottom with mass M and the top with mass m. The coefficients of friction are μ_s between the top and bottom block (assume that they **do not slide for the given force $\vec{F}$**) and μ_k between the bottom block and the table. Remember to show (and possibly evaluate) *all* forces acting on *both* blocks, including internal forces between the blocks.

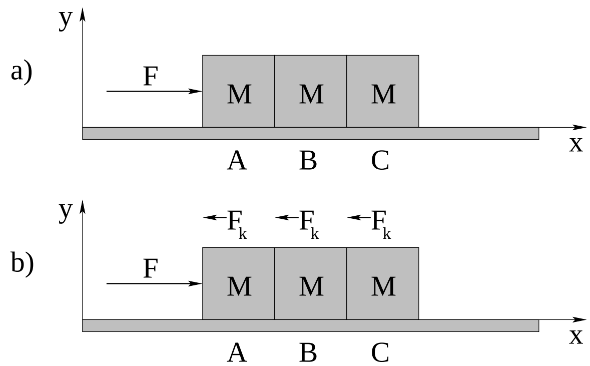
a) Draw a "free body diagram" for **each** mass shown, that is, draw in and label all *real* forces acting on it;

b) Apply **Newton's Second Law** in appropriate coordinates to **each** mass shown;

c) Solve for the acceleration(s) of **each** mass shown and evaluate **all** unknown forces (such as a normal force or the tension in a string) in terms of the given quantities.

Don't forget that the acceleration is a **vector** and must be given as a magnitude and a direction (for example, "along the plane to the right" is ok) or in vector components.

Problem 80. problems-1/force-pr-pushing-three-blocks.tex



Blocks A, B, and C each have mass M and are sitting on a smooth horizontal surface. A horizontal force with magnitude F is applied to block A on the left in the x-direction as shown.

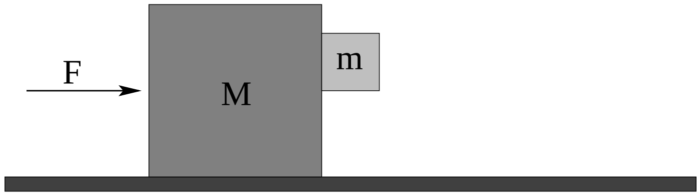
a) Initially, assume that the horizontal surface is frictionless. Determine:

- The acceleration of the system of blocks.
- The normal contact force N_{AB} between block A and block B.
- The normal contact force N_{BC} between block B and block C.

b) Now, assume that in addition to the force F the horizontal surface exerts a kinetic frictional force with magnitude $F_k \ll F$ in the negative x-direction on ***each*** block. Determine:

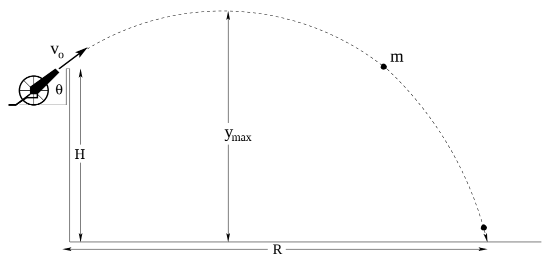
- The acceleration of the system of blocks.
- The normal contact force N_{AB} between block A and block B.
- The normal contact force N_{BC} between block B and block C.
- Evaluate your answers (for this part only) if $F = 100$ N, $M = 10$ kg, and $F_k = 10$ N.

Problem 81. problems-1/force-pr-pushing-vertical-blocks-friction-icp.tex



A force $\vec{\boldsymbol{F}}$ is applied to a large block with mass M, which pushes a smaller block of mass m as shown. The large block is supported by a frictionless table. The coefficient of static friction between the large block and the small block is μ_s. Find the **magnitude** of the minimum force F_{min} such that the small block does not slide down the face of the large one. Draw free body diagrams and show all of your reasoning.

Problem 82. problems-1/force-pr-range-of-cannon-on-hill.tex



A cannon sits on at the top of a rampart of height (to the mouth of the cannon) H. It fires a cannonball of mass m at speed v_0 at an angle θ relative to the ground. Find:

a) The maximum height $y_{\rm max}$ of the cannonball's trajectory.

b) The time the cannonball is in the air.

c) The range of the cannonball.

Problem 83. problems-1/force-pr-terminal-velocity-tom-and-jerry.tex

The script calls for Tom (cat) to chase Jerry (mouse) across the top of a cartoon skyscraper of height H and off the edge where they both fall straight down (their initial x-velocity is negligible as they fall off) towards a soft pile of dirt that will keep either one from getting hurt by the fall no matter how hard they land (no toon animals were injured in this problem).

Your job is to work out the physics of a "realistic" fall for the animation team. You decide to use the following for the drag force acting on either one:

$$\vec{F}_i = -b_i v^2 \hat{\boldsymbol{v}}$$

where hv is a unit vector in the direction of the velocity and $i = \mathrm{t}, \mathrm{j}$ for Tom or Jerry respectively and where:

$$b_i = C L_i^2$$
$$m_i = D L_i^3$$

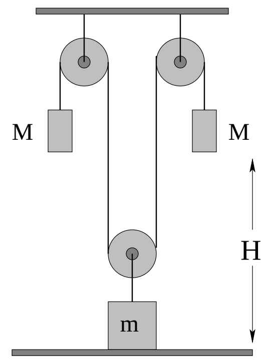
(that is, the drag force is proportional to their cross-sectional area and their mass is proportional to their volume). Their relative size is $L_t = 5 L_j$ (Tom is five times the height of Jerry).

a) Draw a on the back of the preceding page showing the building, Tom and Jerry at the instant that Tom runs off of the top. Jerry (who is ahead) should have fallen a short distance d towards the ground.

b) Using the laws of physics, determine the equation of motion (find an expression for the acceleration and write it as a differential equation) algebraically (so the solution applies to Tom or Jerry equally well). Your answer can be given in terms of b_i and m_i.

c) *Without* solving the equation of motion, find an algebraic expression for the terminal velocity of Tom or Jerry as functions of L_i. Explain/show your *reasoning*, don't just write down an answer.

Problem 84. problems-1/force-pr-triple-atwoods-machine.tex



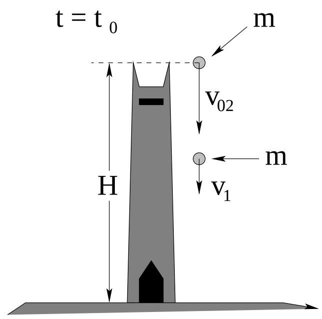
A block and tackle arrangement is built with three massless pulleys and three hanging masses with masses M, m, and M as shown above. The two M masses are a height H off the ground, and m is on the ground. At time $t = 0$ the masses are released from rest from this configuration.

a) Draw a GOOD free body diagram. Clearly label all quantities.

b) Find the acceleration (magnitude and direction) of each block and the tension T in the string as a function of the givens, assuming that $M + M > m$.

c) Find the velocity of each block when the blocks of mass M hit the ground.

Problem 85. problems-1/force-pr-two-balls-1D-2.tex



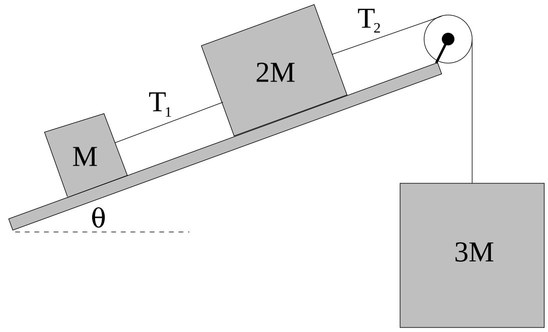
A ball of mass m is dropped at time $t = 0$ from rest ($v_{01} = 0$) from the top of the Duke Chapel (which has height H) to fall freely under the influence of *gravity*. A short time $t = t_0$ later a second ball, also of mass m, is *thrown* down after it at speed v_{02}. Neglect drag.

a) (2 points) Draw a free body diagram for and compute the net force acting on each mass **separately**.

b) (4 points) **From the equation of motion for each mass**, determine their one dimensional trajectory functions, $y_1(t)$ and $y_2(t)$.

c) (3 points) Sketch *qualitatively correct* graphs of $y_1(t)$ and $y_2(t)$ on the same axes in the case where the two collide.

Problem 86. problems-1/force-pr-two-blocks-on-inclined-plane-plus-pulley.tex

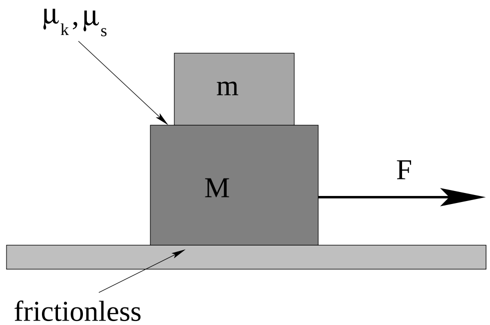


Three blocks of mass M, $2M$ and $3M$ are drawn above. The two smaller blocks sit on a frictionless table tipped at an angle θ with respect to the horizontal as shown. The three blocks are connected by massless unstretchable strings, one of which runs over a massless frictionless pulley to the largest mass. At time $t = 0$ the system is released from rest. Find:

a) The acceleration *vector* in *Cartesian components* of the middle block on the incline. Any correct way of uniquely specifying the Cartesian vector will be accepted, for example $\vec{\boldsymbol{a}} = (a_x, a_y)$, or $\vec{\boldsymbol{a}} = a_x\hat{\boldsymbol{x}} + a_y\hat{\boldsymbol{y}}$.

b) The magnitude of the tensions T_1 and T_2 in the strings as indicated on the diagram.

Problem 87. problems-1/force-pr-two-blocks-with-friction-icp.tex

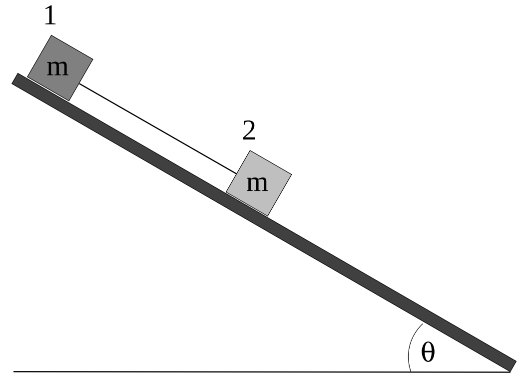


A small block of mass m sits on top of a large block of mass M that sits on a frictionless table. The coefficient of static friction between the two blocks is μ_s and the coefficient of kinetic friction between the two blocks is μ_k. A force $\vec{F} = F\hat{x}$ is directly applied to the *lower* block as shown. All answers should be given in terms of m, M, μ_s, μ_k, and g.

a) What is the largest force F_{max} that can be applied such that the upper block does not slide on the lower block?

b) Suppose that $F = 2F_{\mathrm{max}}$ (so that the upper block slips freely). What is the acceleration of each block?

Problem 88. problems-1/force-pr-two-masses-incline-different-friction.tex



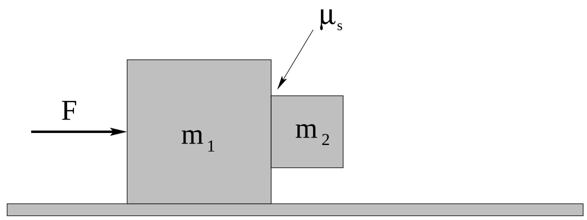
Two blocks, each with the same mass m but ***made of different materials***, sit on a rough plane inclined at an angle θ that is large enough that they will ***definitely slide down***. The first (upper) block has a coefficient of *kinetic* friction of μ_{k1} between block and inclined plane; the second (lower) block has coefficient of kinetic friction μ_{k2}. The two blocks are connected by a massless unstretchable ***string***.

Find the acceleration of the two blocks a_1 and a_2 down the incline:

a) when $\mu_{k2} > \mu_{k1}$.

b) when $\mu_{k1} > \mu_{k2}$;

Problem 89. problems-1/force-pr-vertical-block-friction.tex



A block of mass m_1 is pushed on a frictionless table by a force $\vec{F}$ to the right. A mass m_2 is positioned on the front face as shown. There is a coefficient of static friction μ_s between the big and little block.

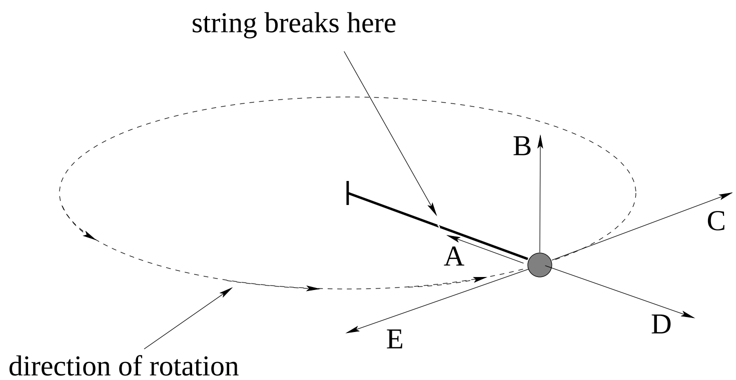
a) What is the horizontal force exerted on block m_2 by the block m_1?

b) Find the ***minimum*** magnitude of force F_{min} that will keep the little block from slipping down the big one.

5.3 Circular Motion

5.3.1 Multiple Choice

Problem 90. problems-1/circular-motion-mc-ball-on-string-breaks.tex



A ball is being whirled on a string. At the instant shown, the string *breaks*. Select the correct trajectory of the ball after it breaks.

a) A

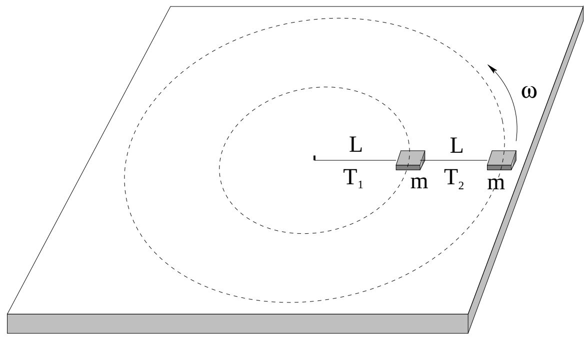
b) B

c) C

d) D

e) E

Problem 91. problems-1/circular-motion-mc-two-masses-on-strings-qual.tex



A block of mass m is tied to a cord of length L that is pivoted at the center of a frictionless table. A second block of mass m is tied to the first block also on a cord of length L, and both are set in motion so that they rotate together at angular speed ω as shown above. The tensions T_1 and T_2 in the cords are:

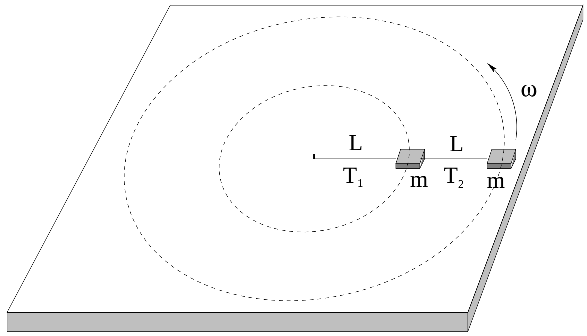
a) $T_1 = T_2$

b) $T_1 > T_2$

c) $T_1 < T_2$

d) $T_1 > T_2$ for $\omega > 0$ and $T_1 < T_2$ for $\omega < 0$

Problem 92. problems-1/circular-motion-mc-two-masses-on-strings-icp.tex



A block of mass m is tied to a cord of length L that is pivoted at the center of a frictionless table. A second block of mass m is tied to the first block also on a cord of length L, and both are set in motion so that they rotate together at angular speed ω as shown above. The tensions T_1 and T_2 in the cords are:

a) $T_1 = 3m\omega^2 L$, $T_2 = 2m\omega^2 L$

b) $T_1 = m\omega^2 L$, $T_2 = 2m\omega^2 L$

c) $T_1 = m\omega^2 L$, $T_2 = m\omega^2 L$

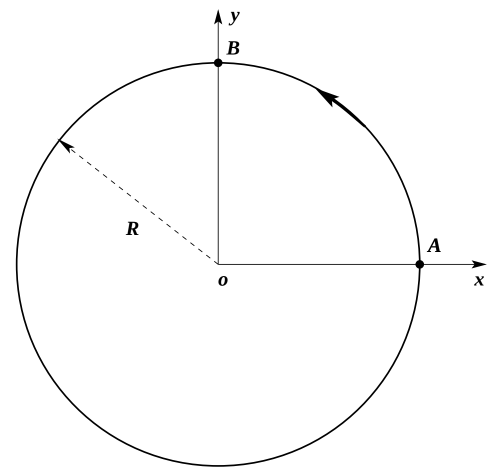
d) $T_1 = 2m\omega^2 L$, $T_2 = 2m\omega^2 L$

e) $T_1 = m\omega^2 L$, $T_2 = 4m\omega^2 L$

5.3.2 Short Answer

Problem 93. problems-1/circular-motion-sa-runner-on-track-icp.tex

Starting from point A, a runner runs at a ***constant speed*** counter-clockwise along a circular race track of radius $R = 20$ m.



a) When the runner has reaches point B, draw and label the runner's velocity $\vec{v}_B$ and acceleration $\vec{a}_B$;

b) If the speed of the runner is 4 m/s, find the angular speed and the magnitude of the acceleration.

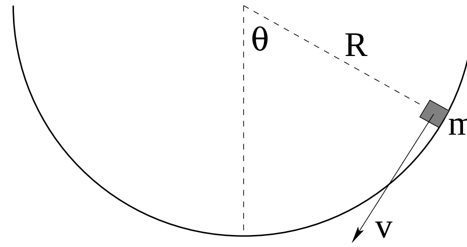
$\omega = \boxed{}$ and $a = \boxed{}$.

c) If the runner runs on the same circular track, but he finished in one-half of the original time, the new angular speed will be ω_2, and the new magnitude of the acceleration will be a_2. Find the following ratios:

$\dfrac{\omega_2}{\omega} = \boxed{}$ and $\dfrac{a_2}{a} = \boxed{}$.

Problem 94. problems-1/circular-motion-sa-sliding-down-curve.tex

In the figure on the right, a small block slides down a frictionless curved track of circular radius R. When it reaches the angle θ as shown, it has speed v (in a later chapter, we'll learn how to find v from initial conditions).



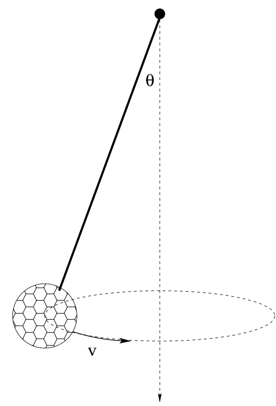
a) Draw a free body diagram for the mass. You may draw it directly on the figure if you wish.

b) Select and draw the best coordinate system to use to analyze its motion. This is tricky!

c) Find the normal force exerted by the track in terms of v and θ.

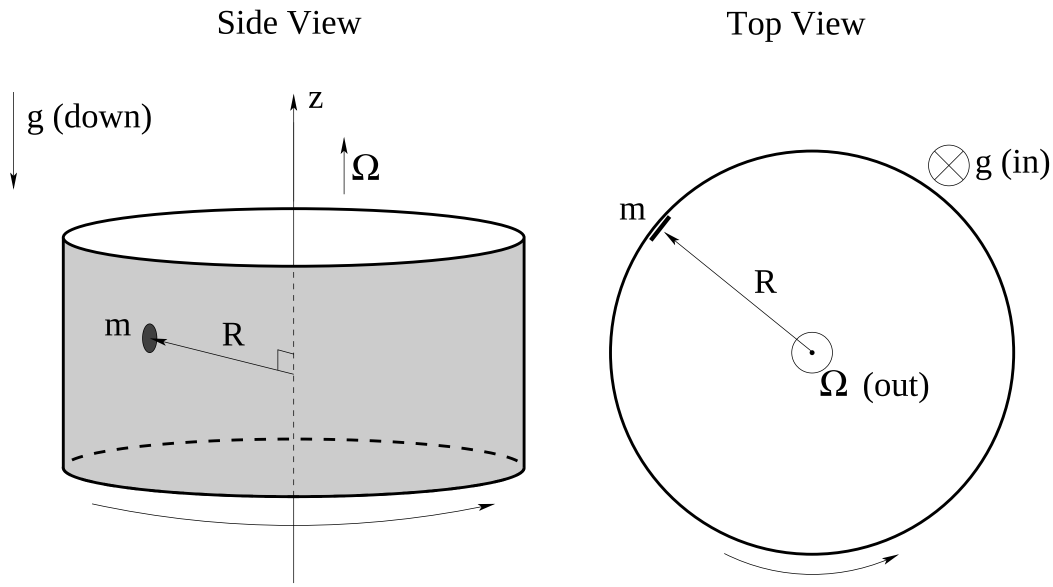
5.3.3 Long Problems

Problem 95. problems-1/circular-motion-pr-conic-pendulum-tether-ball.tex



A tether ball of mass m is suspended by a rope of length L from the top of a pole. A youngster gives it a whack so that it moves in a circle of radius $r = L\sin(\theta) < L$ around the pole. Find an expression for the speed v of the ball as a function of θ.

Problem 96. problems-1/circular-motion-pr-puck-on-cylinder-friction-review.tex



A hockey puck with mass m is placed against the wall of a hollow cylinder of radius r that is ***rotating at a constant angular speed*** Ω around the z-axis as shown in ***side and top views*** above. The coefficient of static fraction between the puck and the wall of the cylinder is μ_s. Gravity points in the negative $-z$ direction: down in the left hand figure and into the page in the right hand figure.

a) On the diagrams above draw in ***all real forces that act on the mass*** while the cylinder rotates. For forces acting vertically, **use the side view**. For forces acting in the horizontal directions, **use the top view**.

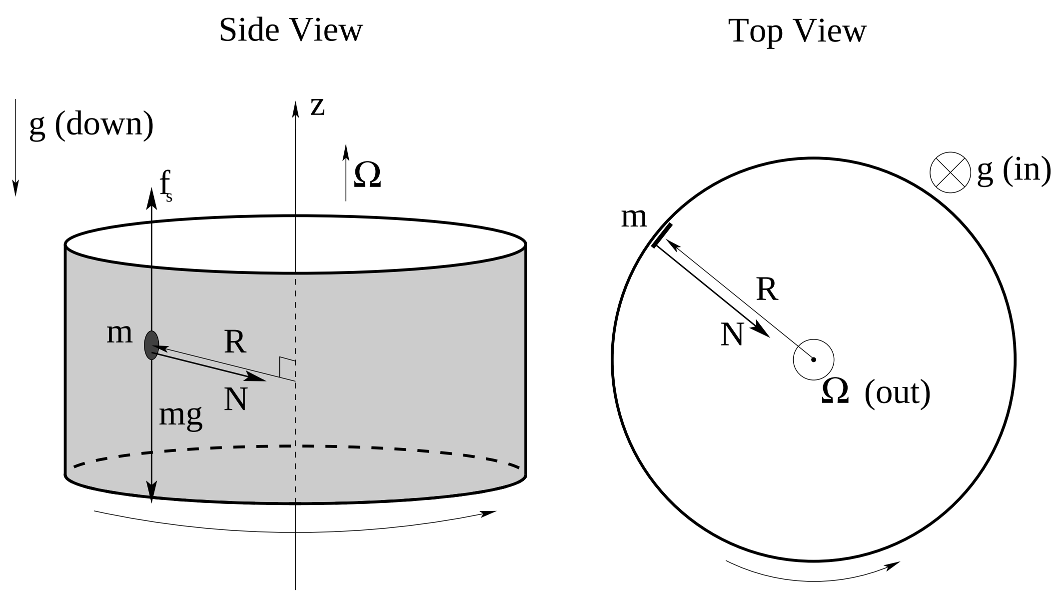
b) If the cylinder is rotating fast enough, the puck does not slide down the wall. What is the value of f_s, the magnitude of static friction, in this case?

$$f_s = \boxed{}$$

c) What is the ***minimum angular speed*** $\mathbf{\Omega_{min}}$ such that the hockey puck does not slide down the wall?

$$\Omega_{\mathrm{min}} = \boxed{}$$

Problem 97. problems-1/circular-motion-pr-puck-on-cylinder-friction-review-soln.tex



A hockey puck with mass m is placed against the wall of a hollow cylinder of radius r that is ***rotating at a constant angular speed*** Ω around the z-axis as shown in ***side and top views*** above. The coefficient of static fraction between the puck and the wall of the cylinder is μ_s. Gravity points in the negative $-z$ direction: down in the left hand figure and into the page in the right hand figure.

a) See above.

b) Comment: f_s is a ***variable*** force and is ***always*** equal to mg if the puck does not slide down. Presumably this means that $\Omega > \Omega_{\text{min}}$, obtained next!

$$f_s = \boxed{mg}$$

c) Here we need to use ***Newton's Second Law and circular motion kinematics*** to find N, use N to find $F_s = \mu_N$ and F_s and $f_s = mg$ to determine Ω_{min}. That is:

$$F_c = N = ma_c = m\Omega^2 R$$

$$F_s = \mu_s N = \mu_s m\Omega^2 R$$

$$f_s = mg < F_s = \mu_s m\Omega^2 R$$

so (cancelling m and rearranging):

$$\Omega > \sqrt{\frac{g}{\mu_s R}} = \Omega_{\text{min}}$$

or:

$$\Omega_{\text{min}} = \boxed{\sqrt{\frac{g}{\mu_s R}}}$$

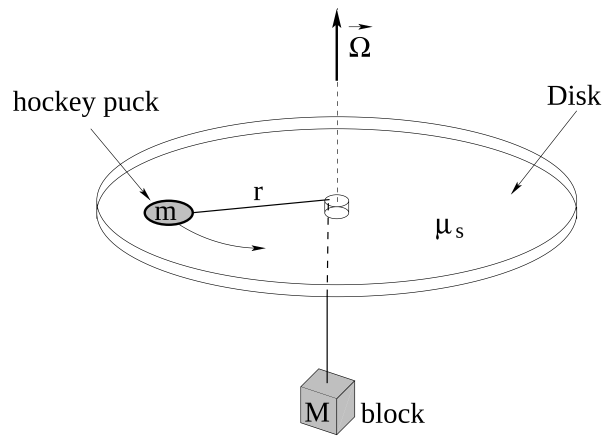
Scoring:

a) +6 total. +2 each for the three forces in the figure. The common mistakes are making f_s tangent to the circle of motion instead of opposite to gravity, and to use $m\Omega^2 R$ outward ("centrifugal force") instead of N inward. These mistakes will likely cost more points later.

b) +5 points straight up, not much room for partial credit. Student loses all five if they write $f_s = \mu_s N$.

c) +9 total. +2 each for each of the three equations above. +3 for the general algebra and final result. Note that they lose at least another 2-3 points if they got b) wrong and hence fail to write $f_s < F_s$ and even if they wrote F_c and F_s correctly.

Problem 98. problems-1/circular-motion-pr-puck-on-wheel-friction.tex



A disk is rotating with a ***constant angular velocity*** $\vec{\Omega}$ (up). A small hockey puck of mass m is placed on the disk at a distance r from the center, and is attached to another block with mass M hanging below by a massless unstretchable string that passes through a tiny (frictionless) hole right in the center of the disk. The static friction coefficient between the hockey puck m and the disk is μ_s.

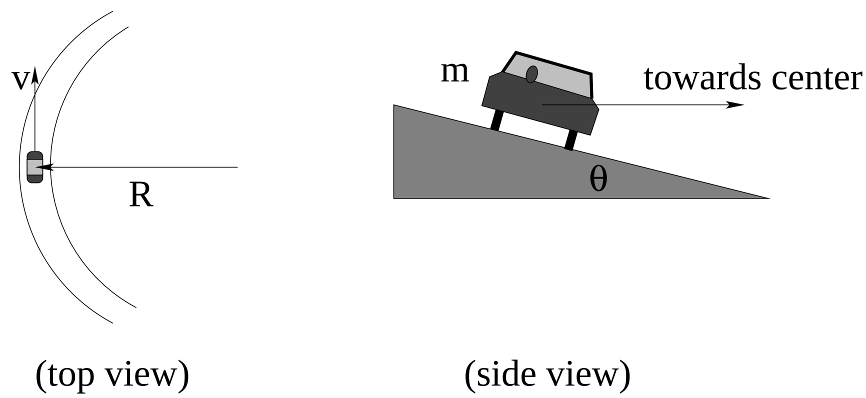
a) Which direction(s) could static friction need to point to keep the puck stationary on the rotating disk (check all that are possible for different/given Ω, r, M, m, μ_s):

☐ In (towards hole) ☐ Out ☐ Tangent to circle of motion

b) Find a formula for the largest M that will ***not move down*** (as the puck slips on the disk), given Ω.

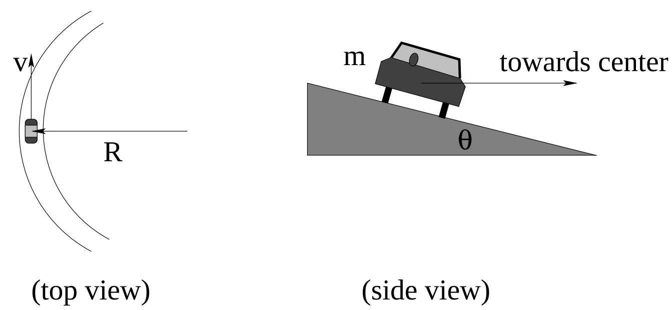
c) Find a formula for the smallest M that will ***not move up*** (as the puck slips on the disk), given Ω.

Problem 99. problems-1/circular-motion-pr-rounding-a-banked-curve-frictionless.tex



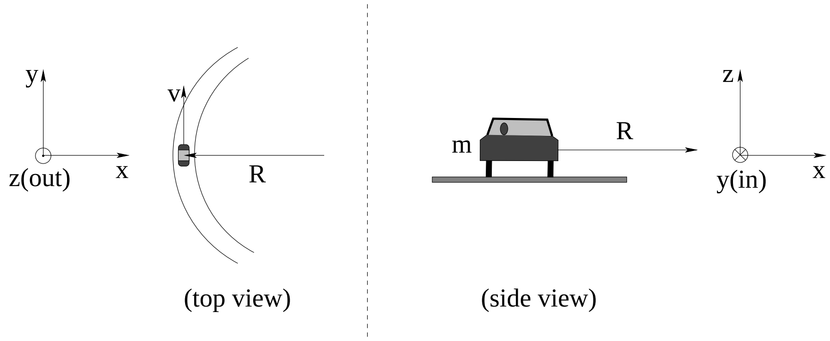
A car of mass m is rounding an icy ***frictionless*** banked curve that has radius of curvature R and banking angle θ. What must the speed v of the car be such that it can succeed in making it around the curve without sliding off of the road uphill or down?

Problem 100. problems-1/circular-motion-pr-rounding-a-banked-curve.tex



A car of mass m is rounding a banked curve that has radius of curvature R and banking angle θ. The coefficient of static friction between the car's tires and the road is μ_s. Find the *range* of speeds v of the car such that it can succeed in making it around the curve without skidding.

Problem 101. problems-1/circular-motion-pr-rounding-a-flat-curve.tex



A car of mass m is rounding a flat (unbanked) curve that has radius of curvature R. The coefficient of static friction between the car's tires and the road is μ_s. We will define $v_{\max}$ to be the ***fastest*** speed that the car can have to make it around the curve without ***without skidding***.

a) Suppose that the car is travelling at some constant speed $v < v_{\max}$ (so it does not skid and successfully rounds the curve). Find the total frictional force $\vec{f}_s$ exerted by the tires as it rounds the curve at that speed. Note that force is a vector – be sure to give its magnitude and direction ***using the provided coordinate frame(s)***.

b) Find the *fastest* speed of the car $v_{\max}$ such that it can make it around the curve without skidding.

Chapter 6

Work and Energy

In the previous problems, you were frequently asked to e.g. find the speed at the bottom of an incline or after a mass falls from some height. Every single time, you had to find the time it reached the bottom and substitute it back into the expression for the velocity/speed.

Work (and the general concept of energy) arise when we eliminate time ***once and for all*** from Newton's Laws, and directly relate speed to position. At first this is just a convenience that simplifies certain kinds of problems. Later, however, we will see that energy is in some sense ***more fundamental*** than force, that we could have started our study of dynamics ***from the beginning*** with energy concepts and worked from various potential energies to forces.

Sadly, certain non-conservative forces are difficult to connect back to potential energies in laws of nature – they do come from them but we'll only understand that after the fact. In the meantime, it makes more sense to go from force to energy than from energy to force, although we'll do a bit of both in the following problems.

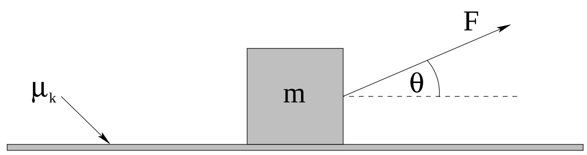
When you you think maybe I should use work or energy..." in a problem?

Whenever the questions asked relate speed to position independent of time (or, of course, when the questions asked directly pertain to the concept of work and energy itself). How fast at the bottom of the incline? Think ***energy***. How high does a mass thrown upward rise before stopping? think ***energy***.

6.1 Work and Kinetic Energy

6.1.1 Multiple Choice

Problem 102. problems-1/wke-mc-block-and-friction-icp.tex



A block of mass m is on a floor. The kinetic friction coefficient between the block and the floor is μ_k. A student pulls a block with a force $\vec{\boldsymbol{F}}$ directed upward at an angle θ with respect to the horizontal as shown. What is the work done ***by friction*** when the block moves a distance L along the floor to the right?

a) $-\mu_k mgL$

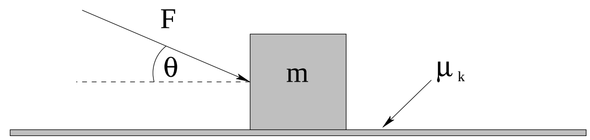
b) $-\mu_k(mg - F\sin(\theta))L$

c) $FL\ \cos(\theta) - \mu_k mgL + \mu_k F\sin(\theta)L$

d) $FL\ \cos(\theta)$

e) $-\mu_k(mg + F\sin(\theta))L$

Problem 103. problems-1/wke-mc-block-and-friction.tex



A block of mass m is on a floor. The kinetic friction coefficient between the block and the floor is μ_k. A student pushes a block with a force $\vec{F}$ directed down at an angle θ with respect to the horizontal as shown that makes the block slide ***to the right***. What is the work done ***by the student*** (the force $\vec{F}$) when the block moves a distance L along the floor?

a) FL

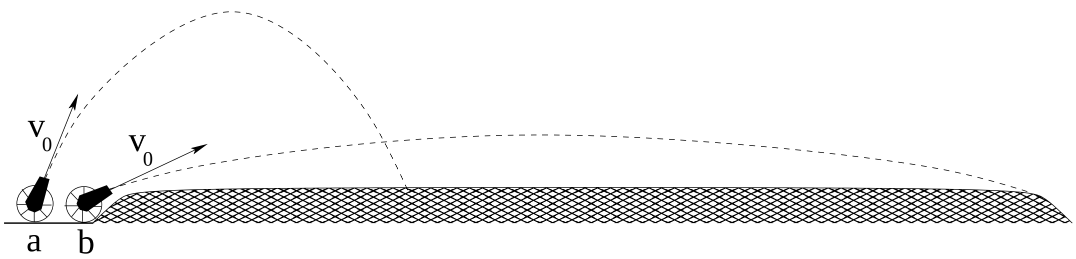
b) $\mu_k mgL$

c) $FL\ \cos(\theta)$

d) $\mu_k(mg + F\sin(\theta))L$

e) Cannot tell from the information given.

Problem 104. problems-1/wke-mc-cannonball-energy-speed-trajectories-icp.tex



Two cannons fire cannonballs ***at the same initial speed*** $\boldsymbol{v_0}$ into the air along the trajectories shown. Neglect the drag force of the air.

Which cannonball strikes the ground faster?

a) Cannonball **a** hits going faster.

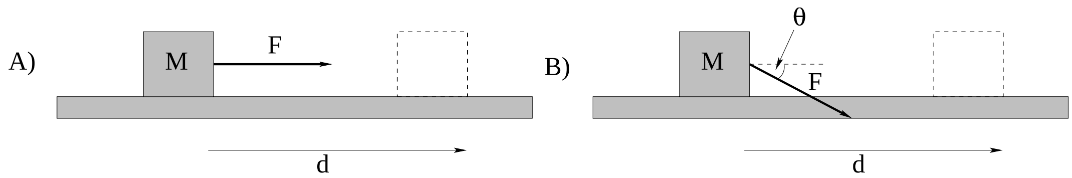
b) Cannonball **b** hits going faster.

c) Cannonball **a** and **b** hit at the same speed

d) We cannot tell which hits the ground going faster without more information than is given in the problem and picture.

6.1.2 Ranking/Scaling

Problem 105. problems-1/wke-ra-work-by-force-2.tex



In the figure above a force with a ***constant magnitude*** F is applied to a block of mass M resting on a table with a rough surface at two different angles as shown. The coefficient of kinetic friction between the block and table is μ_k. As the block slides to the right a distance d, the work done by $\vec{F}$ is W_F, and the work done by friction is W_{f_k}.

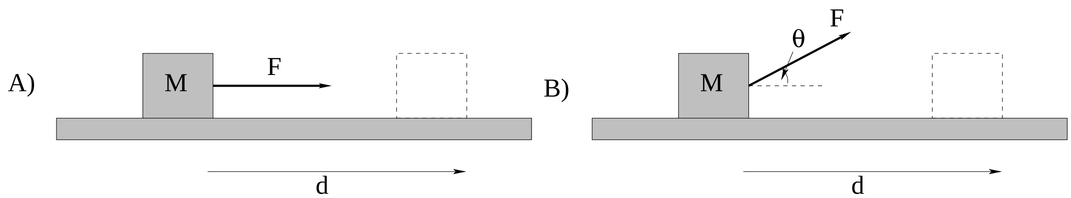
a) Rank the ***magnitude*** of the work done by $\vec{F}$ in the two cases (<>= in box):

$$W_F^A \ \square \ W_F^B$$

b) Rank the ***magnitude*** of the work done by friction in the two cases (<>= in box):

$$W_{f_k}^A \ \square \ W_{f_k}^B$$

Problem 106. problems-1/wke-ra-work-by-force-3.tex



In the figure above a force with a ***constant*** magnitude $F < Mg$ is applied to a block of mass M resting on a table with a rough surface at two different angles as shown. The coefficient of kinetic friction between the block and table is μ_k. As the block slides to the right a distance d, the work done by $\vec{F}$ is W_F, and the work done by friction is W_{f_k}.

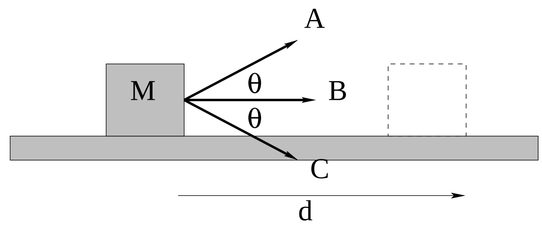
a) Rank the ***magnitude*** of the work done by $\vec{F}$ in the two cases (<>= in box):

$$W_F^A \quad \square \quad W_F^B$$

b) Rank the ***magnitude*** of the work done by friction in the two cases (<>= in box):

$$W_{f_k}^A \quad \square \quad W_{f_k}^B$$

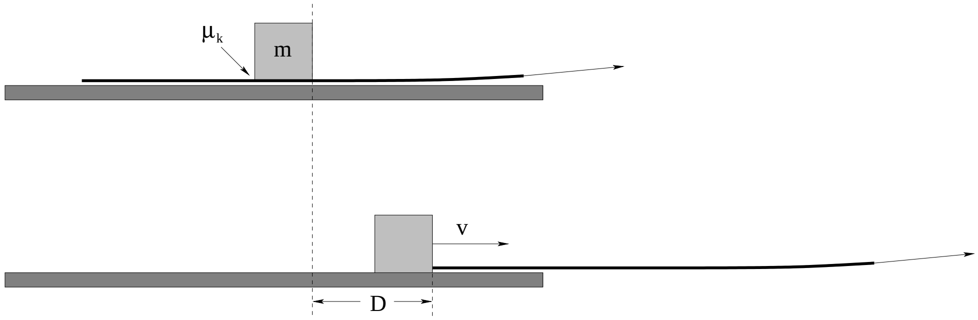
Problem 107. problems-1/wke-ra-work-by-force.tex



In the figure above a force with a *constant* magnitude F is applied to a block of mass M resting on a smooth (frictionless) table at three different angles as shown. Rank the work done by $\vec{F}$ as the block slides to the right a distance d, where equality is allowed. (A possible answer might be $A = B > C$ for example.)

6.1.3 Short Answer

Problem 108. problems-1/wke-sa-block-on-paper-icp.tex



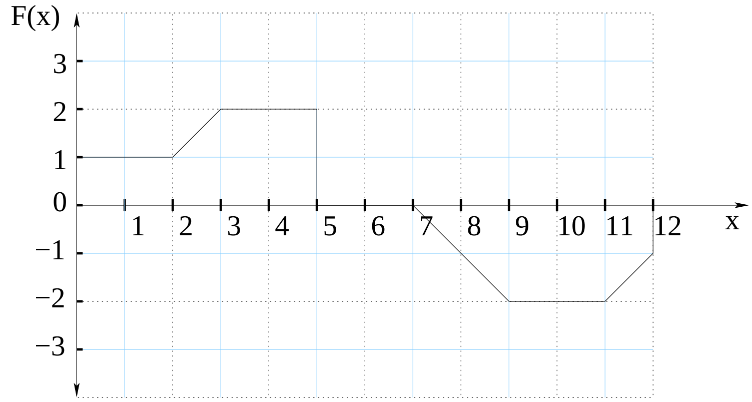
A block of mass m is initially at rest on a long piece of smooth paper on a frictionless table. The block has a coefficient of kinetic friction μ_k with the paper. You pull the paper horizontally out from under the block quickly in the direction indicated by the arrow, such that the block moves a distance D (relative to the ground) while still on the paper.

a) What is the ***magnitude*** of the work done by kinetic friction on the block?

b) Is the work done positive (increasing the kinetic energy of the block) or negative (decreasing the kinetic energy of the block).

c) What is the final velocity of the block when it comes off of the paper and slides along the frictionless table? ***Use +x to the right!***

Problem 109. problems-1/wke-sa-graph-work-1.tex



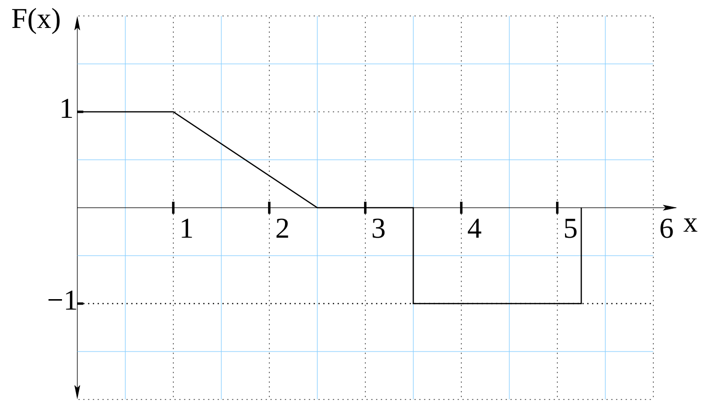
The graph above represents a force in the positive x direction $F(x)$ applied to a mass m as a function of its position. The mass begins at rest at $x = 0$. The force F is given in ***Newtons***, the position x is given in ***meters***.

a) ***How much work*** is done going from $x = 0$ to $x = 6$?

b) ***How much work*** is done going from $x = 6$ to $x = 12$?

c) Assuming $m = 1$ kg, ***what is the final velocity*** of the object at $x = 12$?

Problem 110. problems-1/wke-sa-graph-work-icp.tex



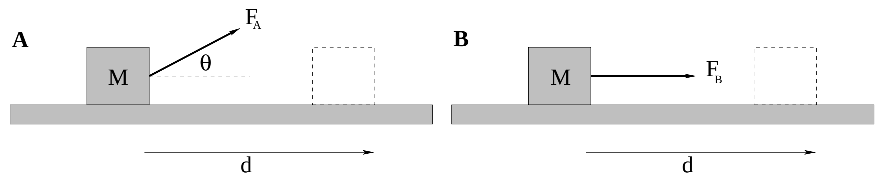
The graph above represents the total one-dimensional force in the x direction $F(x)$ being applied to a mass m as a function of its position. The mass begins at rest at $x = 0$ and moves only along the x axis. The force F is given in Newtons, the position x is given in meters. Answer the following questions (and give the units of your answers):

a) How much work is done going from $x = 0$ to $x = 3$? $W(0 \to 3) =$ $\boxed{}$

b) How much work is done going from $x = 3$ to $x = 6$? $W(3 \to 6) =$ $\boxed{}$

c) Assuming that $m = 1$ kg and that it begins ***at rest*** at the beginning of the motion, what is the speed of the particle at $x = 6$? $v(x = 6) =$ $\boxed{}$

Problem 111. problems-1/wke-sa-work-done-by-force.tex



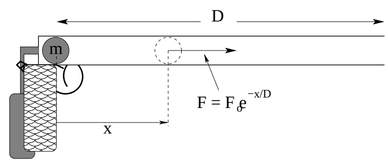
In the figure above a force with a ***constant*** magnitude $F_A = F_B = F$ is applied to a block of mass M resting on a table with a rough surface at two different angles as shown. The coefficient of kinetic friction between the block and table is μ_k. As the block slides to the right a distance d, the work done by $\vec{F}$ is W_F, and the work done by friction is W_{f_k} in the two cases, **A** and **B**.

a) For case **A**, find the work done by $\vec{F}$ and friction, W_F^A and $W_{f_k}^A$, respectively. Your answers should have the correct sign.

b) For case **B**, find the work done by $\vec{F}$ and friction, W_F^B and $W_{f_k}^B$, respectively. Your answers should have the correct sign.

6.1.4 Long Problems

Problem 112. problems-1/wke-pr-painball-gun-exponential-example.tex



A simple schematic for a paintball gun with a barrel of length D is shown above; when the trigger is pulled carbon dioxide gas under pressure is released into the approximately frictionless barrel behind the paintball (which has mass m). The expanding, cooling gas exerts a force on the ball of magnitude:

$$F = F_0 e^{-\frac{x}{D}}$$

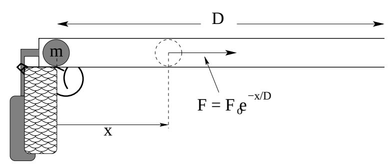
on the ball to the *right*, where x is measured from the paintball's initial position as shown.

a) Find the work done on the paintball by the force as the paintball is accelerated down the barrel.

b) Use the work-kinetic-energy theorem to compute the kinetic energy of the paintball after it has been accelerated.

c) Find the speed with which the paintball emerges from the barrel after the trigger is pulled.

Problem 113. problems-1/wke-pr-painball-gun-exponential-solution.tex



A simple schematic for a paintball gun with a barrel of length D is shown above; when the trigger is pulled carbon dioxide gas under pressure is released into the approximately frictionless barrel behind the paintball (which has mass m). The expanding, cooling gas exerts a force on the ball of magnitude:

$$F = F_0 e^{-\frac{x}{D}}$$

on the ball to the *right*, where x is measured from the paintball's initial position as shown.

a) Find the work done on the paintball by the force as the paintball is accelerated down the barrel.

b) Use the work-kinetic-energy theorem to compute the kinetic energy of the paintball after it has been accelerated.

c) Find the speed with which the paintball emerges from the barrel after the trigger is pulled.

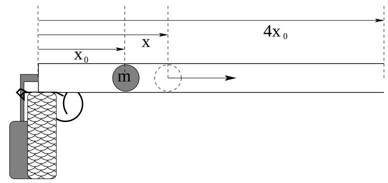
The only force acting on the paintball is the force applied by the prssurized gas (gravity is countered by a normal force from the barrel, and in any case neither does work when the motion is horizontal; we are neglecting friction which may be less realistic). So WKE reads simply

$$\begin{aligned} K_f - K_i &= \int_i^f bfF \cdot d\mathbf{x} \\ &= \int_0^D F_0 e^{-\frac{x}{D}}\,dx \\ &= F_0\left(-De^{-d/D}\right)\Big|_0^D \\ &= -FD(e^{-1} - 1) = FD(1 - 1/e)\ . \end{aligned} \tag{6.1}$$

This is the work done by the gas. Since the paintball starts at rest so $K_i = 0$ it is also the kinetic energy the ball has when it leaves the barrel. To find the speed with which it leaves we set

$$\begin{aligned} \frac{mv^2}{2} &= K_f = FD(1 - 1/e) \\ v &= \sqrt{\frac{2FD(1 - 1/e)}{m}}\ . \end{aligned} \tag{6.2}$$

Problem 114. problems-1/wke-pr-paintball-gun-adiabatic.tex



A simple schematic for a paintball gun is shown above; when the trigger is pulled carbon dioxide gas under pressure is released into the approximately frictionless barrel behind the paintball (which has mass m) initially resting at x_0. The gas expands approximately ***adiabatically*** and exerts a force on the ball of magnitude:

$$F = F_0 \frac{x_0^\gamma}{x^\gamma}$$

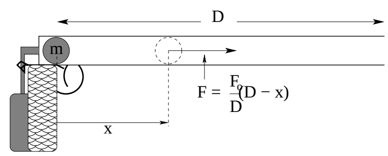
on the ball to the *right*, where F_0 is the initial force exerted at $x = x_0$, and x is measured from the end of the barrel as shown. γ is a constant (equal to 1.4 for carbon dioxide). This force is only exerted up to the end of the barrel at $x = 4x_0$.

a) Find the work done on the paintball by the force as the paintball is accelerated down the barrel.

b) Use the work-kinetic-energy theorem to compute the kinetic energy of the paintball after it has been accelerated.

c) Find the speed with which the paintball emerges from the barrel after the trigger is pulled.

Problem 115. problems-1/wke-pr-paintball-gun-linear.tex



A simple schematic for a paintball gun is shown above; when the trigger is pulled carbon dioxide gas under pressure is released into the approximately frictionless barrel behind the paintball (which has mass m). The gas exerts a force on the ball of magnitude:

$$F = \frac{F_0}{D}(D - x)$$

on the ball to the *right*, where x is measured from the paintball's initial position as shown.

a) Find the work done on the paintball by the force as the paintball is accelerated down the barrel.

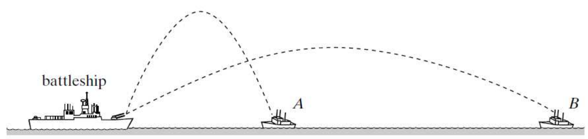
b) Use the work-kinetic-energy theorem to compute the kinetic energy of the paintball after it has been accelerated.

c) Find the speed with which the paintball emerges from the barrel after the trigger is pulled.

6.2 Work and Mechanical Energy

6.2.1 Multiple Choice

Problem 116. problems-1/wme-mc-battleship.tex



A battleship ***simultaneously*** fires two shells at enemy ships along the trajectories shown, such that the shells have the ***same initial speed***. One ship (**A**) is close by; the other ship (**B**) is far away. Ignore drag forces.

a) Which ship is hit first (circle both if they are hit at the same time)?

A **B**

b) Which shell has the greater speed when it hits the ship (circle both if the speeds are equal)?

A **B**

Problem 117. problems-1/wme-mc-zero-of-potential-energy.tex

Tommy is working on a physics problem involving energy. "Look," he says, "the total energy of this block at rest is zero at the top of this incline of height H and therefore must be zero at the bottom."

Sally disagrees. "Impossible. The block is at the *top* of the incline. It has total energy mgH at the top and so its total energy must still be mgH at the bottom."

a) Tommy is right, Sally is wrong.

b) Sally is right, Tommy is wrong.

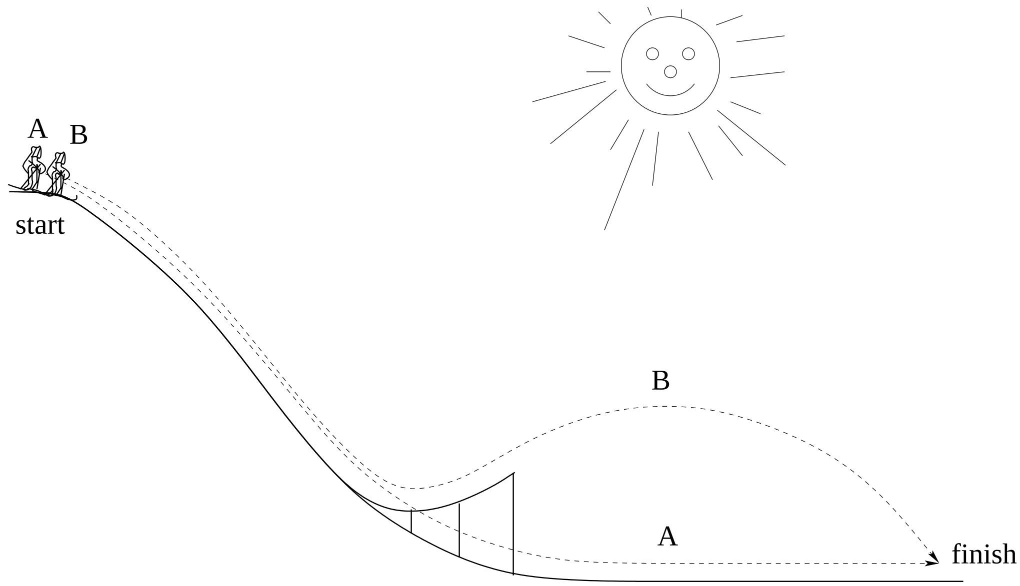
c) Both Tommy and Sally are right.

d) Both Tommy and Sally are wrong.

e) There isn't enough information to tell who is right and who is wrong.

6.2.2 Ranking/Scaling

Problem 118. problems-1/wme-ra-two-skiers-icp.tex



Two skiers start at the same point on a (frictionless) slope. One (A) skis straight down the slope to the finish line. The other (B) passively skis off of a ski jump to arrive at the finish more flamboyantly. ***Rank*** the answers to the following questions, where equality is a possibility, that is, possible answers are $A < B$ or $A = B$. Ignore friction and drag forces and assume that the jumper does not use their leg muscles to "jump".

a) Rank the ***relative speed*** of the two skiers when they reach the finish line.

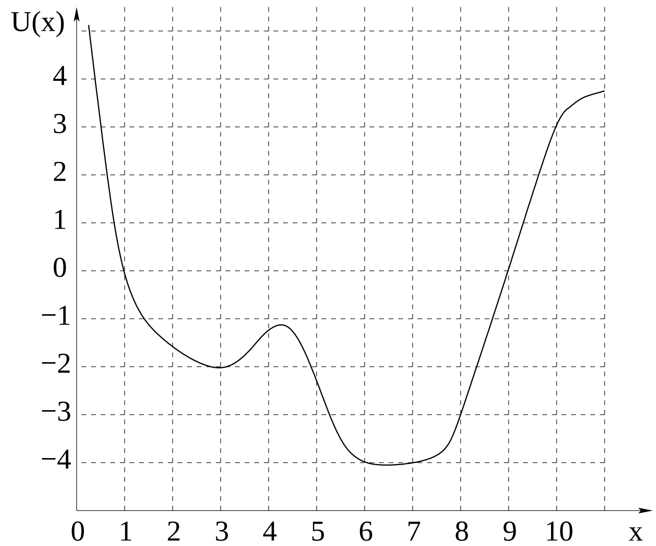
b) Rank the ***finish time*** – who arrives at the finish line first (or is it at the same time)?

6.2.3 Short Answer

Problem 119. problems-1/wme-sa-ball-to-ground.tex

A ball is thrown with some speed v_0 from the top of a cliff of height H. Show that the speed with which it hits the ground is independent of the direction it is thrown (and determine that speed in terms of g, H, and v_0).

Problem 120. problems-1/wme-sa-potential-energy-graph-2.tex



A conservative one-dimensional force $F(x)$ acts on a particle of mass $m = 2$ kg. The potential energy $U(x)$ associated with $F(x)$ is shown in the figure above. The particle is at initially located $x_1 = 3$ m with kinetic energy $K_1 = 2$ J, moving to the left (alone the negative x direction). [*Show your work and make sure that you use the correct units.*]

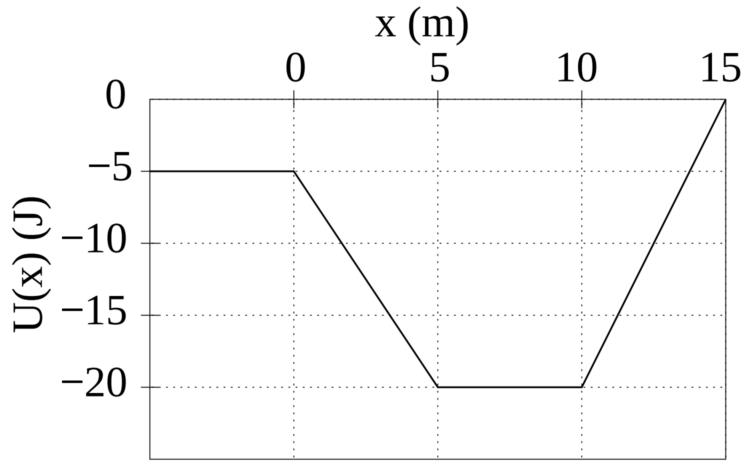
a) Mark all points ***on the*** x***-axis*** where the force on the mass would vanish. Label the points "stable" or "unstable according to the kind of equilibrium point they are.

b) Between what limits of x does the particle move?

c) What is the particle's speed when it is at $x_2 = 6$ meters?

d) What is the magnitude and direction of the force $\vec{\boldsymbol{F}}(x)$ at $x_3 = 9$ meters.

Problem 121. problems-1/wme-sa-potential-energy-to-force-icp.tex



A one-dimensional force $F(x)$ acts on a 2 kg particle which moves along the x axis. The potential energy $U(x)$ associated with $F(x)$ is shown in the graph.

When the particle is at $x = 11$ m, its speed is 2 m/s.

a) What is the magnitude and direction of $F(x)$ at:
   $x = -3$ m:
   $x = 4$ m:
   $x = 6$ m:
   $x = 11$ m:

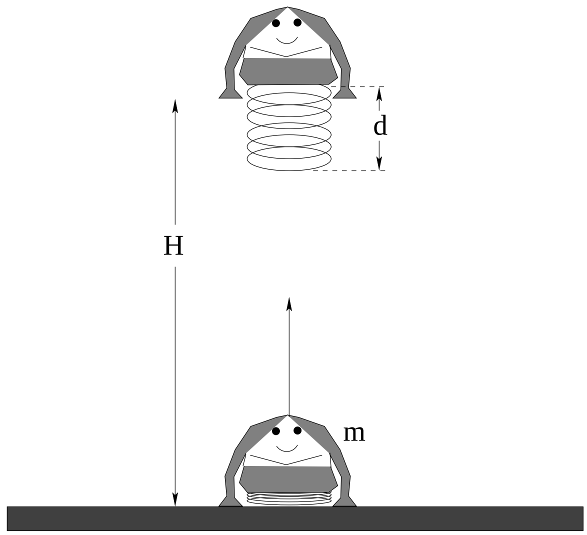
b) Between what limits of x does the particle move?

c) What is its speed at $x = 7$ m?

Problem 122. problems-1/wme-sa-sliding-block-friction.tex

A block of mass m sitting on a horizontal surface is given an initial speed v_0. Travelling in a straight line it comes to rest after sliding a distance d. Show that the coefficient of kinetic friction is given by $\frac{v^2}{2gd}$.

Problem 123. problems-1/wme-sa-spring-jumper-straight-up-soln.tex



A simple child's toy is a jumping frog made up of an approximately massless spring of uncompressed length d and spring constant k that propels a molded "frog" of mass m. The frog is pressed down onto a table (compressing the spring by d) and at $t = 0$ the spring is released so that the frog leaps high into the air.

Use work and/or mechanical energy to determine how high the frog leaps.

It is by far the easiest to use conservation of mechanical energy. Initially, the frog is located, at rest (so its kinetic energy $K = 0$), at $y = 0$ (so $U_g = mgy = 0$) with the spring compressed a distance d (so $U_k = \frac{1}{2}kd^2$). When the frog reaches its maximum height, it is *again* at rest (so $K = 0$), its gravitational potential energy is now $U_g = mgy = mgH$, and the spring is fully expanded so its spring potential energy is $U_k = 0$. Hence:

$$E_i = 0 + 0 + \frac{1}{2}kd^2 = 0 + mgH + 0$$

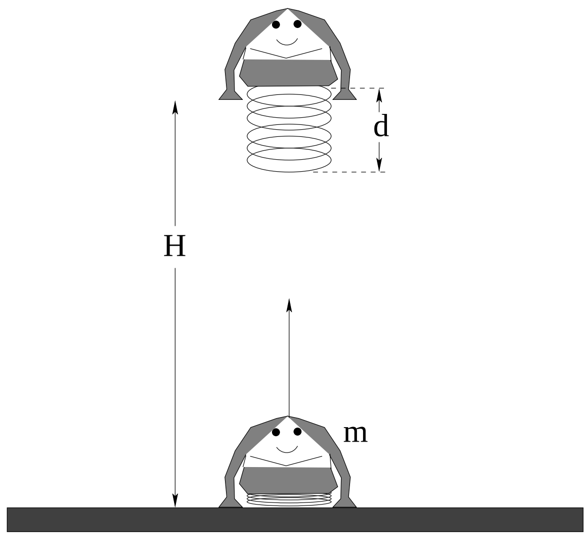
or

$$mgH = \frac{1}{2}kd^2$$

or

$$H = \frac{kd^2}{2mg}$$

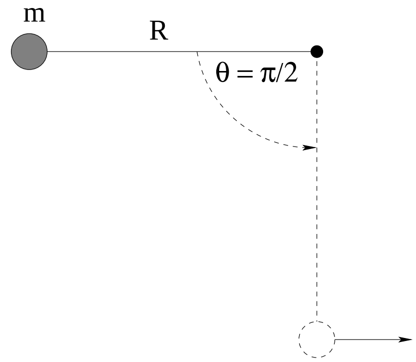
Problem 124. problems-1/wme-sa-spring-jumper-straight-up.tex



A simple child's toy is a jumping frog made up of an approximately massless spring with spring constant k that propels a molded "frog" of mass m. The frog is pressed down onto a table (compressing the spring by a distance d) and at $t = 0$ the spring is released so that the frog leaps high into the air.

Use work and/or mechanical energy to determine how high the frog leaps. Neglect drag forces.

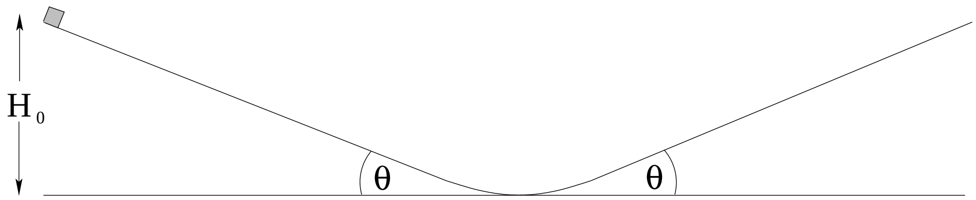
Problem 125. problems-1/wme-sa-tension-pendulum-height-R-icp.tex



In the figure above, a mass m is attached to a massless unstretchable string of length R and held at an initial position at an angle $\theta = \pi/2$ relative to the horizontal as shown. At time $t = 0$ it is released from rest. Find the tension T in the string when it reaches $\theta = 0$.

6.2.4 Long Problems

Problem 126. problems-1/wme-pr-double-inclines-with-friction.tex



A block with mass m is released from rest at a height H_0 on an inclined plane that makes an angle θ with the ground. When it reaches the bottom, it smoothly slides up a second incline, also at an angle θ with respect to the ground as shown. The coefficient of static friction between the block and the inclines is μ_s; the coefficient of kinetic friction between the block and the inclines is μ_k.

a) Find the minimum angle θ_{min} such that the block will be able to slide down the incline after being released from rest.

$\theta_{\text{min}} =$

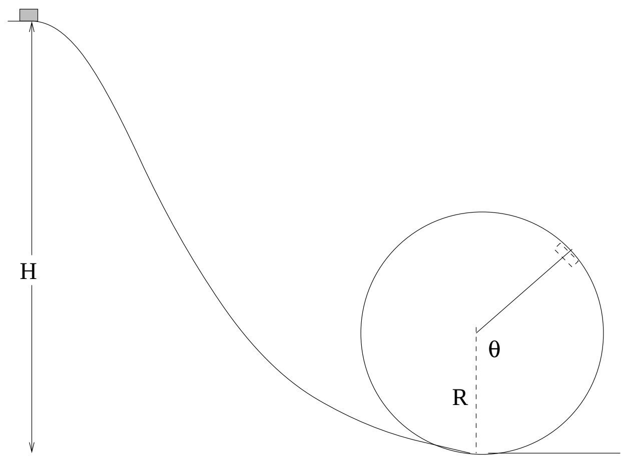
b) Suppose $\theta > \theta_{\text{min}}$. When the block is released from the initial height H_0, what height H_1 will it reach as it slides up the opposite incline before coming momentarily to rest?

$H_1 =$

c) *Bonus question: 5 points*
Suppose that the coefficient of kinetic friction is very small so that it can slide back and forth many times. Approximately how many times will the block slide back and forth before it loses 1/2 of its initial energy?

$N_{\frac{1}{2}} =$

Problem 127. problems-1/wme-pr-loop-the-loop-block.tex

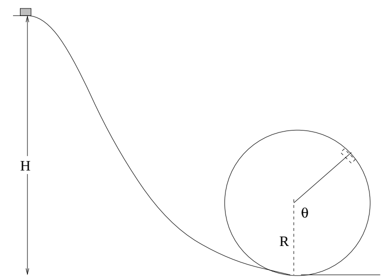


A block of mass M sits at the top of a frictionless loop-the-loop of height H.

a) Find the normal force exerted by the track when the mass is at an angle θ on the loop as shown.

b) Find the minimum height H such that the block loops the loop without coming off of the track.

Problem 128. problems-1/wme-pr-loop-the-loop-classic-example.tex



A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R.

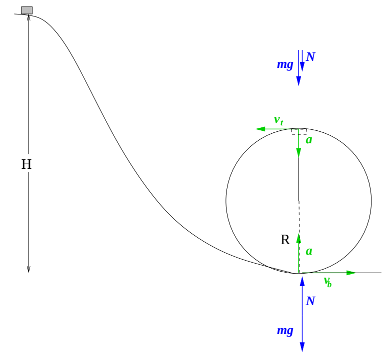
a) Find the minimum height H_{min} for which the block *barely* goes around the loop staying on the track at the top. (Hint: What is the condition on the normal force when it "barely" stays in contact with the track? This condition can be thought of as "free fall" and will help us understand circular orbits later, so don't forget it.).

   Discuss within your recitation group why your answer is a scalar number times R and how this *kind* of result is usually a good sign that your answer is probably right.

b) If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

   If you have ever ridden roller coasters with loops, use the fact that your apparent weight *is* the normal force exerted on you by your seat if *you* are looping the loop in a roller coaster and discuss with your recitation group whether or not the results you derive here are in accord with your experiences. If you haven't, consider riding one *aware* of the forces that are acting on you and how they affect your perception of weight and change your direction on your next visit to e.g. Busch Gardens to be, in a bizarre kind of way, a *physics assignment*. (Now c'mon, how many classes have you ever taken that assign riding roller coasters, even as an optional activity?:-)

Problem 129. problems-1/wme-pr-loop-the-loop-classic-solution.tex



A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R.

a) Find the minimum height H_{min} for which the block *barely* goes around the loop staying on the track at the top. (Hint: What is the condition on the normal force when it "barely" stays in contact with the track? This condition can be thought of as "free fall" and will help us understand circular orbits later, so don't forget it.).

   Discuss within your recitation group why your answer is a scalar number times R and how this *kind* of result is usually a good sign that your answer is probably right.

b) If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

   If you have ever ridden roller coasters with loops, use the fact that your apparent weight *is* the normal force exerted on you by your seat if *you* are looping the loop in a roller coaster and discuss with your recitation group whether or not the results you derive here are in accord with your experiences. If you haven't, consider riding one *aware* of the forces that are acting on you and how they affect your perception of weight and change your direction on your next visit to e.g. Busch Gardens to be, in a bizarre kind of way, a *physics assignment.* (Now c'mon, how many classes have you ever taken that assign riding roller coasters, even as an optional activity?:-)

Let us follow the hint and think about what is going on here. In this problem the block is *not* bound to the looping track. This means that when it goes over the top of the loop nothing is "holding it up." Like any other object not held up by anything, it must accelerate down with an acceleration g. Yet experience with toy cars, roller coasters, and strings tells us that if it is going fast enough it will not fall off the track. The reason is that going around a circular track does involve an acceleration, towards the center of the circle, of magnitude v^2/R where v is the speed. We will reproduce this in the last problem on this set. If this acceleration is at least g then at the top of the track the block can be in free fall without leaving the track. If the speed is higher, the acceleration required to complete the circle will be higher than g. This means if the track broke, the block would in fact fly off *above* the circular trajectory. This is prevented by a normal force applied by the track. The point of all this is that if the block is moving too slowly around the loop it will leave the track. As the speed is reduced past this minimum, the first failure to stay on the track will occur at the very top of the loop. This is intuitively clear, we will work it out in detail in another problem.

To turn these words into equations, consider applying Newton's second law to the block at the instant when it is at the apex of the looping track, moving (horizontally, to the left) at a speed v_t. The figure indicates forces, velocity, and acceleration at this point, including the initially unknown normal force applied by the track. Newton's second law then reads

$$\mathbf{F} = -mg\hat{\mathbf{y}} - N\hat{\mathbf{y}} = m\mathbf{a} \ . \tag{6.3}$$

In order for the block to continue its circular motion along the track this downward vertical acceleration must be equal to the centripetal acceleration, directed downward towards the center of the circle, i.e. we have $\mathbf{a} = -mv_t^2/R\,\hat{\mathbf{y}}$. This requires

$$N = \frac{mv_t^2}{R} - mg \ . \tag{6.4}$$

Since $N \geq 0$ we see that if the block is moving too slowly it will not stay on the track. The minimum speed needed to just maintain contact with the track at the top is the speed at which $N = 0$, i.e.

$$v_t^2 = gR \ . \tag{6.5}$$

Now our job is to find how high the initial ramp must be in order for the block to reach the top of the look with this speed. Of course, as it goes down the ramp the block accelerates under the influence of gravity, but as it goes up the looping track it slows down under the same influence. Since all forces acting on the block are conservative (gravity) or do no work at all (the normal forces, which are everywhere perpendicular to the direction of motion) the total mechanical energy of the block is conserved throughout its travels along our track. We can thus relate its speed at the top of the loop to the height of the ramp where it was released from rest by equating the total mechanical energy in both configurations, including kinetic and gravitational potential energy. Setting $U_g = 0$ at the base of the loop to determine the irrelevant additive constant we have for these initial and final configurations the following expressions for total energy

$$\begin{aligned} E_i &= mgH \\ E_t &= \frac{mv_t^2}{2} + mg\,2R \ . \end{aligned} \tag{6.6}$$

Setting these equal to each other we find that the speed of the block at the top of the loop is determined by H, R as

$$v_t^2 = g(2H - 4R) \ . \tag{6.7}$$

The minimum H needed to clear the loop will lead to a value for v equal to the minimal value (6.5) i.e.

$$\begin{aligned} g(2H - 4R) &= gR \\ H &= \frac{5}{2}R \ . \end{aligned} \tag{6.8}$$

As predicted, we find a number, determined by various geometric factors, times R. This makes sense, because R is the only parameter in the problem with the right dimensions, length. So to determine a length H related in some way to R we expect to find a result like this. That it makes sense does not make it trivial. Neglecting friction, we predict that to double the height of the loop you must double the height of your ramp. And we could have predicted that just using this kind of dimensional reasoning, without doing any calculations at all!

We now want to find the normal force applied by the track at the bottom of the loop when the block is released from this height. The figure indicates forces, velocity, and acceleration at this instant. The total force on the block is now

$$\mathbf{F} = -mg\hat{\mathbf{y}} + N\hat{\mathbf{y}} \ . \tag{6.9}$$

Once more the net acceleration is vertical. In order to be moving around a circle at speed v_b we must have $\mathbf{a} = v_b^2/R\hat{\mathbf{y}}$ directed upward towards the center of the circle. This requires

$$N = mg + m\frac{v_b^2}{R} \ . \tag{6.10}$$

This makes sense. At the bottom, in addition to holding up the block's weight, the track must apply additional normal force to provide the centripetal acceleration.

To find the value of N we again use energy conservation to find v_b. At the bottom of the loop the gravitational potential energy vanishes but the conserved total energy is equal to its value at the top of the ramp (or at any other time during the block's travels). This means

$$E_b = \frac{mv_b^2}{2} = E_i = mgH \ , \tag{6.11}$$

or

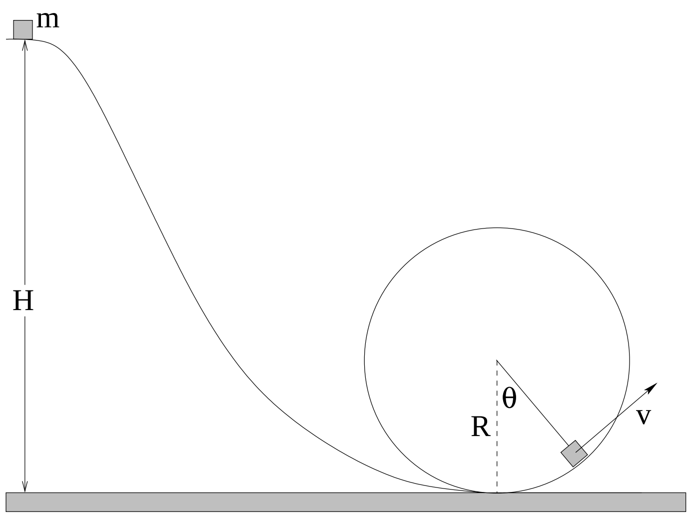
$$v_b^2 = 2gH = 5gR \ , \tag{6.12}$$

where the last equality used (6.8). Inserting this we find

$$N = mg + 5mg = 6mg \ . \tag{6.13}$$

If you are sitting in this block and travelling at the minimal speed needed to traverse the loop, then at the top of the loop (where $N = 0$ you will just barely touch your seat. At the bottom your seat needs to apply six times your weight to your bottom to accelerate you up. Your diaphragm needs to apply six times the force it is accustomed to to hold up the contents of your abdominal cavity, and most importantly your heart must lift your blood out of your feet against an apparent $6g$ of gravity. This is why pilots of WWII planes that first achieved high speeds had trouble with blacking out. Their hearts failed to overcome the increased apparent gravity at the bottom of maneuvers and their oxygen-starved brains lost consciousness. The remedy at the time was inserting wood blocks on the pedals, to raise their feet and put them in a cramped position amenable to tightening their abdominal muscles to restrict blood flow. Modern pressure suits simply squeeze the lower extremities in any configuration so that blood flow is unaffected by acceleration.

Problem 130. problems-1/wme-pr-loop-the-loop-difficult.tex

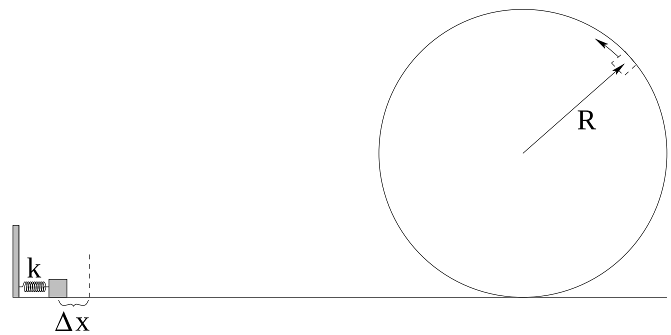


A block of mass m sits at the top of a frictionless hill of height H. It slides down and around a loop-the-loop of radius R to an angle θ as shown.

a) Find the magnitude of the normal force as a function of the angle θ.

b) From this, deduce an expression for the angle θ_0 at which the block will *leave* the track if the block is started at a height $H = 2R$.

Problem 131. problems-1/wme-pr-loop-the-loop-from-spring.tex



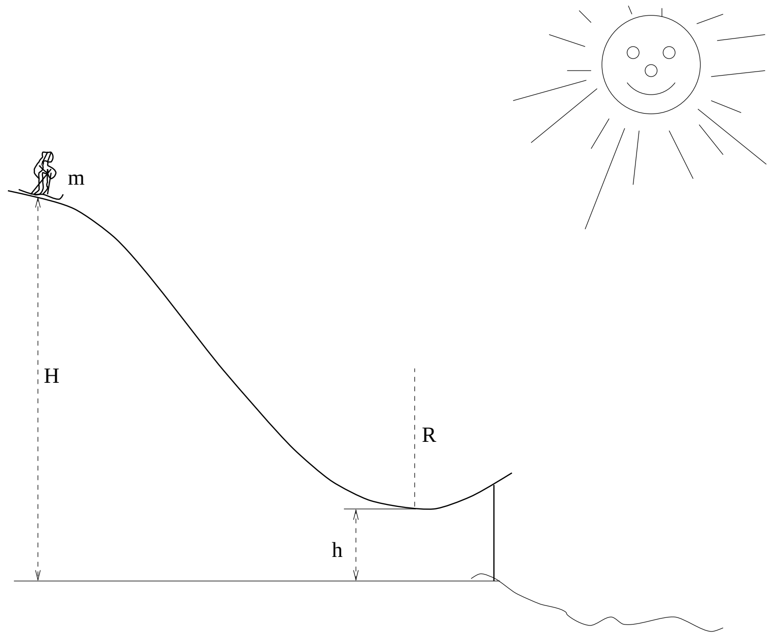
A block of mass M sits in front of a spring with spring constant k compressed by an amount Δx on a frictionless track leading to a circular loop-the-loop of radius R as shown.

a) ***Draw two force diagrams***, one with the block at the top of the loop and one with the block at the bottom of the loop. Clearly label all forces, including ones that you might set to zero or ignore. Use these force diagrams to help answer the following two questions.

b) Find the minimum value of Δx for which the block *barely* goes around the loop staying on the track at the top.

c) If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

Problem 132. problems-1/wme-pr-loop-the-loop-skier-ambitious-amy.tex



Ambitious Amy has a mass m. She skis from initial rest down the (frictionless) ski slope of height H to a ski ramp whose radius of curvature R and whose lowest point is h above the ground (as shown).

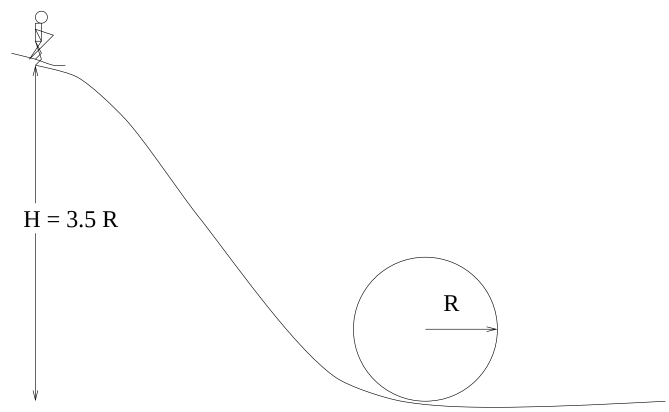
Amy's leg strength must oppose her apparent weight at the bottom of the jump. Is she strong enough? It would be good to know how strong she has to be so that she can work on leg presses if need be before trying the actual jump. So (in terms of the given quantities m, g, R, H, h) :

a) How fast is Amy going when she reaches the lowest point in the curved jump?

b) What is the *total* force that must be directed towards the center of the circle of motion at that point (again, in terms of the given).

c) Using your knowledge of the actual forces acting on her that have to sum to this force, determine her "apparent weight" – the peak force she has to *push down on the ground* with her skis with in order to stay on the circular curve.

Problem 133. problems-1/wme-pr-loop-the-loop-skier.tex

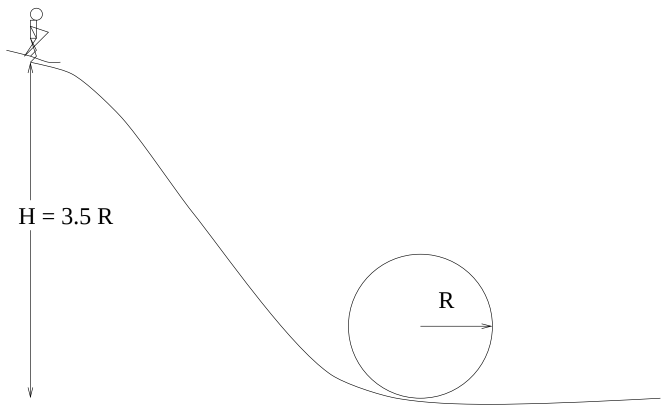


A skier of mass m at an exhibition wants to loop-the-loop on a special (frictionless) ice track of radius R set up as shown. Suppose $H = 3.5R$. All answers should be given in terms of g, m and R. (Note that the skier is really much shorter than R; the picture is not drawn strictly to scale for ease of viewing.)

a) What is her apparent "weight" (the normal force exerted by the track on her skis) when she is upside down at the top of the loop-the-loop?

b) What is her maximum apparent "weight" on the loop-the-loop and where (at what point on the loop-the-loop track) does it occur? Indicate the position on the figure.

Problem 134. problems-1/wme-pr-loop-the-loop-skier-weight.tex

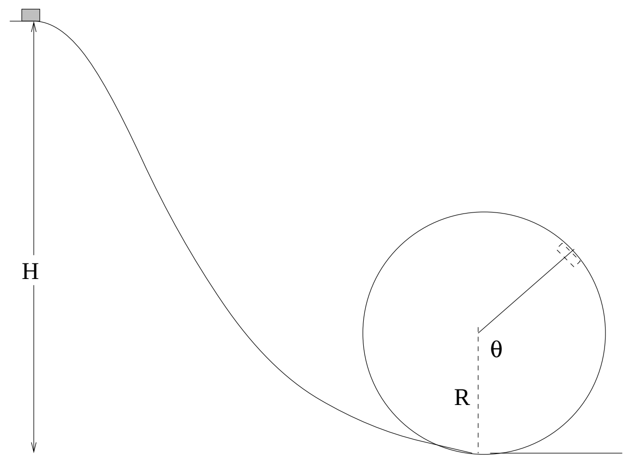


A skier of mass m at an exhibition wants to loop-the-loop on a special (frictionless) ice track of radius R set up as shown. Suppose $H = 3.5R$. All answers should be given in terms of g, m and R. (Note that the picture is not drawn strictly to scale for ease of viewing.)

a) What is her apparent weight (the normal force exerted by the track on her skis) when she is upside down at the top of the loop-the-loop? If she closed her eyes, what direction would she think of as "down"?

b) What is her maximum apparent "weight" on the loop-the-loop and where (at what point on the loop-the-loop track) does it occur? Indicate the position on the figure.

Problem 135. problems-1/wme-pr-loop-the-loop.tex

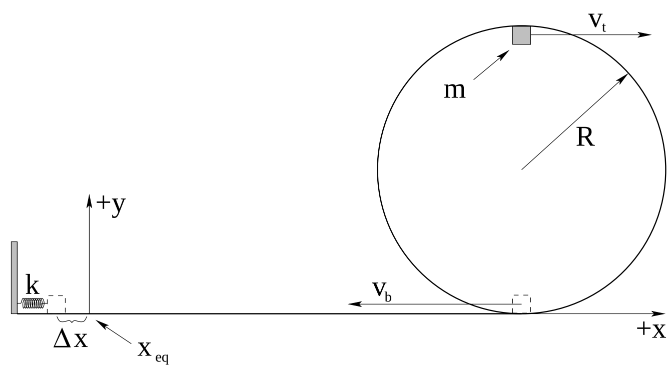


A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R.

a) Find the minimum height $H_{\min}$ for which the block *barely* goes around the loop staying on the track at the top.

b) If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

Problem 136. problems-1/wme-pr-loop-the-loop-to-spring.tex



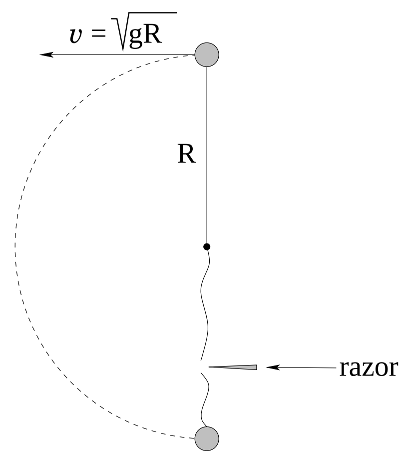
A block of mass m is travelling to the right at the top of a frictionless circular loop-the-loop track of radius R, travelling at speed v_t to the right as shown. v_t is large enough that the mass remains on the track at the top. It then slides around the track to the bottom, slides across the (frictionless) ground, and hits a spring with spring constant k which slows it to rest after the spring has compressed a distance Δx from its initial equilibrium length.

a) What is the speed v_b at the bottom of the circular loop?

b) What is the normal force exerted by the track at the bottom just before/as it leaves the circular loop?

c) By what distance Δx is the spring compressed at the instant the block comes to rest?

Problem 137. problems-1/wme-pr-razor-cuts-loop-string-1.tex



A small ball of mass m (which can be treated as a point particle for this problem) is attached to a massless, unstretchable string whose other end is attached to a fixed, frictionless pivot. The ball swings in a vertical circle, with gravity acting downward as usual.

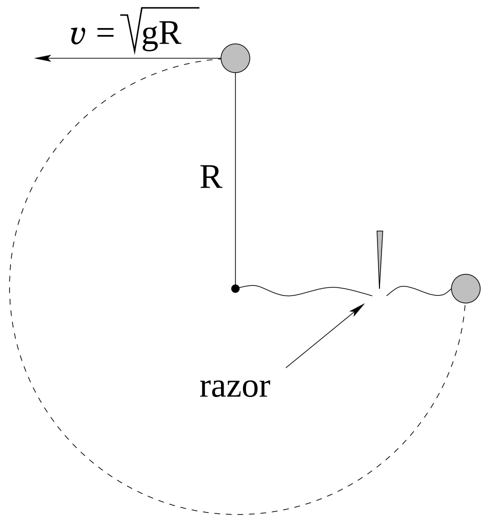
When the ball is at the top of the circle it has velocity $\sqrt{gR}$ to the left as shown, the minimum needed to keep the particle moving in a circle. After the ball has gone half way around the circle and the string is again vertical, a razor blade cuts the string. You can assume that the impulse delivered to the string by the very sharp razor is small enough that it can be ignored.

a) Find the velocity of the ball ***just before*** the string is cut.

b) Find the tension in the string ***just before*** the string is cut.

c) On the diagram, qualitatively show the path of the ball ***just after*** the string is cut.

Problem 138. problems-1/wme-pr-razor-cuts-loop-string.tex



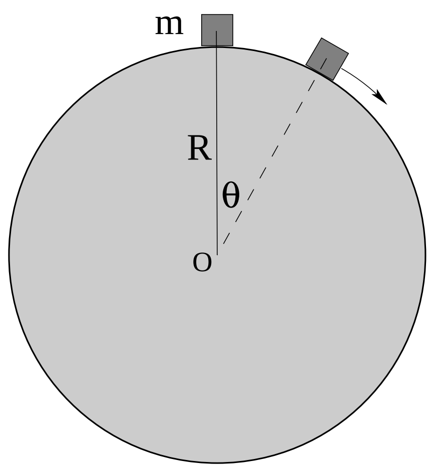
A small ball (which can be treated as a point particle for this problem) is attached to an Acme (massless, unstretchable) string whose other end is attached to a fixed, frictionless pivot. The ball swings in a vertical circle, with gravity acting downward as usual.

When the ball is at the top of the circle it has velocity $\sqrt{gR}$ to the left as shown. After the ball has gone three quarters of the way around and the string is horizontal, a razor blade cuts the string. You can assume that the impulse delivered to the string by the razor is small enough that it can be ignored.

a) Find the tension in the string ***just before*** it is cut.

b) On the diagram, show the path of the ball ***after*** the string is cut. Describe in words any features of the path that you intended to illustrate and be sure to indicate the maximum height you expect the ball to reach relative to the center of the circle of motion.

Problem 139. problems-1/wme-pr-sliding-off-a-cylinder-review.tex



In the figure above, a ***small*** (treat as a point mass) block of mass m is on top of a frictionless cylinder so that its center of mass is a distance R from the axis of the cylinder. It is given a nudge so that it slides with negligible initial speed down the side of the cylinder.

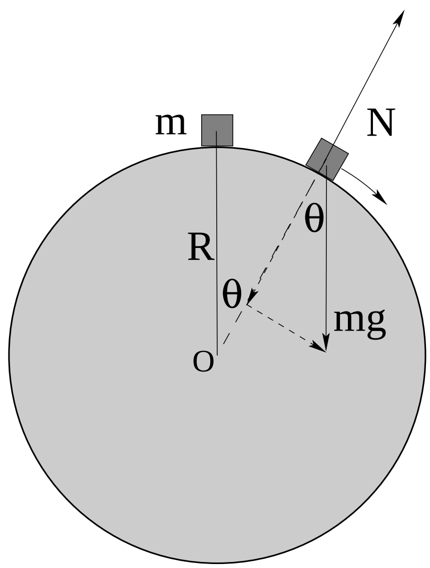
a) When its angular position is θ as shown, what is its speed (assuming that it is still on the cylinder)?

b) What is the magnitude of the normal force exerted on the block by the cylinder at this point?

c) For what value of θ will the block leave the cylinder?

Express your answers in m, R, g and θ.

Problem 140. problems-1/wme-pr-sliding-off-a-cylinder-review-soln.tex



In the figure above, a ***small*** (treat as a point mass) block of mass m is on top of a frictionless cylinder so that its center of mass is a distance R from the axis of the cylinder. It is given a nudge so that it slides with negligible initial speed down the side of the cylinder.

a) When its angular position is θ as shown, what is its speed (assuming that it is still on the cylinder)?

   Use ***Energy Conservation***. Let's set $U_g = 0$ at the top of the slope. As it slides, it falls by a vertical height $H = R(1 - \cos\theta)$ (why?). Then $E_i = U_i + K_i = 0$ Hence:

$$E_f = \frac{1}{2}mv^2 - mgR(1 - \cos\theta) = 0 = E_i$$

   or (rearranging):

$$v(\theta) = \boxed{\sqrt{2gR(1 - \cos\theta)}}$$

   As a quick check, this correctly predicts $v = \sqrt{2gR}$ at $\theta = \pi/2$ and $v = 0$ at $\theta = 0$, and has the right units.

b) What is the magnitude of the normal force exerted on the block by the cylinder at this point?

   We need to write Newton's Second Law for the component of $\vec{F}$ or $\vec{a}$ ***towards the center of the circle***. As long as one uses this component only, any coordinate frame above can be made to work. That is:

$$F_c = mg\cos\theta - N = ma_c = \frac{mv^2}{R}$$

   From the energy conservation result:

$$mv^2 = 2mgR(1 - \cos\theta)$$

   Finally we substitute and rearrange:

$$N = mg\cos\theta - 2mg(1 - \cos\theta) = 3mg\cos\theta - 2mg$$

c) For what value of θ will the block leave the cylinder?

Note that $N \to 0$ at the *specific* angle θ_a where it comes off of the cylinder. Using the result for b), θ_a is easily found:

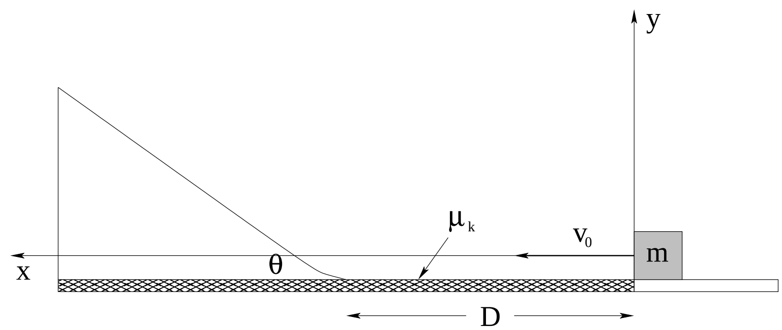
$$3mg\cos\theta_a = 2mg$$

or:

$$\theta_a = \boxed{\cos^{-1}\left(\frac{2}{3}\right)} = 48.2^\circ$$

Note that because the use of calculators is discouraged, this angle is perfectly well and uniquely (enough) expressed as an inverse cosine, but I put down the angle in degrees just for fun.

Problem 141. problems-1/wme-pr-smooth-inclined-plane-friction-table-2.tex

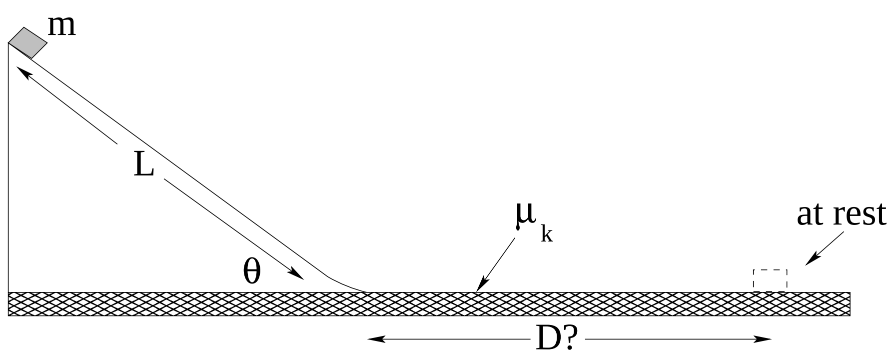


A block of mass m is given a push so that it begins sliding at a speed v_0 from the right to the left over a rough surface of length D leading up to a smooth (frictionless) incline. The incline makes an angle θ with the horizontal as shown. The coefficient of friction between the block and the rough surface is μ_k.

a) What is the minimum speed $v_{0,\text{min}}$ the block must have at the right-hand end of the rough surface such that the block will reach the bottom of the incline a distance D away?

b) Assuming that the block is travelling at some $v_0 > v_{0,\text{min}}$ when it starts at the right-hand end of the rough patch as drawn, how high (to what maximum height y_{max}) will the block slide up the incline (use the coordinate system given)?

Problem 142. problems-1/wme-pr-smooth-inclined-plane-friction-table.tex

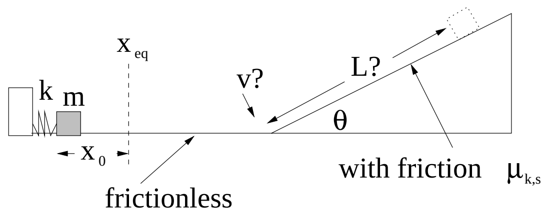


A block of mass m slides down a ***smooth*** (frictionless) incline of length L that makes an angle θ with the horizontal as shown. It then reaches a ***rough*** surface with a coefficient of kinetic friction μ_k.

a) How fast is the block going as it reaches the bottom of the incline?

b) What distance D does the block slide across the rough surface before coming to rest?

Problem 143. problems-1/wme-pr-spring-to-inclined-plane-friction-numbers.tex

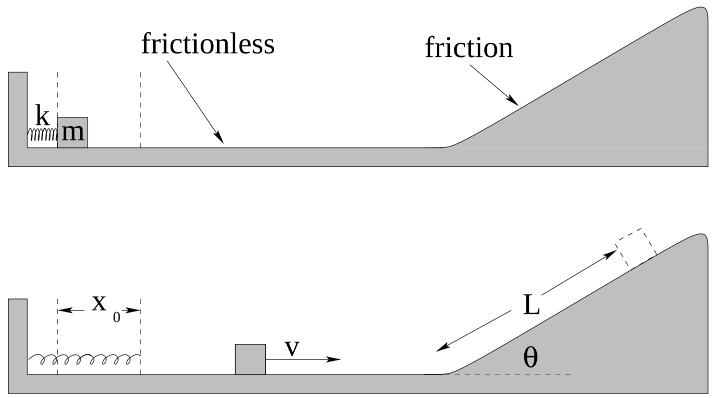


A block of mass $M = 1$ kg is propelled by a spring with spring constant $k = 10$ N/m onto a smooth (frictionless) track. The spring is initially compressed a distance of 0.5m from its equilibrium configuration ($x_i - x_0 = 0.5$ m). At the end of the track there is a rough inclined plane at an angle of 45∘ with respect to the horizontal and with a coefficient of kinetic friction $\mu_k = 0.5$.

a) How far up the incline will the block slide before coming to rest (find H_f)?

b) The coefficient of static friction is $\mu_s = 0.7$. Will the block remain at rest on the incline? If not, how fast will it be going when it reaches the bottom again?

Problem 144. problems-1/wme-pr-spring-to-inclined-plane-friction-icp.tex

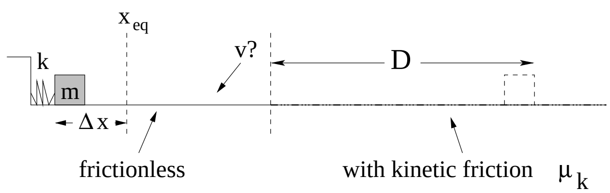


A spring with spring constant k is initially compressed a distance of x_0 from its equilibrium configuration as shown in the top diagram above. A block of mass m is placed against it and the spring is released, propelling it forward on a smooth (frictionless) track. At the end of the track there is a rough inclined plane at an angle of θ with respect to the horizontal and with a coefficient of kinetic friction μ_k.

a) How far up the incline will the block slide before coming momentarily to rest (find $L_{\max}$)?

b) Suppose the coefficient of static friction is μ_s. Find the maximum angle $\theta_{\max}$ such that the block will remain at rest at the top of the incline instead of sliding back down.

Problem 145. problems-1/wme-pr-spring-to-plane-friction.tex



A block of mass m sits against a spring with spring constant k that is initially compressed a distance Δx. At some time the block is released and slides across a frictionless surface until it reaches a *rough* surface with a coefficient of kinetic friction μ_k as shown.

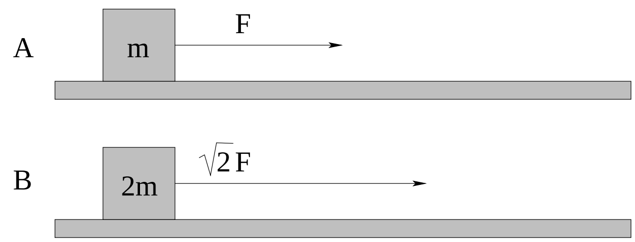
a) How fast is the block going as it leaves the spring at $x_{\rm eq}$?

b) What distance D down the rough surface does the block slide before coming to rest?

6.3 Power

6.3.1 Multiple Choice

Problem 146. problems-1/power-mc-two-blocks-hard.tex



In the figures **A** and **B** above a force of magnitude F is applied to mass m and a force of magnitude $\sqrt{2}F$ is applied to mass 2m as shown, where both masses are sitting ***initially at rest*** on a frictionless table. For all values of $t > 0$, identify the true statement in the list below:

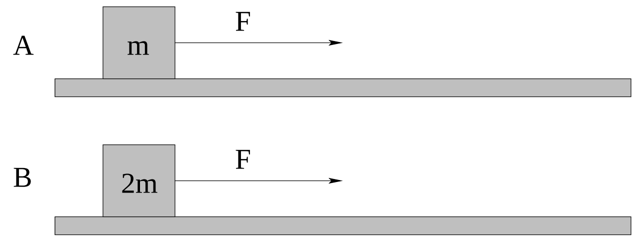
a) The power provided to block m in **A** is larger than that provided to block $2m$ in **B**, and mass m in **A** ends up with more kinetic energy at time t.

b) The power provided to the block $2m$ in **B** is larger than that provided to block m in **A**, but block m in **A** travels further in time t.

c) The power provided to both blocks is identical throughout this time, and they end up with the same final kinetic energy at time t.

d) It is impossible to tell from the information given which block receives more power from the forces.

Problem 147. problems-1/power-mc-two-blocks.tex



In the figures **A** and **B** above an identical magnitude of force is applied to two masses m and $2m$ respectively, sitting ***initially at rest*** on a frictionless table. For all values of $t > 0$ identify the true statement in the list below:

a) The power provided to both blocks is identical throughout this time, and they end up with the same final kinetic energy at time t.

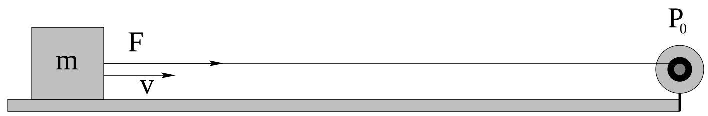
b) The power provided to block m in **A** is larger than that provided to block $2m$ in **B**, and mass m in **A** ends up with more kinetic energy at time t.

c) The power provided to the block $2m$ in **B** is larger than that provided to block m in **A**, but block m in **A** travels further in time t.

d) It is impossible to tell from the information given which block receives more power from the forces.

6.3.2 Long Problems

Problem 148. problems-1/power-pr-constant-power-v-of-t.tex



In the figure above, a mass m is pulled along on a frictionless table by a motor with ***constant power*** $\boldsymbol{P_0}$. At the instant shown, the mass has been previously accelerated to a speed v towards the motor.

a) Find F as a function of P_0 and v (in the direction of the motor).

b) Write Newton's second law for the mass m in terms of your answer to a), using $a = dv/dt$ for the acceleration.

c) Solve the equation of motion you get for $v(t)$, assuming that $v(0) = 0$.

d) Qualitatively sketch what you expect to get for $v(t)$ (or what you did get in the previous section). Note that you can do this one even if you fail to do the integral correctly, if you think about what happens to the force as the speed gets bigger and bigger.

Chapter 7

Center of Mass and Momentum

With basic dynamics and kinetics (force and energy) under our belts for massive particles taken one or two at a time (or even three or four at a time) we need to move on and see what happens when we treat ***lots*** of particles at a time – arbitrarily many. After all, eventually we want to understand solids that can rotate as well as translate, and fluids that, well, act like fluids. Neither one can be thought of *precisely* like a particle.

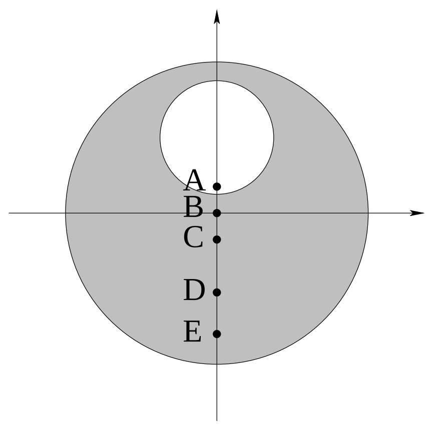
Or can they? As long as we consider a system of particles to be a single particle" located at its center of mass, nearly everything we've done so far ***can*** be applied to the entire system, even if the system is a liquid or a gas with many, many particles and no particular structure!

When do we want to use the concepts of center of mass and momentum conservation? Momentum conservation works for *isolated* systems (or not-so-isolated systems in the impulse approximation) with no net external force acting on it, especially to understand ***collisions***. Center of mass is a useful concept both then and when a collection of particles is being acted on in a uniform way by an external force (such as near-Earth gravity).

7.1 Center of Mass

7.1.1 Multiple Choice

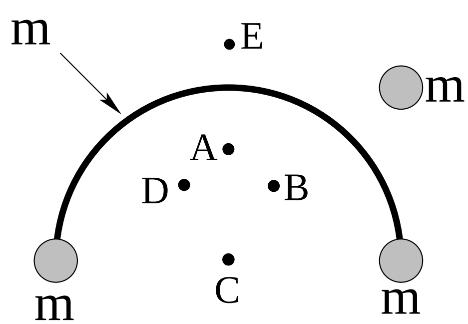
Problem 149. problems-1/center-of-mass-mc-disk-with-hole-icp.tex



A uniform circular disk has a circular hole cut out of it as shown above. Which letter represents the best estimate for the position of its new center of mass?

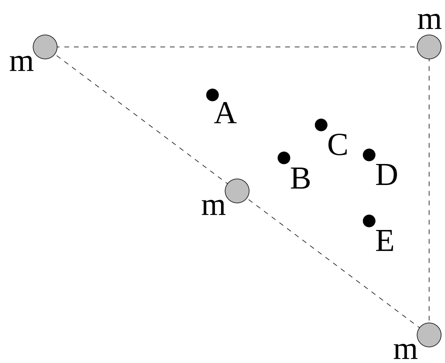
A **B** **C** **D** **E**

Problem 150. problems-1/center-of-mass-mc-particles-1.tex



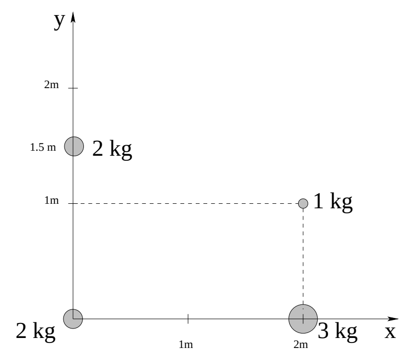
A collection of four equal masses m (including the uniform half-circle of wire) is shown above. Which of the points A-E is a plausible location of the center of mass?

Problem 151. problems-1/center-of-mass-mc-particles-2.tex



Pick (circle) the point A-E ***closest*** to the center of mass of the system above, given four equal masses m arranged as shown. Note that the dashed lines are drawn simply as a guide to the eye.

Problem 152. problems-1/center-of-mass-mc-particles-3.tex



In the figure above, various given masses (in kilograms) are located at the positions shown shown. The center of mass of this system is at:

a) $x = 5/4m$, $y = 1/2m$

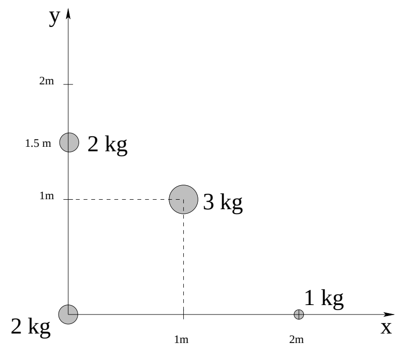
b) $x = 1m$, $y = 1/2m$

c) $x = 1/2m$, $y = 1/4m$

d) $x = 3/4m$, $y = 1m$

e) $x = 1/2m$, $y = 1m$

Problem 153. problems-1/center-of-mass-mc-particles-4.tex



In the figure above, various given masses (in kilograms) are located at the positions shown shown. The center of mass of this system is at:

a) $x = 5/4$ m, $y = 1/2$ m

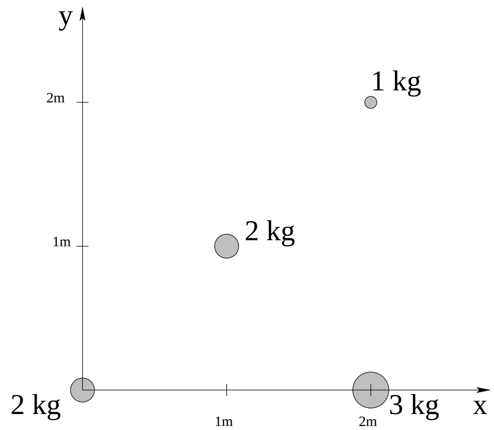
b) $x = 3/4$ m, $y = 5/8$ m

c) $x = 1/2$ m, $y = 1/4$ m

d) $x = 1$ m, $y = 1/2$ m

e) $x = 5/8$ m, $y = 3/4$ m

Problem 154. problems-1/center-of-mass-mc-particles-icp.tex

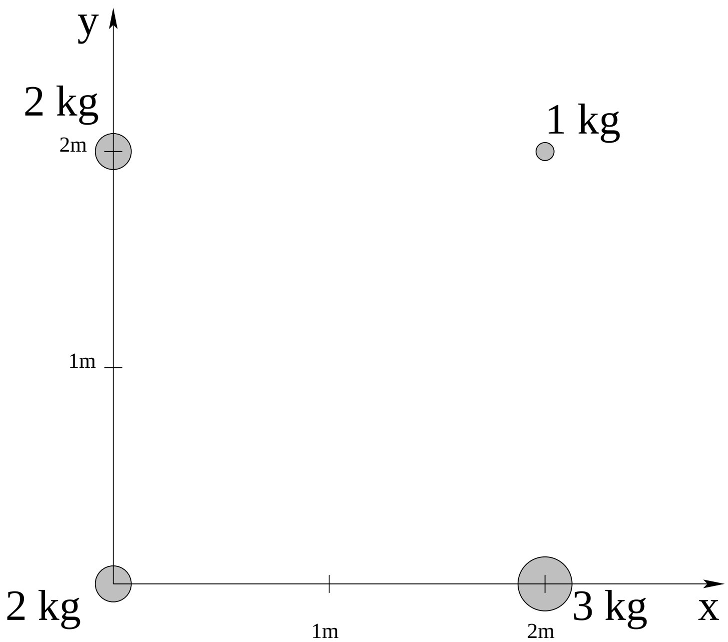


In the figure above, various given masses (in kilograms) are located at the corners of a square with sides of length 2 meters as shown. The center of mass of this system is at:

a) $x = 5/4m$, $y = 1/2m$ b) $x = 1m$, $y = 1/2m$ c) $x = 3/2m$, $y = 3/4m$

d) $x = 5/4m$, $y = 3/4m$ e) $x = 3/2m$, $y = 1/2m$

Problem 155. problems-1/center-of-mass-mc-particles-6.tex



In the figure above, various given masses (in kilograms) are located at the positions shown above. The center of mass of this system is at:

a) $x = 5/4\ m$, $y = 3/4\ m$

b) $x = 1\ m$, $y = 3/4\ m$

c) $x = 1\ m$, $y = 1\ m$

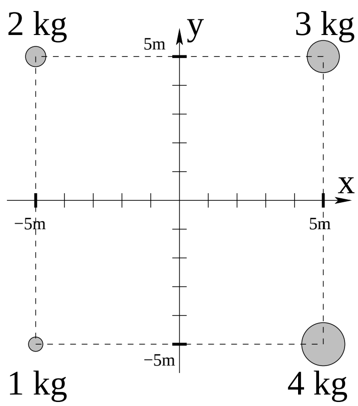
d) $x = 5/3\ m$, $y = 5/4\ m$

e) $x = 3/4\ m$, $y = 1\ m$

7.1.2 Short Answer

Problem 156. problems-1/center-of-mass-sa-particles.tex

figure In the figure below, various given masses (in kilograms) are located at the corners of a square with sides of length 10 meters as shown. Fill in the coordinates of the center of mass of this system below ***and*** place an "x" on the the graph at its location.

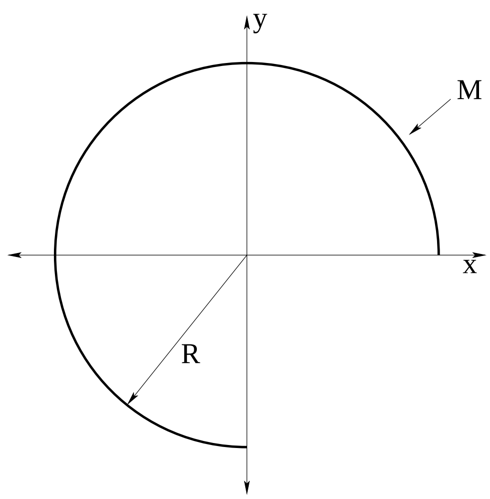


$x_{\mathrm{CoM}} =$

$y_{\mathrm{CoM}} =$

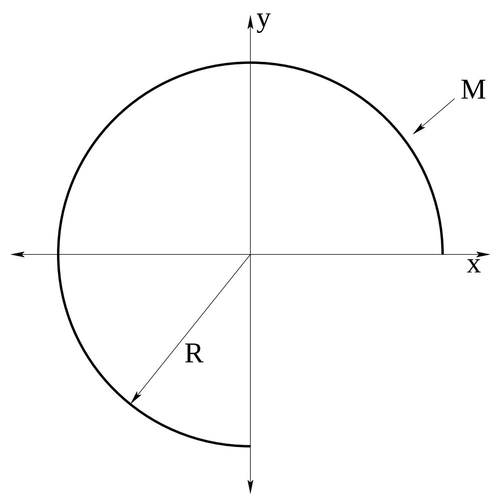
7.1.3 Long Problems

Problem 157. problems-1/center-of-mass-pr-arc-270-example.tex



In the figure above, a uniformly thick piece of wire is bent into 3/4 of a circular arc as shown. Find the center of mass of the wire in the coordinate system given, using integration to find the $x_{\rm cm}$ and $y_{\rm cm}$ components separately.

Problem 158. problems-1/center-of-mass-pr-arc-270-solution.tex



This problem will help you learn required concepts such as:

- Center of Mass
- Integrating a Distribution of Mass

so please review them before you begin.

In the figure above, a uniformly thick piece of wire is bent into 3/4 of a circular arc as shown. Find the center of mass of the wire in the coordinate system given, using integration to find the $x_{\rm cm}$ and $y_{\rm cm}$ components separately.

The uniform wire can be assumed to have a uniform *mass per unit length* μ. We will assume that the wire's thickness is far smaller than R so to compute the center of mass we can take the entire mass of the wire to lie along one circle (neglecting the different positions of various parts of the wire's cross-section).

We can then parameterize points on the wire using the angle θ from the positive x-axis as shown, just as we did for motion along a circle. The coordinates of points on the circle are, in Cartesian coordinates,

$$\mathbf{r} = R\cos(\theta)\hat{\mathbf{x}} + R\sin(\theta)\hat{\mathbf{y}} \ , \tag{7.1}$$

and the wire extends over the range $0 \le \theta \le 3\pi/2$. The length of the segment of wire represented by the angular interval from θ to $\theta + d\theta$ is μ times the length $Rd\theta$ of the interval, i.e.

$$dM = R\mu d\theta \ . \tag{7.2}$$

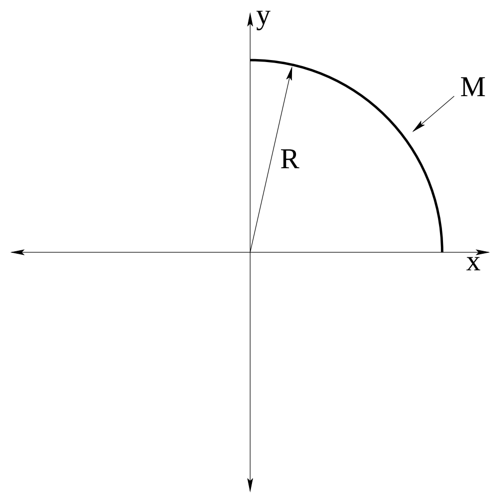
The total mass of the wire is thus

$$M = \int dM = \int_0^{3\pi 2} R\mu d\theta = \frac{3\pi}{2}\mu R \ . \tag{7.3}$$

The center of mass position is then

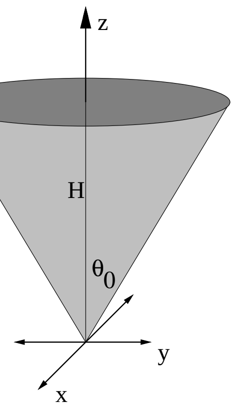
$$
\begin{aligned}
\mathbf{r}_{\mathrm{COM}} &= (1/M)\int \mathbf{r}dm \\
&= (1/M)\int_0^{3\pi/2} \left(R\cos(\theta)\hat{\mathbf{x}} + R\sin(\theta)\hat{\mathbf{y}}\right) R\mu d\theta \\
&= \frac{R^2\mu}{M}\int_0^{3\pi/2} \left(\cos(\theta)\hat{\mathbf{x}} + \sin(\theta)\hat{\mathbf{y}}\right) d\theta \\
&= \frac{2R}{3\pi}\left(-\hat{\mathbf{x}} + \hat{\mathbf{y}}\right) \ .
\end{aligned}
$$

Problem 159. problems-1/center-of-mass-pr-arc-90.tex



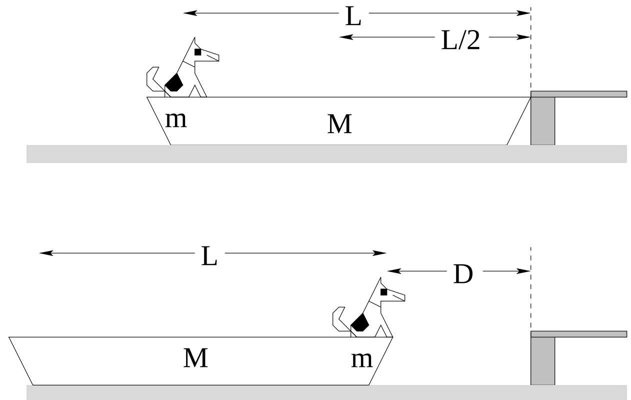
In the figure above, a uniformly thick piece of wire is bent into 1/4 of a circular arc as shown. Find the center of mass of the wire in the coordinate system given, using integration to find the $x_{\rm cm}$ and $y_{\rm cm}$ components separately.

Problem 160. problems-1/center-of-mass-pr-circular-cone.tex



Above is drawn a circular cone with uniform mass density ρ. The cone side makes an angle θ_0 with the positive z axis. The cone height is H. Find the center of mass of the cone in terms of the quantities given above. Hint: Consider circular slabs of thickness dz a height z above the origin.

Problem 161. problems-1/center-of-mass-pr-dog-in-a-boat.tex

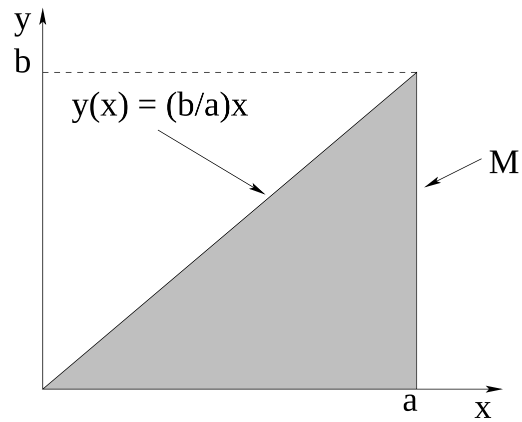


A dog of mass m is sitting at one end of a boat of mass M and length L that is sitting next to a dock as shown. The dog decides he wants some tasty dog chunks that are waiting for him at home and walks to the other end of the boat, expecting to step out onto the dock. Sadly, when he gets there he finds himself a distance D away from the dock.

a) What is D in terms of m, M, and L. You may assume that the boat is symmetric, so that its center of mass is at $L/2$, although this is not strictly necessary to get the answer.

b) The dog can successfully jump to the dock from the boat if $D < L/2$, but otherwise he'll have to swim. Find the ratio m/M for which the dog (first, barely) can't make the leap and has to take a bath to get to the chunks.

Problem 162. problems-1/center-of-mass-pr-right-triangle.tex



In the figure above, a uniformly thick sheet of plastic is cut into the $a \times b$ right triangle shown. Find the center of mass of the triangle in the coordinate system given, using integration to find the $x_{\rm cm}$ and $y_{\rm cm}$ components separately.

Suggested solution strategy:

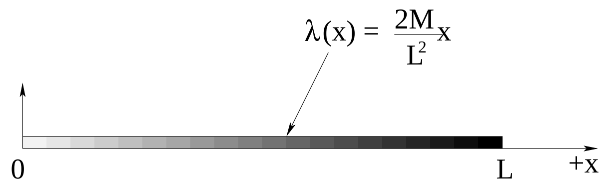
a) Form $\sigma = M/A$ where A is the area of the triangle.

b) Form $dm = \sigma\ dA$ where $dA = dx\ dy$.

c) Do the integrals $\int x\ dm$ and $\int y\ dm$ separately, using the provided functional form of the hypotenuse to set up the limits of integration in both cases.

d) Divide out the M to obtain $x_{\rm cm}$ and $y_{\rm cm}$.

Problem 163. problems-1/center-of-mass-pr-rod-variable-lambda-icp.tex



In the figure above a rod of total mass M and length L is portrayed (with shading that increases with mass density) that has been machined so that it has a mass per unit length that increases *linearly* along the length of the rod:

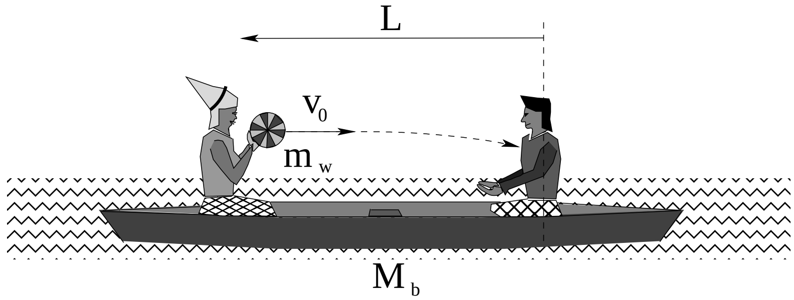
$$\lambda(x) = \frac{2M}{L^2}x$$

This might be viewed as a very crude model for the way mass is distributed in something like a human leg or a baseball bat. The rod is so thin that $y_{\rm cm} = z_{\rm cm} \approx 0$ by inspection.

a) verify that the total mass of the rod is indeed M for this mass distribution;

b) find $x_{\rm cm}$, the x-coordinate of the center of mass of the rod.

Problem 164. problems-1/center-of-mass-pr-romeo-and-juliet.tex



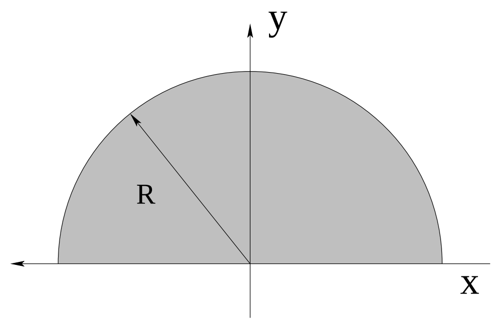
Romeo and Juliet are out in their damn boat again, this time for a picnic on the lake. The boat is ***initially at rest***. Juliet decides she wants a piece of tasty watermelon, and throws the watermelon at horizontal speed v_0 to Romeo at the other end of the boat a distance L away so he can cut her a piece with his ever-handy bodkin (dagger). The combined mass of Romeo, Juliet and the boat is M_b; the mass of the watermelon is m_w. Assume that the boat can move horizontally on the water without drag or friction.

a) What is the horizontal speed of the boat while the watermelon is in the air (neglect its vertical motion – assume that Juliet has thrown it on a flat trajectory as shown).

b) What is the horizontal speed of the boat after Romeo catches the watermelon?

c) How long is the watermelon in the air?

Problem 165. problems-1/center-of-mass-pr-semicircular-sheet.tex



Find the center of mass of the two-dimensional semicircular sheet drawn above. It has a uniform mass per unit area σ and radius R. You may invoke symmetry for one of the two vector components of the center of mass location.

7.2 Momentum

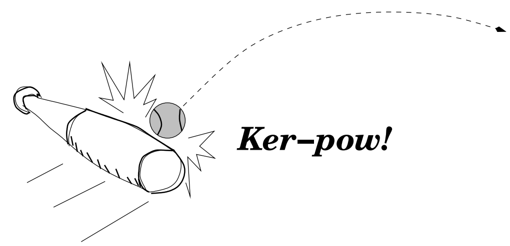
7.2.1 Multiple Choice

Problem 166. problems-1/momentum-mc-also-energy-football-icp.tex

Two football players, one large (L – bigger mass) and one small (S – smaller mass) are running in a straight line directly at one another. They have the ***same magnitude of momentum***. Rank their mechanical energies and speeds right ***before*** they collide.

a) $E_S < E_L$, $v_S < v_L$ b) $E_S < E_L$, $v_S > v_L$ c) $E_S > E_L$, $v_S > v_L$ d) $E_S > E_L$, $v_S < v_L$

Problem 167. problems-1/momentum-mc-baseball-and-bat-icp.tex



Michelle is playing baseball and hits a home run with a solid wood bat (mass of 3 kg). The baseball (mass of 0.5 kg) is knocked clean out of the park. The ***magnitude*** of the force exerted by the bat on the baseball is:

a) Greater than the magnitude of the force exerted by the baseball on the bat.

b) Less than the magnitude of the force exerted by the baseball on the bat;

c) The same as the magnitude of the force exerted by the baseball on the bat.

Problem 168. problems-1/momentum-mc-cement-truck-and-bug.tex

A cement truck with a mass M_t is travelling at speed v_t collides with a bug of mass m_b that is hovering above the road (so $v_b \approx 0$). One can safely assume that $M_t \gg m_b$. Which of the following statements are unambiguously true (circle all definitely true statements)?

a) If the bug recoils off of the windshield ***elastically***, its final speed is roughly $2v_t$ (in the same direction as the truck).

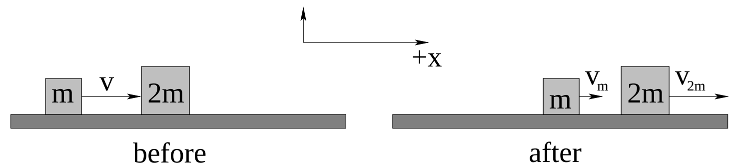
b) The magnitude of the momentum change of the truck is much smaller than the magnitude of the momentum change of the bug.

c) If the bug splatters and sticks to the windshield of the truck, the total kinetic energy of the bug and truck will be conserved.

d) At all times during the collision, the bug exerts exactly the same magnitude of force on the truck that the truck exerts on the bug.

e) The final speed of the bug as it recoils off of the windshield is roughly $\frac{M_t}{m_b} v_t$ (in the same direction as the truck).

Problem 169. problems-1/momentum-mc-elastic-collision-blocks-2.tex



A block of mass m slides on a frictionless table at velocity $\vec{\boldsymbol{v}} = v\hat{\boldsymbol{x}}$ to the right (positive x-direction) to collide with a block of mass $2m$ initially ***at rest*** as shown. Assuming that the collision is ***one dimensional*** and ***elastic***, the velocities of the two blocks after the collision are:

a) $\vec{\boldsymbol{v}}_m = -\frac{v}{2}\hat{\boldsymbol{x}}$ $\quad \vec{\boldsymbol{v}}_{2m} = \frac{3v}{4}\hat{\boldsymbol{x}}$

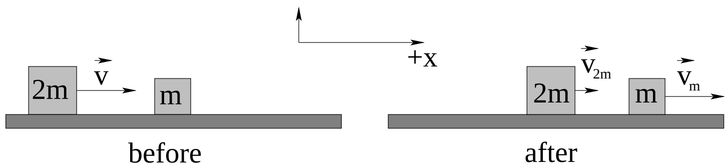
b) $\vec{\boldsymbol{v}}_m = 0\hat{\boldsymbol{x}}$ $\quad \vec{\boldsymbol{v}}_{2m} = \frac{v}{6}\hat{\boldsymbol{x}}$

c) $\vec{\boldsymbol{v}}_m = -\frac{v}{3}\hat{\boldsymbol{x}}$ $\quad \vec{\boldsymbol{v}}_{2m} = \frac{4v}{3}\hat{\boldsymbol{x}}$

d) $\vec{\boldsymbol{v}}_m = \frac{v}{3}\hat{\boldsymbol{x}}$ $\quad \vec{\boldsymbol{v}}_{2m} = \frac{2v}{3}\hat{\boldsymbol{x}}$

e) $\vec{\boldsymbol{v}}_m = -\frac{v}{3}\hat{\boldsymbol{x}}$ $\quad \vec{\boldsymbol{v}}_{2m} = \frac{2v}{3}\hat{\boldsymbol{x}}$

Problem 170. problems-1/momentum-mc-elastic-collision-blocks.tex



A block of mass $2m$ slides on a frictionless table at velocity $\vec{\boldsymbol{v}} = v\hat{\boldsymbol{x}}$ to the right (positive x-direction) to collide with a block of mass m initially ***at rest*** as shown. Assuming that the collision is ***one dimensional*** and ***elastic***, the velocities of the two blocks after the collision are:

a) $\vec{\boldsymbol{v}}_{2m} = \frac{v}{2}\hat{\boldsymbol{x}}$ $\quad\quad$ $\vec{\boldsymbol{v}}_m = \frac{3v}{2}\hat{\boldsymbol{x}}$

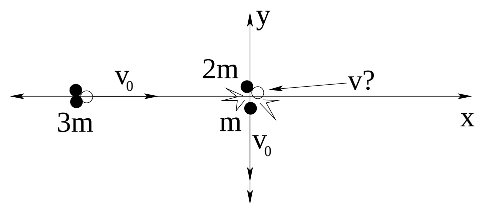
b) $\vec{\boldsymbol{v}}_{2m} = 0\hat{\boldsymbol{x}}$ $\quad\quad$ $\vec{\boldsymbol{v}}_m = \sqrt{2}v\hat{\boldsymbol{x}}$

c) $\vec{\boldsymbol{v}}_{2m} = -\frac{v}{3}\hat{\boldsymbol{x}}$ $\quad\quad$ $\vec{\boldsymbol{v}}_m = \frac{4v}{3}\hat{\boldsymbol{x}}$

d) $\vec{\boldsymbol{v}}_{2m} = \frac{v}{3}\hat{\boldsymbol{x}}$ $\quad\quad$ $\vec{\boldsymbol{v}}_m = \frac{4v}{3}\hat{\boldsymbol{x}}$

e) The answers above are all incorrect.

Problem 171. problems-1/momentum-mc-fission-1.tex



An atomic nucleus of mass $3m$ is travelling to the right at velocity $\vec{\boldsymbol{v}}_{\text{initial}} = v_0\hat{\boldsymbol{\imath}}$ as shown. It spontaneously fissions into two fragments of mass m and $2m$. The smaller fragment m travels straight down at velocity $\vec{\boldsymbol{v}}_m = -v_0\hat{\boldsymbol{\jmath}}$ after the fission. What is the velocity of the larger fragment?

a) $\vec{\boldsymbol{v}}_{2m} = \frac{3}{2}v_0\hat{\boldsymbol{\imath}}$

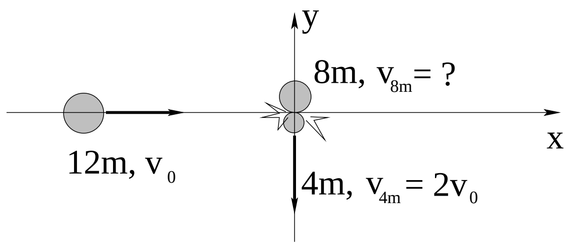
b) $\vec{\boldsymbol{v}}_{2m} = 2v_0\hat{\boldsymbol{\jmath}}$

c) $\vec{\boldsymbol{v}}_{2m} = \frac{3}{2}v_0\hat{\boldsymbol{\imath}} + \frac{1}{2}v_0\hat{\boldsymbol{\jmath}}$

d) $\vec{\boldsymbol{v}}_{2m} = -\frac{3}{2}v_0\hat{\boldsymbol{\imath}} - \frac{1}{2}v_0\hat{\boldsymbol{\jmath}}$

e) $\vec{\boldsymbol{v}}_{2m} = 3v_0\hat{\boldsymbol{\imath}} + 2v_0\hat{\boldsymbol{\jmath}}$

Problem 172. problems-1/momentum-mc-fission-2.tex



An atomic nucleus of mass $12m$ is travelling to the right at velocity $\vec{\boldsymbol{v}}_{\text{initial}} = v_0\hat{\boldsymbol{x}}$ as shown. It spontaneously fissions into two fragments of mass $4m$ and $8m$ (releasing energy). The smaller fragment $4m$ travels straight down at velocity $\vec{\boldsymbol{v}}_{4m} = -2v_0\hat{\boldsymbol{y}}$ after the fission. What is the velocity of the larger fragment?

a) $\vec{\boldsymbol{v}}_{8m} = \frac{1}{2}v_0\hat{\boldsymbol{x}} + v_0\hat{\boldsymbol{y}}$

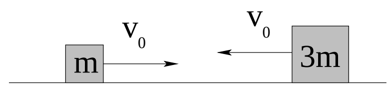
b) $\vec{\boldsymbol{v}}_{8m} = 2v_0\hat{\boldsymbol{x}} + 2v_0\hat{\boldsymbol{y}}$

c) $\vec{\boldsymbol{v}}_{8m} = -\frac{3}{2}v_0\hat{\boldsymbol{x}} - v_0\hat{\boldsymbol{y}}$

d) $\vec{\boldsymbol{v}}_{8m} = \frac{3}{2}v_0\hat{\boldsymbol{x}} + v_0\hat{\boldsymbol{y}}$

e) $\vec{\boldsymbol{v}}_{8m} = \frac{3}{2}v_0\hat{\boldsymbol{x}} + 2v_0\hat{\boldsymbol{y}}$

Problem 173. problems-1/momentum-mc-simple-inelastic-collision-icp.tex



A mass m travelling at (one-dimensional) velocity v_0 to the right collides with mass $3m$ travelling at velocity $-v_0$ to the left and sticks to it. The final velocity v_f of the blocks after the collision is:

a) $v_f = -2v_0$ b) $v_f = v_0/2$ c) $v_f = -v_0$ d) $v_f = -2v_0/3$ e) $v_f = -v_0/2$

Problem 174. problems-1/momentum-mc-two-masses-spring-2.tex

Two masses, m and $3m$, are separated by a compressed spring as shown above. At time $t = 0$ they are released from rest and the (massless) spring expands. There is no gravity or friction. As they move apart, which statement about the **magnitude** of each mass's relative kinetic energy K_i and the **magnitude** of each mass's relative momentum p_i is true?

a) $K_m = 3K_{3m}, \quad p_m = p_{3m}/3$

b) $K_m = K_{3m}/3, \quad p_m = 3p_{3m}$

c) $K_m = K_{3m}/3, \quad p_m = p_{3m}$

d) $K_m = 3K_{3m}, \quad p_m = p_{3m}$

e) $K_m = K_{3m}/4, \quad p_m = p_{3m}$

Problem 175. problems-1/momentum-mc-two-masses-spring.tex

Two masses, $m_2 = 2m_1$ are separated by a compressed spring. At time $t = 0$ they are released from rest and the (massless) spring expands. There is no gravity or friction. As they move apart, which statement about the magnitude of each mass's kinetic energy K_i and momentum p_i is true?

a) $K_1 = 2K_2$, $p_1 = 2p_2$

b) $K_1 = 2K_2$, $p_1 = p_2$

c) $K_1 = K_2$, $p_1 = 2p_2$

d) $K_1 = K_2/2$, $p_1 = p_2/2$

e) $K_1 = K_2/4$, $p_1 = p_2$

Problem 176. problems-1/momentum-mc-two-trucks-collide.tex

A fully laden dump truck (mass of maybe 10,000 kg) slams into a small pickup truck (mass around 2,000 kg). The two trucks exert a collision force on one another, and momentum is transferred during the short collision.

Let $F_D, \Delta p_D, a_D$ represent the magnitude of the force exerted ***by*** the dump truck on the pickup truck, the magnitude of the dump truck's momentum change, and the magnitude of the dump truck's average acceleration during the collision. Let $F_p, \Delta p_p, a_p$ represent the magnitude of the force exerted ***by*** the pickup truck on the dump truck, the magnitude of the pickup truck's momentum change, and the magnitude of the pickup truck's average acceleration during the collision.

Select the correct/true description of these magnitudes below:

a) $F_D = F_p$, $\Delta P_D > \Delta P_p$, $a_D < a_p$

b) $F_D > F_p$, $\Delta P_D = \Delta P_p$, $a_D < a_p$

c) $F_D = F_p$, $\Delta P_D > \Delta P_p$, $a_D > a_p$

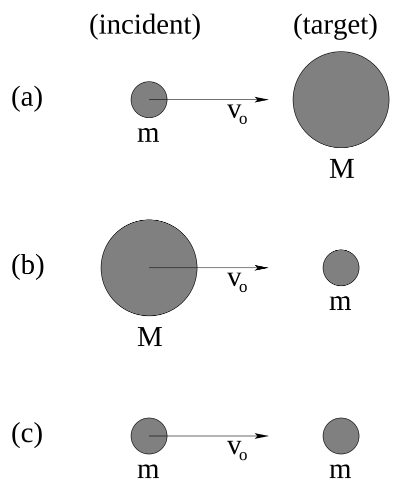
d) $F_D = F_p$, $\Delta P_D = \Delta P_p$, $a_D < a_p$

e) $F_D = F_p$, $\Delta P_D = \Delta P_p$, $a_D = a_p$

f) None of the above (enter the correct answer here):

7.2.2 Short Answer

Problem 177. problems-1/momentum-sa-elastic-recoil.tex



In the three figures above, mass $M > m$. The mass on the left is incident at speed v_0 on the target mass (initially at rest in all three cases) on the right . The two particles undergo an **elastic** collision in one dimension and the target mass recoils to the **right** in all three cases. In the spaces provided below you are asked to provide a *qualitative* estimate of the speed *and* direction of the *incident* particle **after the collision**.

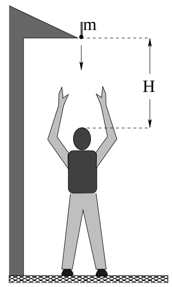
Your answer should be given relative to v_0 and should look like "$v_x > v_0$, to the left" or "$v_x = 0$" or $v_x < v_0$, to the right" where $x = a, b, c$. In other words, specify the speed qualitatively compared to v_0 and then the direction, per figure.

a)

b)

c)

Problem 178. problems-1/momentum-sa-hammer-impacts-head.tex



A hammer of mass m falls from rest off of a roof and drops a height H onto your head. Ouch!

a) Assuming that the tool is in actual contact with your head for a time Δt before it stops (thud!) and slides off, what is the algebraic expression for the average force it exerts on your hapless skull while ***stopping***?

b) *Estimate* the magnitude of this force using $m = 1$ kg, $H = 1.25$ meters, $\Delta t = 10^{-2}$ seconds and $g = 10\frac{\text{m}}{\text{sec}^2}$. Compare this force to the weight of the hammer of 10 Newtons!

Problem 179. problems-1/momentum-sa-impact-orc-spears-frodo.tex

An Orc throws a 2 kg spear at Frodo Baggins at point blank range, but it is stopped by his hidden *mithril* mail shirt. Assuming that the spear was travelling at 20 m/sec when it hit and that it stopped in 0.1 seconds, what was the average force exerted on the spear by the mail coat (and the hobbit underneath)? Ouch!

Problem 180. problems-1/momentum-sa-shark-eats-fish.tex

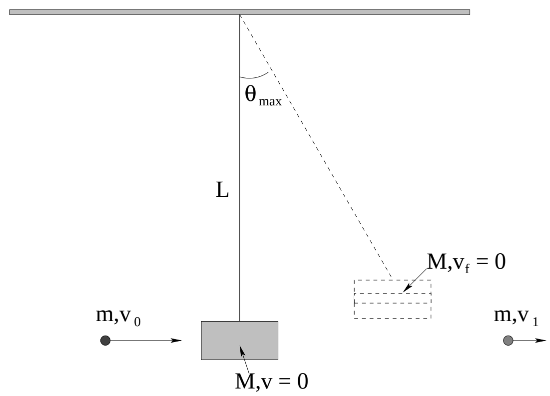
A great white shark of mass m_1, coasting through the water in a nearly frictionless way at speed v_1, engulfs a tuna of mass $m_2 < m_1$ travelling in the same direction at speed $v_2 < v_1$, swallowing it in one bite.

a) What is the speed of the shark after its tasty meal, sadly eaten without wasabi (mmm, sashimi!)?

b) Did the shark gain (kinetic) energy, lose energy, or have its energy remain the same in the process.

7.2.3 Long Problems

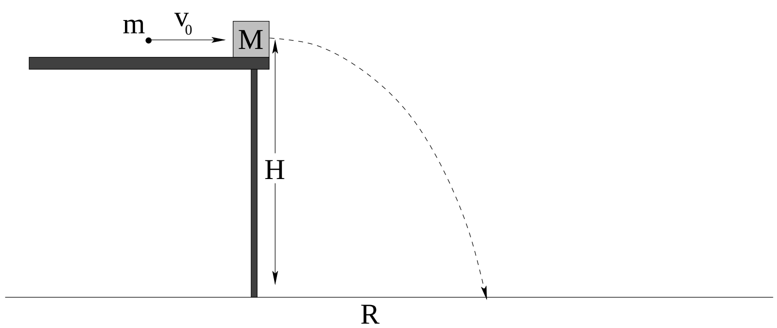
Problem 181. problems-1/momentum-pr-ballistic-pendulum-partially-inelastic.tex



In the figure above, a bullet of mass m and initial velocity v_0 passes through a block of mass M suspended by an unstretchable, massless string of length L from an overhead support as shown. It emerges from the **collision** on the far side travelling at $v_1 < v_0$. This happens extremely quickly (before the block has time to swing up) and the mass of the block is **unchanged** by the passage of the bullet (the mass removed making the hole is negligible, in other words). After the collision, the block swings up to a maximum angle $\theta_{\max}$ and then stops.

Find $\theta_{\max}$.

Problem 182. problems-1/momentum-pr-bullet-block-free-fall-1.tex



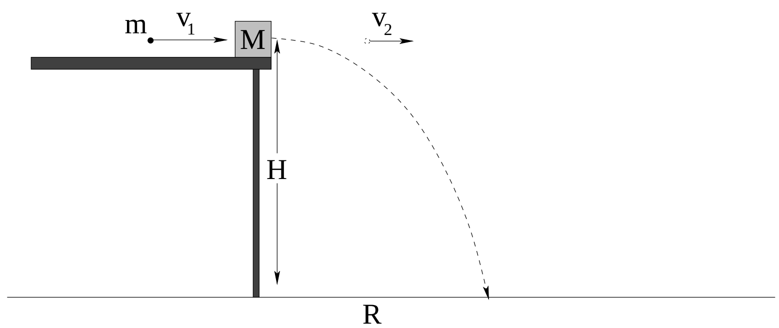
A bullet of mass m travelling at speed v_0 in the direction shown above strikes a block of mass M and embeds itself in it. The block is sitting on the edge of a frictionless table of height H and is knocked off of the table by the collision.

a) What is the speed v_b of the block immediately after the bullet sticks?

b) What distance R from the base of the table does the block land?

Note: If you cannot solve a), just use the symbol v_b where needed to get possibly full credit for b). Do not just use a memorized formula for b): Clearly state the physical principle(s) you are using and work out the answers.

Problem 183. problems-1/momentum-pr-bullet-block-free-fall-2.tex



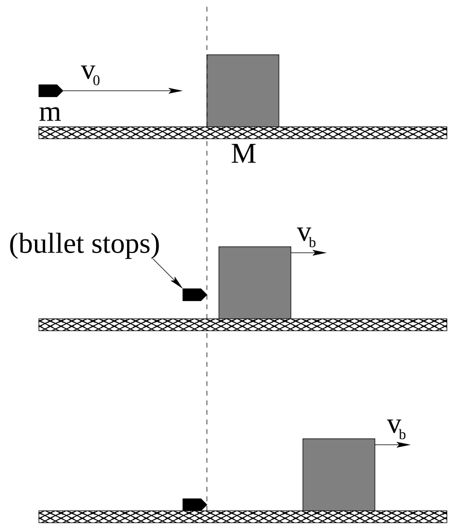
A bullet of mass m travelling at speed v_1 in the direction shown above strikes a block of mass M and passes through, emerging at speed v_2. The block is sitting on the edge of a frictionless table of height H and is knocked off of the table by the collision.

a) What is the speed v_b of the block immediately after the bullet emerges?

b) What distance R from the base of the table does the block land?

Note: If you cannot solve a), just use the symbol v_b where needed to get possibly full credit for b). Do not just use a memorized formula for b): Clearly state the physical principle(s) you are using and work out the answers.

Problem 184. problems-1/momentum-pr-bullet-stops-at-block.tex



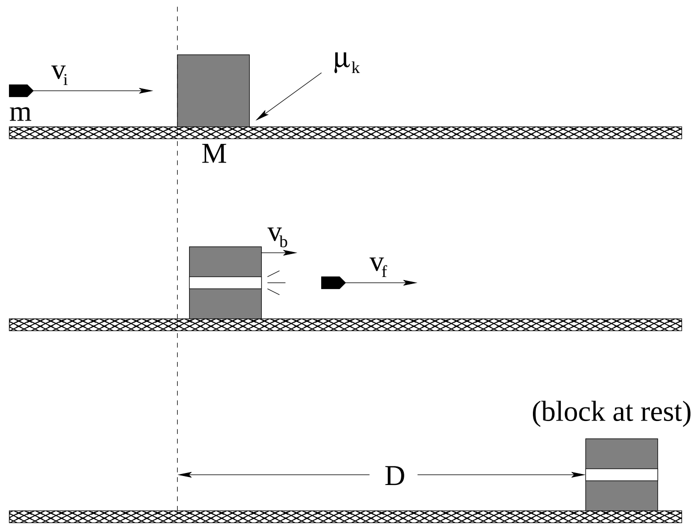
In the figure above a bullet of mass m is travelling at initial speed v_0 to the right when it strikes a larger block of mass M that is resting on a smooth (frictionless) horizontal table.

Instead of "sticking" in the block, the bullet is stopped cold by the block and falls to the ground, while the block recoils from the collision to the right. Note that this collision is *partially inelastic*, so *some* mechanical energy will be lost.

a) What is the velocity of the block v_b immediately after the collision.

b) How much energy is lost in the collision?

Problem 185. problems-1/momentum-pr-bullet-through-block-rough-surface.tex



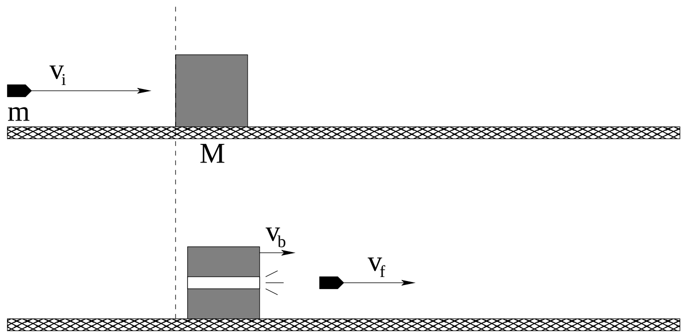
In the figure above a bullet of mass m is travelling at initial speed v_i to the right when it strikes a larger block of mass M that is resting on a rough horizontal table (with coefficient of friction between block and table of μ_k). Instead of "sticking" in the block, the bullet blasts its way *through* the block (without changing the mass of the block significantly in the process). It emerges with the smaller speed v_f, still to the right.

a) Find the speed of the block v_b *immediately after* the collision (but before the block has had time to slide any significant distance on the rough surface).

b) Find the (kinetic) energy lost during this collision. Where did this energy go?

c) How far down the rough surface D does the block slide before coming to rest?

Problem 186. problems-1/momentum-pr-bullet-through-block.tex

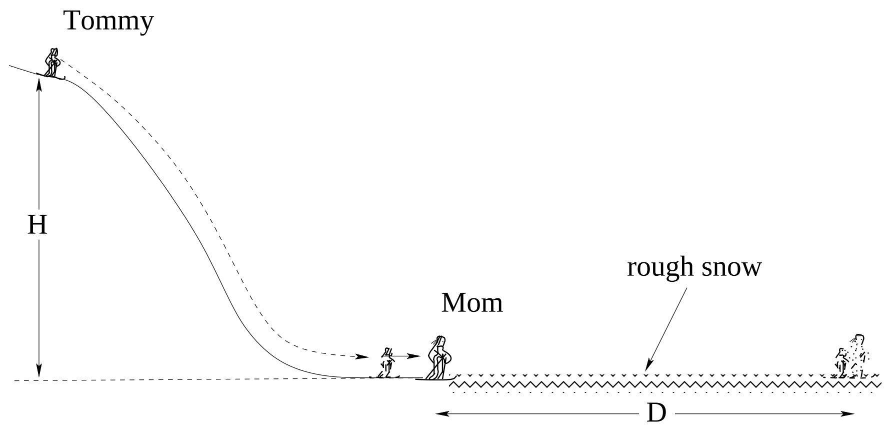


In the figure above a bullet of mass m is travelling at initial speed v_i to the right when it strikes a larger block of mass M that is resting on a horizontal table. Instead of "sticking" in the block, the bullet blasts its way *through* the block (without changing the mass of the block significantly in the process). It emerges with the smaller speed v_f, still to the right.

a) What is the velocity of the block v_b immediately after the collision.

b) How much energy is lost in the collision?

Problem 187. problems-1/momentum-pr-collision-icy-hill-to-rough-snow.tex



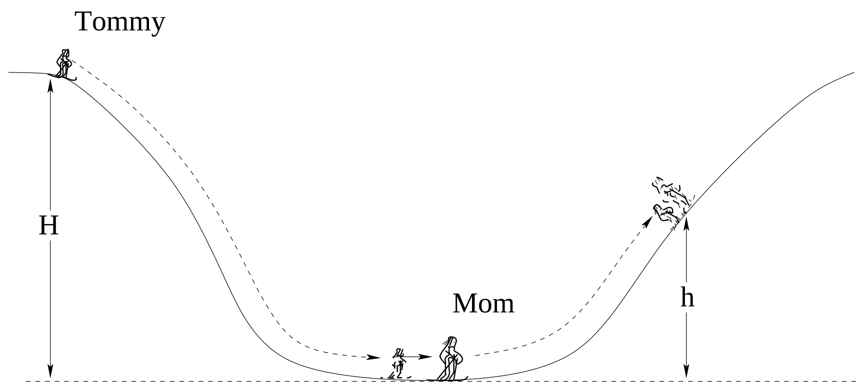
Tommy is learning to ski, but he isn't very good at it. Starting from rest, he skiis down a ***frictionless*** slope of height H above a flat plane, where he runs into Mom. The two of them stick together and slide forward a distance D across a patch of rough snow with coefficient of kinetic friction μ_k until they come to rest. The mass of Tommy is m_t, the mass of his mother is M_m. Answer the following algebraic questions in terms of H, μ_k, m_t, M_m, and g:

a) How fast is Tommy going immediately before he collides with his mother?

b) Find D.

c) How much energy is gained or lost ***during the collision*** between Tommy and his mother? Indicate clearly whether the energy is gained or lost.

Problem 188. problems-1/momentum-pr-collision-on-icy-hills.tex



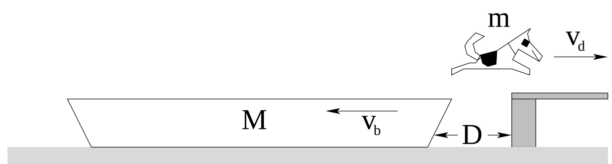
Tommy is learning to ski, but he isn't very good at it. Starting from rest, he skiis down a ***frictionless*** slope of height H above a small valley, where he runs into Mom. The two of them stick together and slide up the slope on the far side to a new height h. The mass of Tommy is m_t, the mass of his mother is M_m. ***Ignore all drag and friction***, and answer the following algebraic questions in terms of H, m_t, M_m, and g:

a) How fast is Tommy going immediately before he collides with his mother?

b) Find h.

c) How much energy is gained or lost ***during the collision*** between Tommy and his mother? Indicate clearly whether the energy is gained or lost.

Problem 189. problems-1/momentum-pr-dog-jumps-from-boat.tex

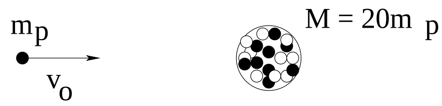


A dog of mass m gets hungry while ***sitting*** at the end of a boat of mass M and length L that is ***at rest*** on the water of a lake. He jumps out onto the dock to go get some tasty dog chunks that are waiting for him at home when the boat is a distance D away from the dock as shown. The dog travels at a horizontal speed v_d ***relative to the ground/lake*** as he flies through the air.

a) What is the recoil speed of the boat, v_b, while the dog is in the air? Assume that dog and boat are both at rest before the jump.

b) How much work did the dog's legs do during the jump?

Problem 190. problems-1/momentum-pr-elastic-collision-proton-neon.tex



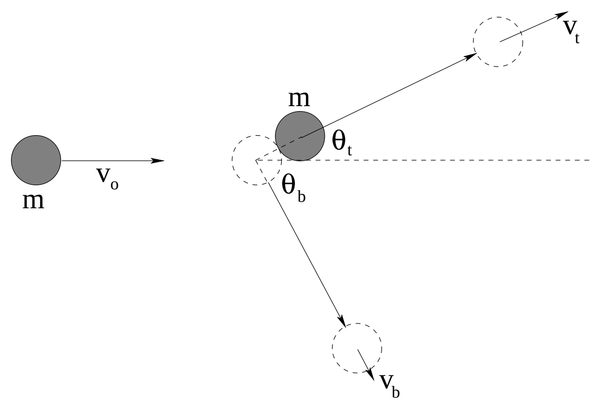
A Proton of mass m_p is directly incident on a Neon nucleus with mass $20m_p$. It is initially (far away from the nucleus) travelling with speed v_0. The two particles repel each other (like charges repel) as they approach, and the force of repulsion is strong enough to prevent the particles from touching. The "collision" that takes place gradually between the two particles is elastic.

a) At some distant time in the future (after the collision) is the proton moving to the left or to the right?

b) What is the speed of the proton when it and the Neon nucleus are at the distance of closest approach?

c) What is the speed of the Neon nucleus at a distant time in the future (after the collision) when they are once again far apart.

Problem 191. problems-1/momentum-pr-elastic-two-balls.tex



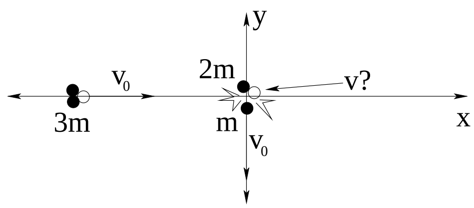
In the figure above, a ball with mass $m = 1kg$ and speed $v_0 = 5$ m/sec ***elastically*** collides with a stationary, identical ball (all resting on a frictionless surface so gravity is irrelevant). A student measures the top ball emerging from the collision at a speed $v_t = 4$ m/sec at an angle $\theta_t \approx 37^\circ$ as shown.

a) Find the speed v_b of the other ball.

b) Find the angle θ_b of the other ball. (Hint: Draw a triangle with sides of length v_0, v_t, v_b.)

c) What does $\theta_t + \theta_b$ add up to? (This is a characteristic of all elastic collisions between identical masses in 2 dimensions.)

Problem 192. problems-1/momentum-pr-fission.tex

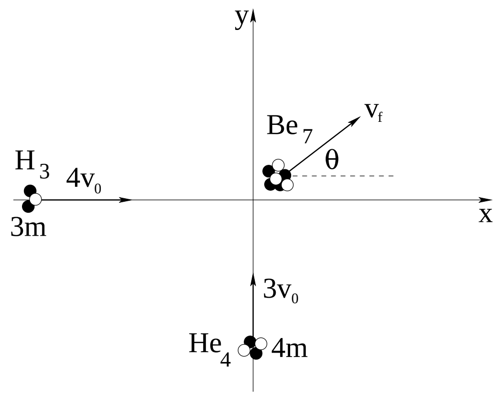


An atomic nucleus of mass $3m$ is travelling to the right at velocity $\vec{\boldsymbol{v}}_{\mathrm{initial}} = v_0\hat{\boldsymbol{x}}$ as shown. It spontaneously fissions into two fragments of mass m and $2m$. The smaller fragment m travels straight down at velocity $\vec{\boldsymbol{v}}_m = -v_0\hat{\boldsymbol{y}}$ after the fission.

a) What is the ***velocity*** of the larger fragment?

b) What is the ***net*** energy ***gain or loss*** (indicate which!) from the fission process, in terms of the initial kinetic energy?

Problem 193. problems-1/momentum-pr-h3-he4-fusion.tex



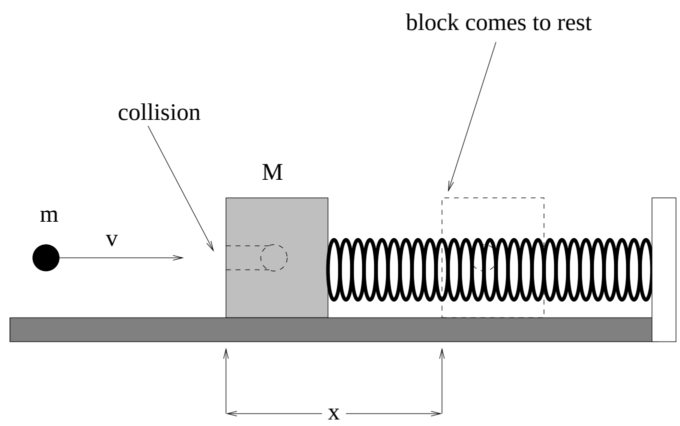
In the figure above, a Tritium (H_3) nucleus and a Helium (He_4) nucleus collide and fuse inelastically into Beryllium (Be_7), an important nucleosynthesis process in the early Universe. The velocity of the H_3 is $4v_0\hat{\boldsymbol{i}}$, the velocity of the He_4 is $3v_0\hat{\boldsymbol{j}}$ as drawn. **Show your work and reasoning** to answer the following questions in terms of the given quantities m and v_0:

a) Find the final velocity vector of the combined object, expressed using vector notation (e.g. $\vec{A} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}}$);

b) Find the magnitude of the final velocity v_f and its angle θ with respect to the x-direction;

c) Find the change of momentum vectors $\Delta\vec{\boldsymbol{p}}_H$ for the H_3 nucleus ***and*** $\Delta\vec{\boldsymbol{p}}_{He}$ for the He_4 nucleus. (Recall that $\Delta\vec{\boldsymbol{p}} == \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i$.) Briefly discuss how they are related and what this means.

Problem 194. problems-1/momentum-pr-inelastic-collision-ball-bearing-spring.tex

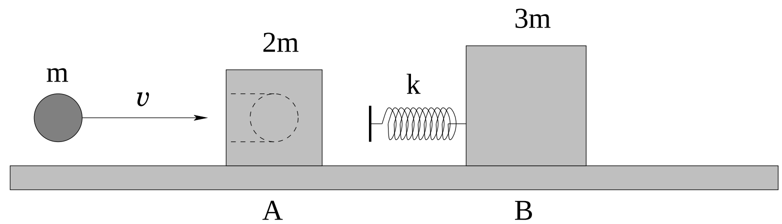


A ball bearing of mass $m = 50$ grams travelling at 200 m/sec smacks into a block of mass $M = 950$ gms and sticks in a hole drilled therein. The block is initially at rest on an Acme frictionless table and is also connected to an Acme spring with spring constant $k = 400$ N/m at its equilibrium position (see figure).

a) What is the maximum distance x the spring is compressed by the recoiling ball bearing-block system?

b) How much mechanical energy is lost in the collision (noting that an answer of 'none' is one possibility)?

Problem 195. problems-1/momentum-pr-inelastic-elastic-collision.tex



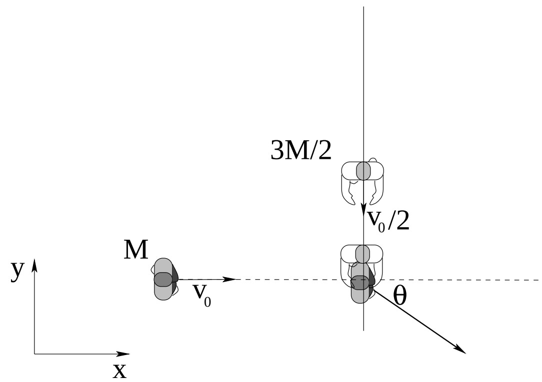
Two masses A and B rest on a frictionless surface, with a massless spring with spring constant k connected to B. A bullet coming from the left with speed v hits A and becomes embedded in it. The masses of the bullet, A and B are m, $2m$ and $3m$ respectively.

a) What is the speed $v_{\rm cm}$ of the center of mass of the system consisting of A, B and the bullet?

b) Immediately after A gets hit by the bullet, what is the speed v_A of A (with the bullet embedded in it) before it hits the spring?

c) In the subsequent motion of the system, what is the maximum compression $\Delta x_{\rm max}$ of the spring?

Problem 196. problems-1/momentum-pr-inelastic-football-collision.tex



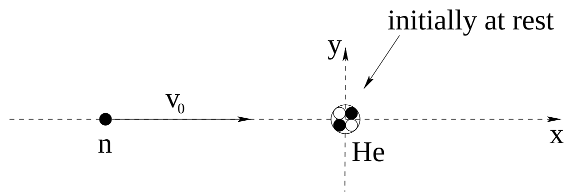
A running back of mass M is running at speed v_0 upfield. A linebacker of mass $3M/2$ is running along one of the yardlines (so his velocity is at a right angle to the running back's) at a speed $\frac{1}{2}v_0$. The linebacker ***tackles the running back in mid-air*** so that the two bodies stick together. Answer the questions below in terms of the givens above and show your reasoning.

a) By what angle θ is the running back deflected from his original direction?

b) What is the horizontal velocity of the two right after the collision in the x-y plane while they are still in the air?

c) Who experiences the ***greatest magnitude of acceleration*** during the collision, the running back (M) or linebacker ($3M/2$)?

Problem 197. problems-1/momentum-pr-neutron-collides-with-helium.tex



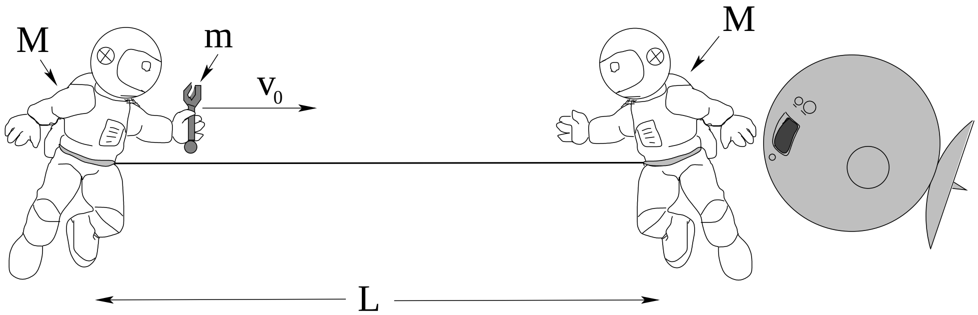
In the figure above, a neutron of mass m collides elastically with a helium nucleus of mass $4m$, striking it head on so that the collision is one dimensional. The initial speed of the the neutron is v_0; the helium nucleus is initially at rest. In answering the following questions you may *either* find *or* just remember the solution for one dimensional elastic collisions – you do not have to derive it, although you may if you wish or cannot remember it.

a) What is the final ***velocity*** of the neutron (magnitude and direction) after the collision.

b) What is the final ***velocity*** of the helium nucleus after the collision.

c) Is the helium nucleus moving faster or slower than the neutron is moving after the collision? (Does your answer make sense?)

Problem 198. problems-1/momentum-pr-two-astronauts.tex



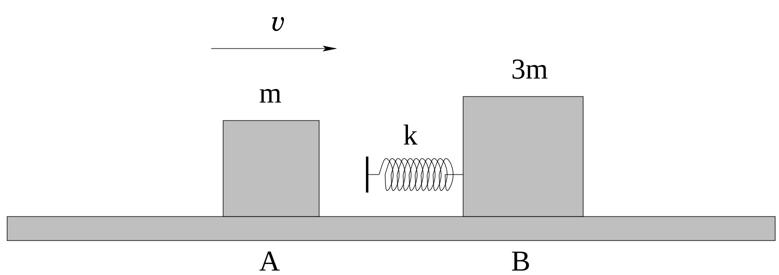
Two astronauts with identical mass M (including their spacesuits) in free-fall are working on a satellite. They are connected by a taut tether rope of length L. The first astronaut on the needs a tool of mass m that the second astronaut is carrying (also initially a distance L away), so the second one tosses the tool to the first at speed v_0.

a) What is the speed of the two astronauts while the tool is in space flying freely between them?

b) What is the speed of the two astronauts after the first one catches the tool?

c) The first astronaut cannot reach the satellite if he has drifted a distance d further away while the tool was in flight. Find the maximum length L that the tether can have such that the first astronaut can still reach the satellite. Express your answer in terms of M, m, v_0 and d as needed.

Problem 199. problems-1/momentum-pr-two-block-elastic-collision.tex



Two blocks A and B collide elastically on a frictionless surface. A massless spring with spring constant k is connected to block B. Initially, block A moves to the right with a speed v to collide with block B which is initially stationary.

Show your work.

a) Before the collision, what is the speed v_{cm} of the center of mass of the system consisting of blocks A and B?

$v_{\mathrm{cm}} =$ $\boxed{}$

b) During the collision, what is the maximum compression Δx_{max} of the spring?

$\Delta x_{\mathrm{max}} =$ $\boxed{}$

Chapter 8

One Dimensional Rotation and Torque

OK, so whole systems behave like particles, as long as the particle" is located at the center of mass of the system of particles. But solid objects have a ***second*** kind of motion. They can ***rotate*** around an axis through their center of mass (or possibly around other axes if they are suitably constrained).

A bit of algebra and thought transforms Newton's Second Law and the notion of kinetic energy into *rotational forms* involving rotations around a single axis, although that axis is itself arbitrary so that we know that eventually we'll have to make this a ***vector*** theory. For the moment, though, we'll just add a single plane-polar angle to the regular coordinate description of the center of mass of objects that rotate, or translate and rotate.

You can understand rotation in terms of the simple 1-D stuff you learned the first week as a *analogous system* by thinking of torque as the rotational equivalent of force, moment of inertia the rotational equivalent of mass, the angle of rotation θ the equivalent of a spatial coordinate like x, and so on.

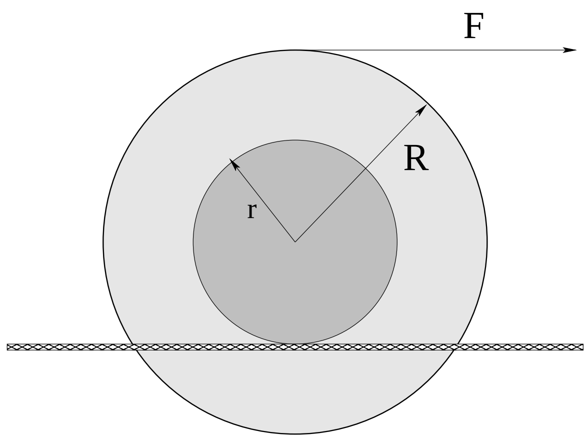
Don't forget the ***rolling constraint*** used in many of the problems! Also don't forget to choose your coordinate system – for translation and for rotation – ***consistently*** so that angular acceleration can be related to translational acceleration (and so on) ***without a spurious minus sign***.

Torque is a bit twisted (as physics subjects go), sorry...

8.1 Rotation

8.1.1 Multiple Choice

Problem 200. problems-1/rotation-mc-cable-spool-rolls-on-line.tex



A cable spool of mass M, radius R and moment of inertia $I = \beta M R^2$ around an axis through its center of mass is wrapped around its ***outer*** disk with fishing line and set on a rough rope as shown. The fishing line is then pulled with a force of magnitude F to the right as shown so that it rolls down the rope on the spool at radius r to the right ***without slipping***.

What is the direction of static friction as it rolls?

a) To the right.

b) To the left.

c) Not enough information to tell (depends on e.g. the size of r relative to R, the numerical value of β, or other unspecified data).

Problem 201. problems-1/rotation-mc-K-scaling.tex

Sphere **A** has mass M and radius R. Sphere **B** has mass M and radius $2R$. In order for the two spheres to have the ***same kinetic energy***, the ratio of their angular velocities must be:

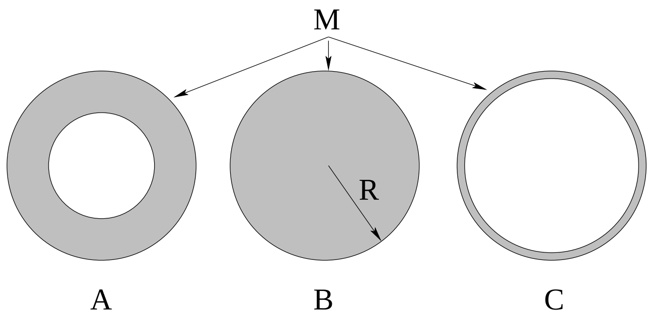
a) $\dfrac{\omega_A}{\omega_B} = 4$

b) $\dfrac{\omega_A}{\omega_B} = 2$

c) $\dfrac{\omega_A}{\omega_B} = 1/2$

d) $\dfrac{\omega_A}{\omega_B} = 1/4$

Problem 202. problems-1/rotation-mc-K-scaling-with-I.tex

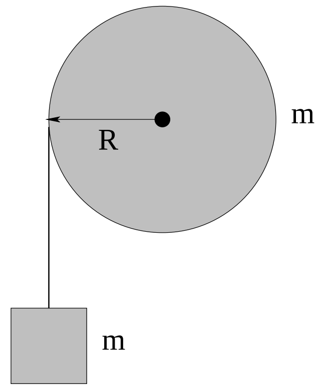


Three wheels have the ***same mass M and outer radius R***. This mass is distributed uniformly (but differently) in each wheel, with the inner radius of the uniform distribution varying as illustrated in the figure above. Each is rotating about its axis of symmetry through its center of mass in the center, and all three have the ***same kinetic energy***. Which wheel is rotating the ***fastest***?

a) ☐ Wheel **A** b) ☐ Wheel **B**

c) ☐ Wheel **C** d) ☐ Two or more of these wheels are tied for fastest

Problem 203. problems-1/rotation-mc-K-sharing-with-m.tex



A very light string is wrapped many times around a disk of mass m and radius R that is pivoted in the center and free to rotate. A block with the ***same*** mass m is attached to the end of the string and released from rest so that the disk spins as the string unrolls as it falls.

At the instant that the mass m has fallen to where it has kinetic energy K, the disk has kinetic energy:

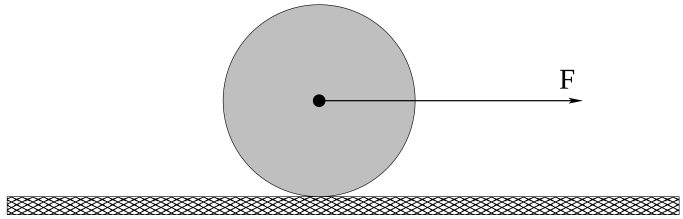
a) K

b) $K/2$

c) $2K$

d) None of these.

Problem 204. problems-1/rotation-mc-rolling-disk-friction-1.tex



In the figure above a force is applied to the center of a disk (initially at rest) sitting on a rough table by means of a rope attached to its frictionless axle ***in the direction shown***. The disk then accelerates and ***rolls without slipping***. The net horizontal force exerted by the table on the disk is:

a) kinetic friction to the right.

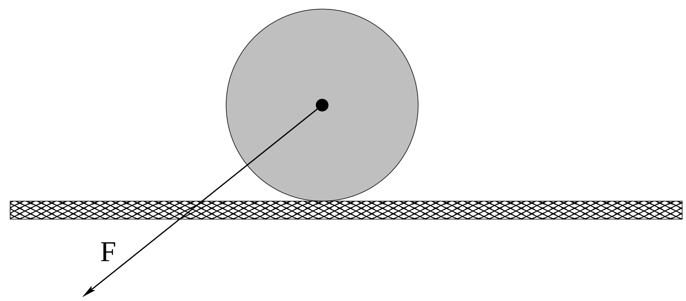
b) kinetic friction to the left.

c) static friction to the right.

d) static friction to the left.

e) Cannot tell from the information given.

Problem 205. problems-1/rotation-mc-rolling-disk-friction.tex



In the figure above a force is applied to the center of a disk (initially at rest) sitting on a rough table by means of a rope attached to its frictionless axle in the direction shown. The disk then accelerates and ***rolls without slipping***. The net horizontal force exerted by the table on the disk is:

a) kinetic friction to the right.

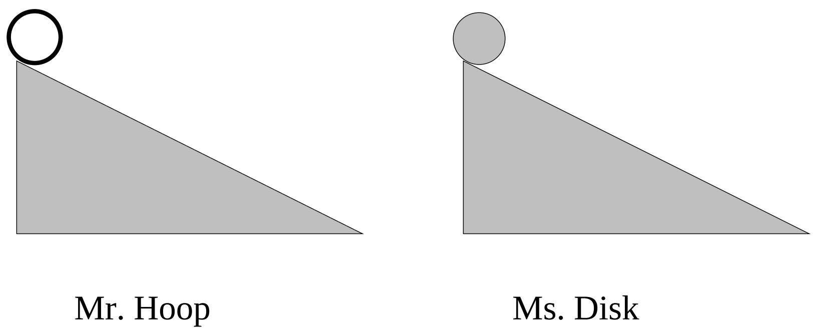
b) kinetic friction to the left.

c) static friction to the right.

d) static friction to the left.

e) Cannot tell from the information given.

Problem 206. problems-1/rotation-mc-rolling-race-1.tex



Mr. Hoop and Ms. Disk had a race rolling down two identical hills ***without slipping***. They both started at the top at the same time. Who won?

a) Mr. Hoop

b) Ms. Disk

8.1.2 Ranking/Scaling

Problem 207. problems-1/rotation-ra-hoop-and-disk.tex

A hoop and a disk of identical mass and radius are rolled up two identical inclined planes ***without slipping*** and reach a maximum height of H_{hoop} and H_{disk} respectively before coming momentarily to rest and rolling back down.

Use one of the three signs $<$, $>$ or $=$ in the boxes below to correctly complete each statement.

a) If both hoop and disk start with the same total ***kinetic energy*** then:

$$H_{\mathrm{hoop}} \boxed{\quad} H_{\mathrm{disk}}$$

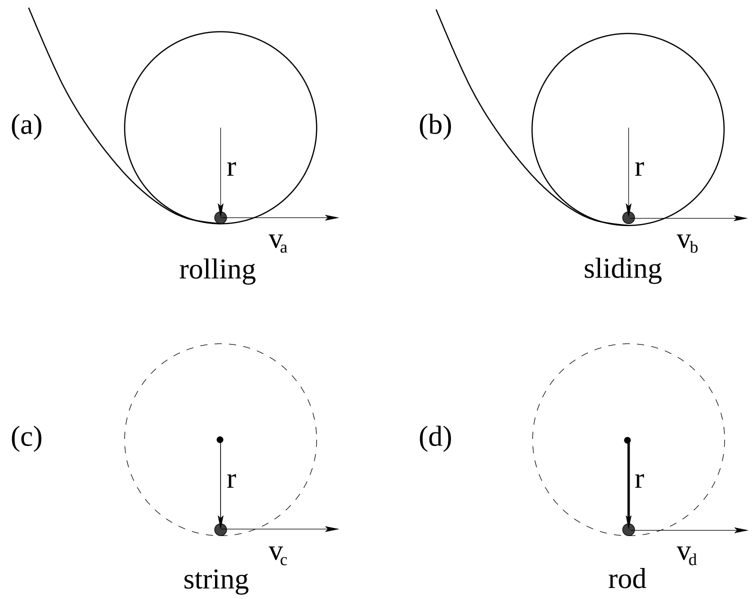
b) If both hoop and disk start with the same total ***center of mass speed*** then:

$$H_{\mathrm{hoop}} \boxed{\quad} H_{\mathrm{disk}}$$

c) If both hoop and disk start with the same total ***center of mass speed*** then comparing the ***magnitude*** of the ***work done by gravity*** when they have reached their maximum height:

$$|W_{\mathrm{gravity,hoop}}| \boxed{\quad} |W_{\mathrm{gravity,disk}}|$$

Problem 208. problems-1/rotation-ra-loop-the-loops-balls.tex



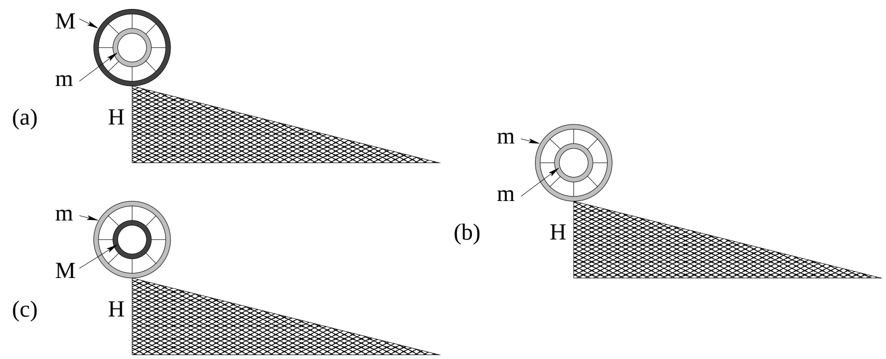
In the figures above, (a) shows a ball rolling without slipping on a track; (b) shows the ball sliding on a frictionless track; (c) shows the ball on a string; (d) shows the ball attached to a rigid massless rod attached to a frictionless pivot. In all four figures the ball has the *smallest* velocity at the bottom of its circular trajectory that will suffice for the ball to reach the top while still moving in a circle (note that the velocities are not drawn to scale).

Correctly ordinally rank these minimum velocities, for example $v_a = v_b < v_c < v_d$ is a possible (but probably incorrect) answer.

Note: You must *either* justify your answer with simple physical arguments *or* just solve for the minimum velocity needed at the bottom in terms of m, r, $\beta = 2/5$ (for a ball), g and then order the results. You can't just put down a "guess" for an order with no valid physical reasoning backing it and have it count, but it is possible to reason your way all or most of the way to an answer without doing all of the algebra.

8.1.3 Short Answer

Problem 209. problems-1/rotation-sa-rolling-race-1.tex



Its a race! Three wheels made out of two concentric rings of mass connected by light spokes are identically placed at the top of an inclined plane of height H as shown. At time $t = 0$ they are all three released from rest to roll without slipping down the incline. You are given the following information about each double ring:

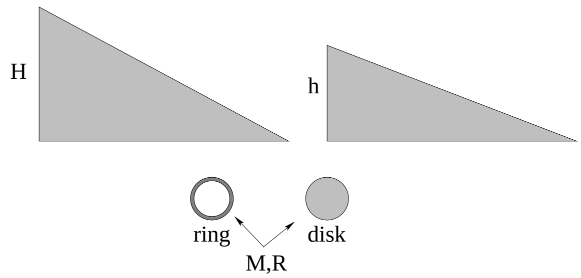
a) Inner ring mass m is less than outer ring mass M.

b) Inner ring mass m is the same as outer ring mass m.

c) Inner ring mass M is greater than outer ring mass m.

In what a, b, c order do the rings arrive at the bottom of the incline? (4 points)

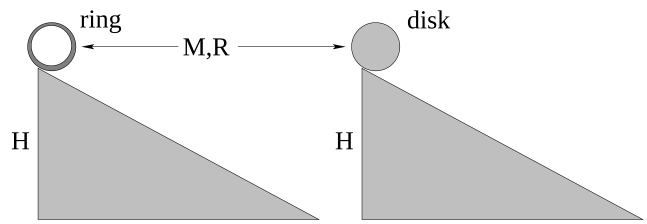
Problem 210. problems-1/rotation-sa-rolling-race-2.tex



You are given two inclined planes with different maximum heights $H > h$ as shown. If a ring and disk of identical radius R and mass M are each placed at the top of one of the two planes and released, they will roll without slipping to arrive at the bottom travelling ***at the same speed***. If placed at the top of the planes in the other order, they will not.

Draw and label the ring and disk at the tops of the correct planes such that they will roll to the bottom and arrive travelling ***at the same speed***.

Problem 211. problems-1/rotation-sa-rolling-race-3.tex



You are given two inclined planes with the same height H as shown. A ring and disk of identical radius R and mass M are each placed at the top of one of the two planes and released at the same instant to roll without slipping to the bottom of their respective inclines..

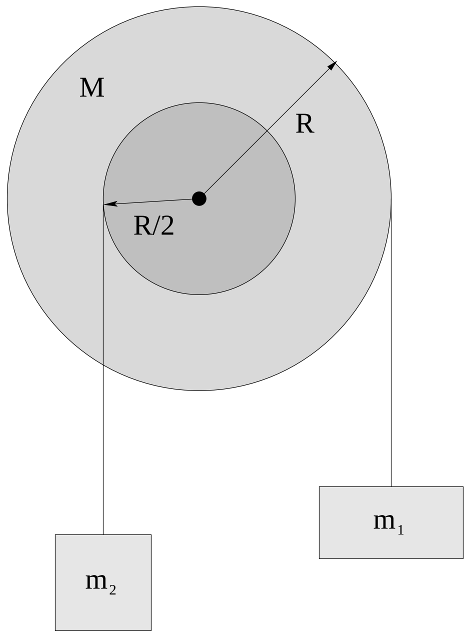
a) Which one gets to the bottom first? (Circle) **ring** **disk**

b) Which one has the greatest speed at the bottom? **ring** **disk**

c) Which one has the greatest rotational kinetic energy at the bottom? **ring** **disk**

8.1.4 Long Problems

Problem 212. problems-1/rotation-pr-asymmetric-atwoods-2.tex

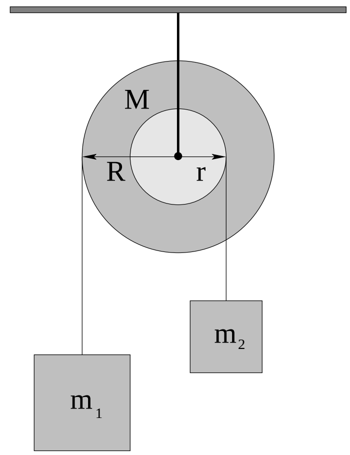


A pulley of mass M, radius R and moment of inertia $I = \beta M R^2$ has a massless, unstretchable string wrapped around it many times and has a mass m_1 suspended from the string. A second massless, unstretchable string is wrapped the opposite way around a massless, frictionless axle with radius $R/2$ as shown and has mass m_2 suspended from the string. The system begins at rest.

a) What must m_2 be in terms of m_1 for the system to remain stationary?

b) Suppose $m_1 = m_2 = M$. Find $\vec{\boldsymbol{\alpha}}$, the angular acceleration of the pulley about its center of mass. This is a vector! ***Indicate the direction*** of the angular acceleration on the figure or in your answer.

Problem 213. problems-1/rotation-pr-asymmetric-atwoods.tex



Two objects m_1 and m_2 (with $m_1 > m_2$) are attached to massless unstretchable ropes that are attached to wheels on a common frictionless axle as shown in the figure. The total moment of inertia of the two wheels is given as I. The radii of the wheels are R (for mass m_1 and r (for mass m_2 as shown.

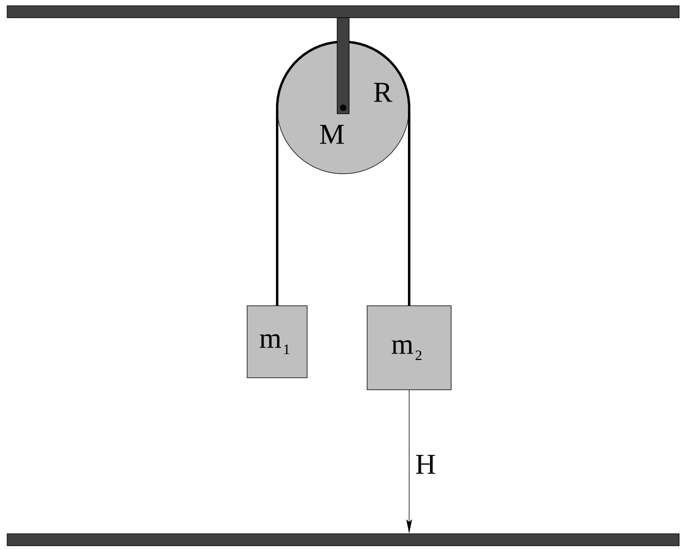
a) Show all forces on the two masses and the system of wheels using free-body diagrams; or drawing the forces in on the provided figure. You do not need to include the force in the strap that connects the wheel axle to the ceiling – you may assume that it is large enough to keep the axle perfectly fixed.

b) When mass m_1 falls a distance x_1, by what distance x_2 does mass m_2 rise?

c) Find the tensions T_1 and T_2 in the ropes supporting m_1 and m_2.

d) After mass m_1 falls a height H, what is its speed?

Problem 214. problems-1/rotation-pr-atwoods-machine.tex



In the figure above Atwood's machine is drawn – two masses m_1 and m_2 hanging over a *massive* pulley which you can model as a disk of mass M and radius R, connected by a massless unstretchable string. The string rolls on the pulley without slipping.

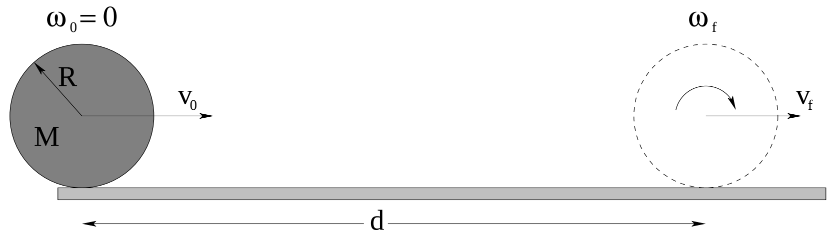
a) Draw three free body diagrams (isolated diagrams for each object showing just the forces acting on that object) for the three masses in the figure above.

b) Convert each free body diagram into a statement of Newton's Second Law (linear or rotational) for that object.

c) Using the *rolling constraint* (that the pulley rolls without slipping as the masses move up or down) find the acceleration of the system and the tensions in the string on *both* sides of the pulley in terms of m_1, m_2, M, g, and R.

d) Suppose mass $m_2 > m_1$ and the system is released from rest with the masses at equal heights. When mass m_2 has descended a distance H, find the velocity of each mass and the angular velocity of the pulley.

Problem 215. problems-1/rotation-pr-bowling-ball-friction.tex



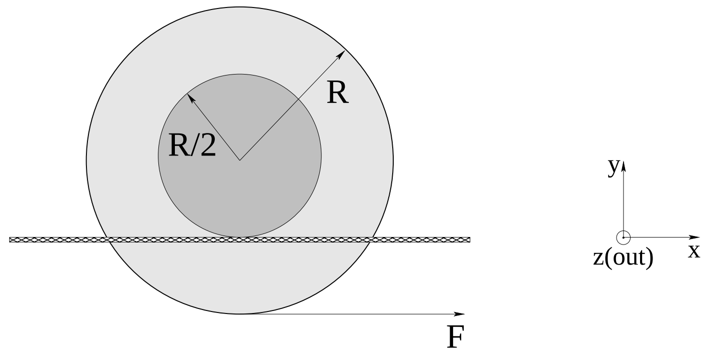
A bowling ball of mass M and radius R is released horizontally moving at a speed v_0 so that it initially slides without rotating on the bowling lane floor. μ_k is the coefficient of kinetic friction between the bowling ball and the lane floor. It slides for a time t and distance d before it rolls without slipping the rest of the way to the pins at speed v_f.

a) Find t.

b) Find d.

c) Find v_f.

Problem 216. problems-1/rotation-pr-cable-spool-rolls-on-line-2.tex



A cable spool of mass M, radius R and moment of inertia $\boldsymbol{I = \frac{1}{3}MR^2}$ is wrapped around its ***outer*** disk radius R with fishing line and set on a rough rope as shown. The fishing line is then pulled with a force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{x}}$ to the right so that it rolls down the rope at its ***inner*** radius $R/2$ ***without slipping***.

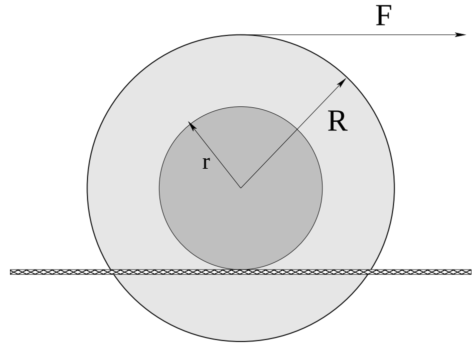
a) Find the acceleration of the spool (***magnitude and direction***): $\vec{\boldsymbol{a}} =$ $\boxed{}$

b) Find the force the friction of the rope exerts on the spool (***magnitude and direction***).

$$\vec{\boldsymbol{f}}_s = \boxed{}$$

Show your work. To make your statements of direction(s) unambiguous, please use the coordinate axes given to express all directions, and draw them in on the figure for good measure.

Problem 217. problems-1/rotation-pr-cable-spool-rolls-on-line.tex



A cable spool of mass M, radius R and moment of inertia $I = \beta M R^2$ around an axis through its center of mass is wrapped around its OUTER disk with fishing line and set on a rough rope as shown. The fishing line is then pulled with a force F to the right as shown so that it rolls down the rope on the spool at radius r **without slipping**.

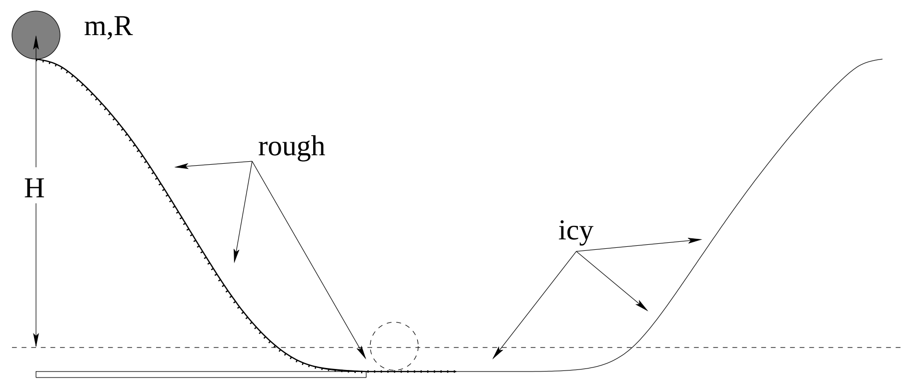
a) Which way does the spool roll (left or right)?

b) Find the magnitude of the acceleration of the spool.

c) Find the force the friction of the rope exerts on the spool.

d) Is there a value of the radius r relative to R for which *friction exerts no force on the spool?* If so, what is it?

Problem 218. problems-1/rotation-pr-disk-on-ice-example.tex



This problem will help you learn required concepts such as:

- Conservation of Mechanical Energy
- Rotational Kinetic Energy
- Rolling Constraint.

so please review them before you begin.

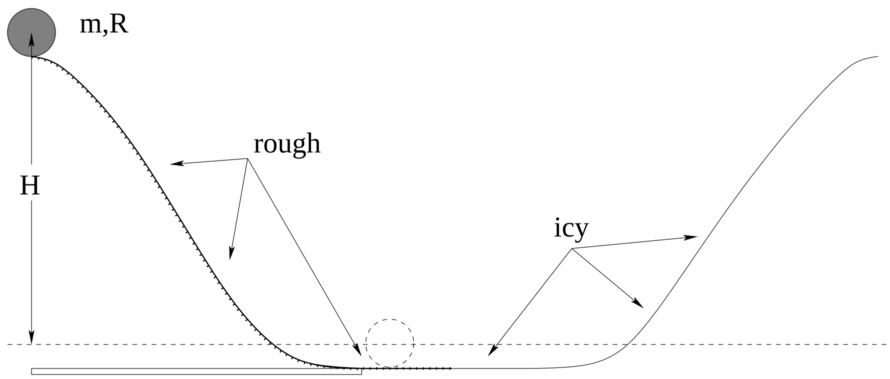
A disk of mass m and radius R rolls without slipping down a rough slope of height H onto an icy (frictionless) track at the bottom that leads up a second icy/frictionless hill as shown.

a) How fast is the disk moving at the bottom of the first incline? How fast is it rotating (what is its angular velocity)?

b) Does the disk's angular velocity change as it leaves the rough track and moves onto the ice (in the middle of the flat stretch in between the hills)?

c) How far up the second hill (vertically, find H') does the disk go before it stops rising?

Problem 219. problems-1/rotation-pr-disk-on-ice-solution.tex



This problem will help you learn required concepts such as:

- Conservation of Mechanical Energy
- Rotational Kinetic Energy
- Rolling Constraint.

so please review them before you begin.

A disk of mass m and radius R rolls without slipping down a rough slope of height H onto an icy (frictionless) track at the bottom that leads up a second icy/frictionless hill as shown.

a) How fast is the disk moving at the bottom of the first incline? How fast is it rotating (what is its angular velocity)?

   As the disk rolls down the incline without slipping, the velocity of its center and the angular velocity with which it rotates are related by the ***rolling without slipping*** constraint:

$$v = \omega R \ . \tag{8.1}$$

   Because it is not slipping, friction does no work, so that the total mechanical energy is conserved during the descent. Initial kinetic energy is zero, so $E_i = U_i = mgH$, where I am setting $U = 0$ at the bottom of the incline. With this choice, final potential energy vanishes and total energy in final state is kinetic. This, in turn, is a sum of a ***translational*** term representing the motion of the center of mass and a ***rotational*** contribution representing the motion as seen by an observer moving with the center of mass. Thus:

$$E_f = K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \ . \tag{8.2}$$

   Inserting the constraint as well as the value of the moment of inertia for a uniform disk $I = \frac{1}{2}mR^2$ we have

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2\left(1 + 1/2\right) = \frac{3}{4}mv^2 \ , \tag{8.3}$$

   whence

$$v = \sqrt{\frac{4}{3}gH} \ . \tag{8.4}$$

Note that this is less than the $\sqrt{2gH}$ we would find were the disk sliding down a frictionless incline. This makes sense, because friction has been acting to enforce the constraint, but has also slowed the disk. Alternatively, the expression for the kinetic energy above shows that some of the work done by gravity was converted into *rotational* kinetic energy, leaving less of it to be converted to translational kinetic energy.

Using the constraint we then have

$$\omega = \frac{v}{R} = \sqrt{\frac{4gH}{3R^2}}. \tag{8.5}$$

b) Does the disk's angular velocity change as it leaves the rough track and moves onto the ice (in the middle of the flat stretch in between the hills)?

During the disk's accelerating descent down the incline, friction acted to retard the acceleration and increase the angular acceleration, in order to maintain the condition of no slipping, but it ***did no work*** because it acts at the one point on the wheel that is ***always stationary with respect to the ground***. Instead it served to ***redistribute*** the gravitational potential energy between translational and rotational kinetic energy.

Once the horizontal stretch is reached, the disk continues at the *constant* translational and angular velocity given by the values we computed above. Since these satisfy the rolling constraint and no energy is entering the system, friction ***does not act*** on the disk as it rolls along the horizontal rough stretch.

This is an important fact! A perfectly round wheel (with frictionless bearings) rolling without slipping on a level surface experiences no friction and does not slow down. ***This is why we use wheels!***

When the disk moves onto the ice the change in the coefficient of friction thus produces *no change* in its motion, since friction was not applying any force on the rough surface anyway.

c) How far up the second hill (vertically, find H') does the disk go before it stops rising?

As it begins to climb the second incline, the disk's velocity decreases as kinetic energy is converted to potential energy. As its motion acquires a vertical component gravity is doing negative work on the disk and this force slows the disk. On the other hand, with no friction the only forces on the disk, gravity and the normal force, exert no torque about the disk's center so its angular velocity remains constant at the value found above. The disk slows as it climbs but continues spinning at a constant rate. When it comes to a stop at the highest point it can reach, its total mechanical energy is:

$$E_{H'} = mgH' + \frac{1}{2}I\omega^2 \ . \tag{8.6}$$

where its ***rotational*** kinetic energy is ***unchanged***.

This is equal to the total energy found above since during the climb all work was done by gravity, whence we find

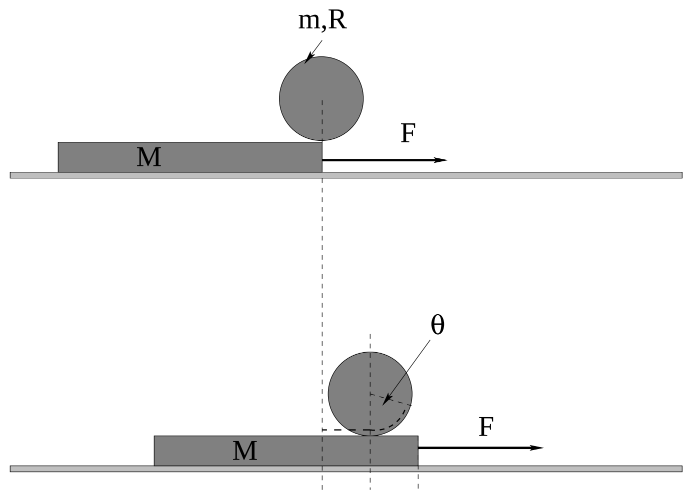
$$mgH' = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{4}{3}gH\right) \ , \tag{8.7}$$

or

$$H' = \frac{2}{3}H \ . \tag{8.8}$$

The disk does not recover its original height, though energy is conserved, because the energy converted to rotational kinetic energy cannot, without friction, be converted to potential energy. If we throw sand on the ice as the disk comes to a halt, the resulting friction will act to propel the disk farther up the hill, slowing its rotation and recovering this stored energy.

Problem 220. problems-1/rotation-pr-disk-rolling-on-slab-difficult.tex



This problem will help you learn required concepts such as:

- Newton's Second Law (linear and rotational)
- Rolling Constraint
- Static and Kinetic Friction

so please review them before you begin.

A disk of mass m is resting on a slab of mass M, which in turn is resting on a frictionless table. The coefficients of static and kinetic friction between the disk and the slab are μ_s and μ_k, respectively. A small force $\vec{F}$ to the right is applied to the slab as shown, then gradually increased.

a) When $\vec{F}$ is small, the slab will accelerate to the right and the disk will roll on the slab without slipping. Find the acceleration of the slab, the acceleration of the disk, and the *angular* acceleration of the disk as this happens, in terms of m, M, R, and the magnitude of the force F.

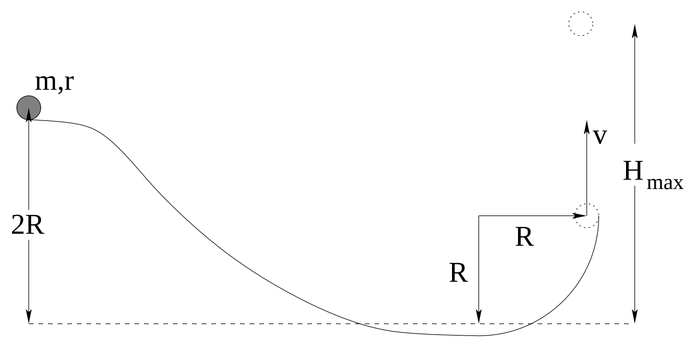
b) Find the maximum force F_{max} such that it rolls without slipping.

c) If F is greater than this, solve once again for the acceleration and angular acceleration of the disk and the acceleration of the slab.

Hint: The hardest single thing about this problem isn't the physics (which is really pretty straightfoward). It is visualizing the coordinates as the center of mass of the disk moves with a different acceleration as the slab. I have drawn *two* figures above to help you with this – the lower figure represents a possible position of the disk after the slab has moved some distance to the right and the disk has rolled back (relative to the slab! It has moved *forward* relative to the ground! Why?) without slipping. Note the dashed radius to help you see the angle through which it has rolled and the various dashed lines to help you relate the distance the slab has moved x_s, the distance the center of the disk has moved x_d, and the angle through which it has rolled θ. Use this relation to connect the acceleration of the slab to the acceleration and angular acceleration of the disk.

If you can do this one, good job!

Problem 221. problems-1/rotation-pr-disk-rolls-to-loop.tex



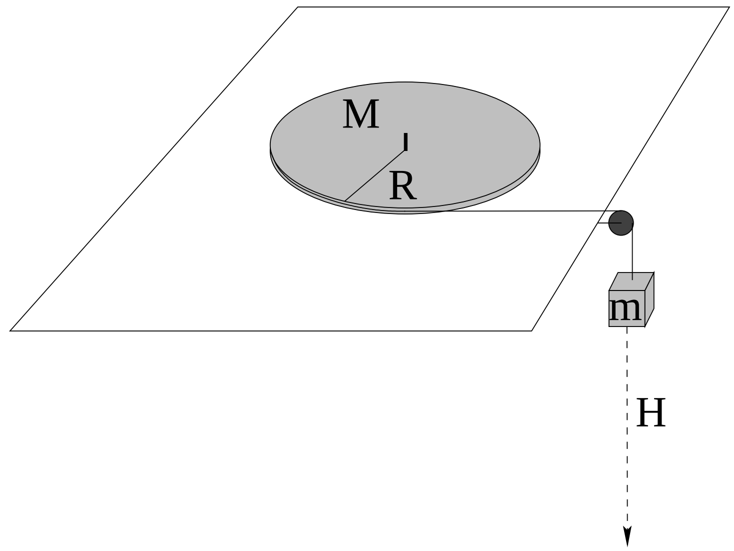
A disk of mass m and radius r (and moment of inertia $I = \frac{1}{2}mr^2$) sits at the top of slope of height $2R$ and ***rolls without slipping*** down the hill to a circular track curving upwards. Ignore drag forces and answer the following questions:

a) How fast is the disk travelling when it reaches the top/end of the curved track (as shown)?

b) Find the normal force acting on the disk due at this point, *just before* it comes off of the circular curve of the track.

c) How high (relative to the lower dashed line) will the disk go above the point where the disk leaves the track before falling back?

Problem 222. problems-1/rotation-pr-falling-mass-spins-disk.tex



A disk of mass M and radius R placed on a frictionless table can rotate freely about a fixed frictionless spindle as shown in the figure. An Acme (massless, unstretchable) string is tightly wound around the disk and then passes over a small frictionless pulley, where it is attached to a hanging mass m. At time $t = 0$ the hanging mass and disk are released from rest.

a) Find the tension T in the string ***while the mass is falling and the disk is rotating***.

b) Find the speed v_m of the mass m when it has fallen a height H from its initial position.

Problem 223. problems-1/rotation-pr-flat-plane-two-blocks-massive-pulley.tex

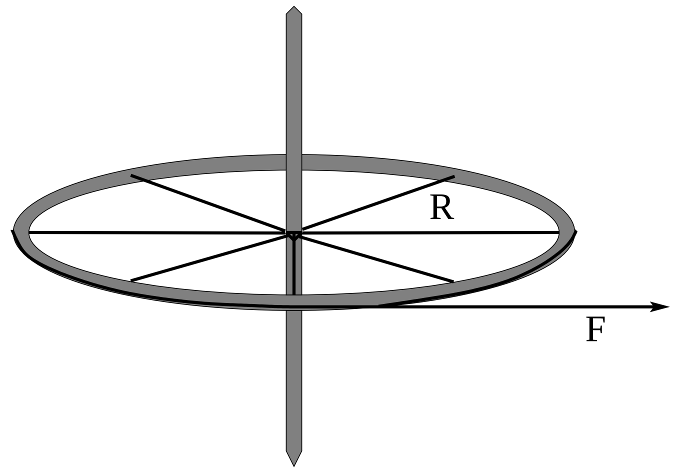
A mass m_1 is attached to a second mass m_2 by an Acme (massless, unstretchable) string. m_1 sits on a frictionless table; m_2 is hanging over the ends of a table, suspended by the taut string from pulley of mass M and radius R. At time $t = 0$ both masses are released.

a) Draw the force/free body diagram for this problem.

b) Find the acceleration of the two masses.

c) Find the angular acceleration of the pulley.

Problem 224. problems-1/rotation-pr-gyroscope-torque.tex

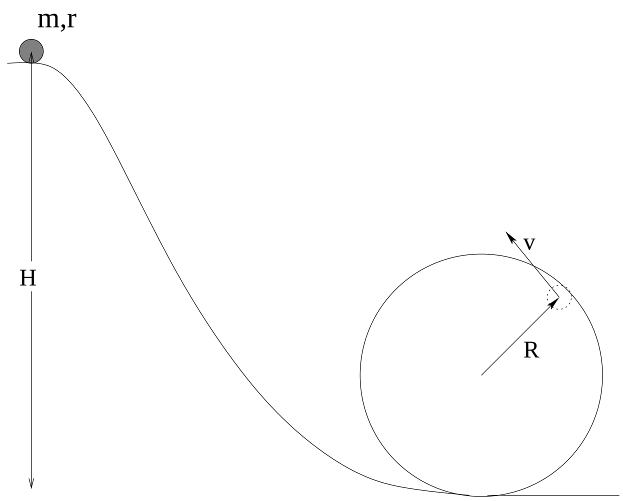


A child spins a gyroscope with moment of inertia I and a frictionless pivot by wrapping a (massless, unstretchable) string of length L around it at a radius R and then pulling the string with a constant force F as shown. Find:

a) The angular acceleration of the gyroscope while the string is being pulled.

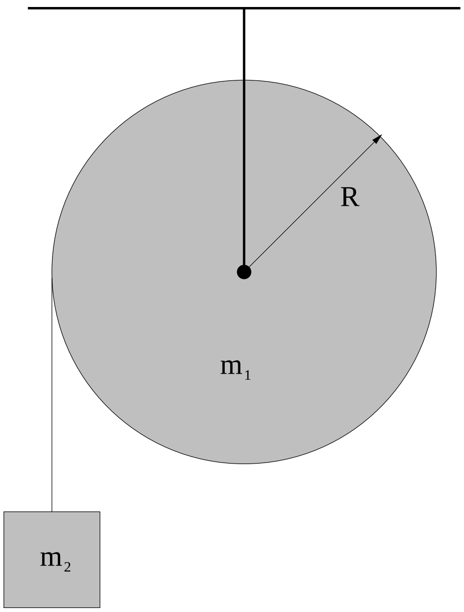
b) The angular speed of the gyroscope as the string comes free (assume that the force F is exerted through the entire distance L).

Problem 225. problems-1/rotation-pr-loop-the-rolling-disk.tex



A disk of mass m and radius r sits at the top of a loop-the-loop of height H. Find the minimum height H such that the disk goes around the loop the loop without coming off of the track, assuming that it rolls without slipping the entire way. Ignore drag forces.

Problem 226. problems-1/rotation-pr-mass-unrolls-spool.tex



A pulley of mass m_1, radius R and moment of inertia $I = \beta m_1 R^2$ has a massless, unstretchable string wrapped around it many times and has a mass m_2 suspended from it so that the string unrolls on the disk without slipping as the mass falls. The system begins at rest and is released at time $t = 0$.

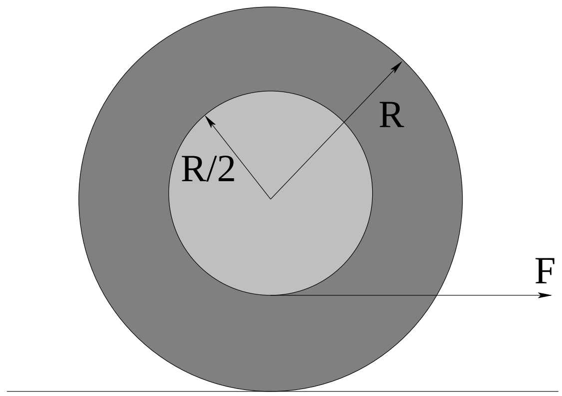
a) Find α, the magnitude of the angular acceleration of the pulley about its center of mass as the mass falls.

b) Find the tension T in the string as the mass falls..

c) After the mass m_2 has fallen through a height H, how fast is it moving?

d) **Bonus question** (2pt): What force is exerted on the axle of the pulley by the support strap ***as the mass falls***?

Problem 227. problems-1/rotation-pr-rolling-spool-pulled-right.tex

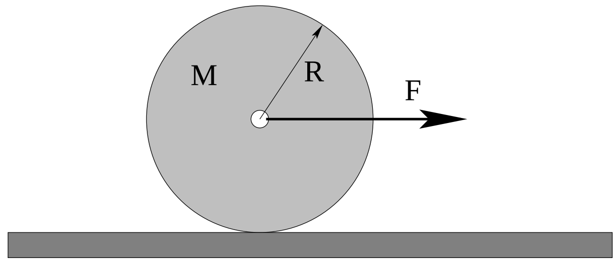


A spool of mass M, radius R and moment of inertia $I = \frac{1}{3}MR^2$ is wrapped around its spindle (radius $R/2$) with fishing line and set on a rough table as shown. The line is then pulled with a force F as shown so that it rolls without slipping.

a) Which way does the spool roll (left or right)? Put another way, does it roll up the string or unroll the string?

b) Find the magnitude of the acceleration of the spool and the force exerted by the table on the spool.

Problem 228. problems-1/rotation-pr-rolling-wheel-static-friction-review.tex



A force of magnitude F (to the right) is applied to the frictionless axle of a wheel made of a ***uniform disk*** with mass M and radius R. It ***rolls without slipping*** on a rough table (with a coefficient of static friction given by μ_s). Find:

a) What is the moment of inertia of this disk about its center of mass? If you cannot remember, use the form $I_cm = \beta MR^2$ to answer the remaining questions.

$$I_{\rm cm} = \boxed{}$$

b) The magnitude of the acceleration of the wheel.

$$a = \boxed{}$$

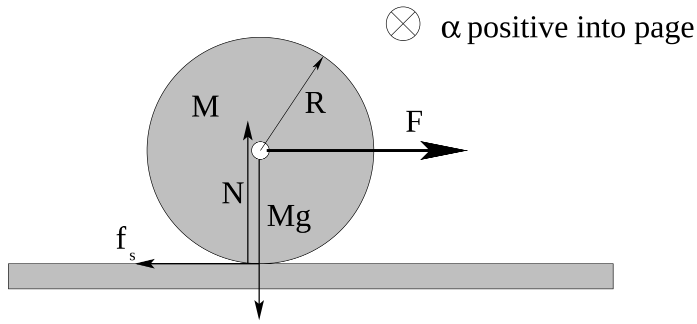
c) The magnitude of the force exerted by static friction. **Indicate its direction on the figure above.**

$$f_s = \boxed{}$$

d) The minimum coefficient of static friction μ_s such that the wheel does not slip for this force.

$$\mu_{s,\rm min} = \boxed{}$$

Problem 229. problems-1/rotation-pr-rolling-wheel-static-friction-review-soln.tex



A force of magnitude F (to the right) is applied to the frictionless axle of a wheel made of a ***uniform disk*** with mass M and radius R. It ***rolls without slipping*** on a rough table (with a coefficient of static friction given by μ_s). Find:

a) What is the moment of inertia of this disk about its center of mass? If you cannot remember, use the form $I_c m = \beta M R^2$ to answer the remaining questions.

$$I_{\rm cm} = \boxed{\frac{1}{2} M R^2}$$

b) The magnitude of the acceleration of the wheel.

This follows from using Newton's Second Law twice, once for translation and once for rotation, plus the rolling constraint $\alpha = a/R$. There is no net vertical force, so $N = Mg$. N, mg, and F exert no torque (about the center of mass – there is an entirely different solution possible using the point of contact with the ground as the pivot, using the parallel axis theorem). So N2 for rotation is:

$$f_s R = I\alpha = \frac{1}{2} M R^2 \frac{a}{R}$$

If we divide this by R on both sides and line it up with N2 for translation:

$$F - f_s = Ma$$

$$f_s = \frac{1}{2} Ma$$

and add them to eliminate f_s, we get:

$$a = \boxed{\frac{2}{3} \frac{F}{M}}$$

c) The magnitude of the force exerted by static friction. **Indicate its direction on the figure above.**

Backsubstitute this into the expression above for f_s:

$$f_s = \frac{1}{2} Ma = \boxed{\frac{1}{3} F}$$

Note Well! Any answer such as $f_s = \mu_s M g$ is ***wrong!***

d) The minimum coefficient of static friction μ_s such that the wheel does not slip for this force.

Here we use:

$$f_s = \frac{1}{3}F < F_s = \mu_s N = \mu_s M g$$

Note Well the inequality! We rearrange this to obtain:

$$\mu_s > \frac{1}{3}\frac{F}{Mg}$$

or

$$\mu_{s,\mathrm{min}} = \boxed{\frac{1}{3}\frac{F}{Mg}}$$

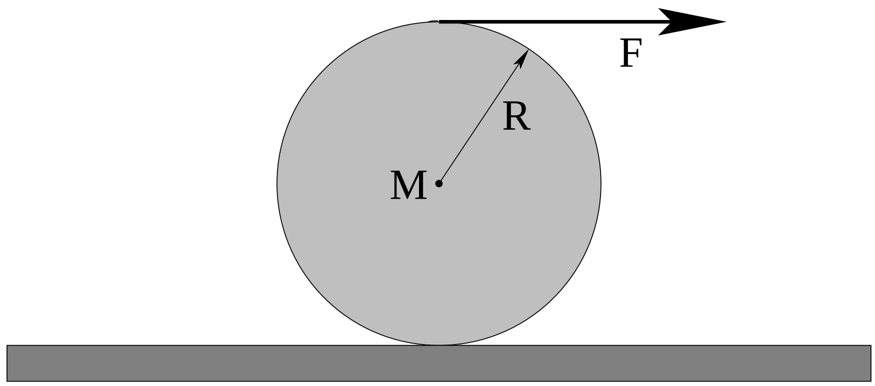
Scoring:

a) +2 points. This is something they were told they should know.

b),c) +12 points together. +4 for each N2, +2 for rolling constraint, +2 for algebra including back substitution. Instant -4 if they assert $f_s = \mu_s M g$.

d) +6 points. If they wrote the wrong answer for c) above, they may well lose more points here, but we'll cap it at an additional -3 (basically giving them +3 total for knowing that $\mu_s N$ is relevant in some way to the problem. But we'll give successful students +4 for writing down the correct inequality, and save the last +2 for algebra or confusion.

Problem 230. problems-1/rotation-pr-rolling-wheel-static-friction-top.tex



A massless rope that is wrapped around a a uniform disk with mass M and radius R is pulled to the right with a force F as shown. As long as F is less than some maximum value it will ***roll without slipping*** on a rough table (with a coefficient of static friction given by μ_s).

a) Choose a ***direction*** for the force of static friction when the disk rolls without slipping by drawing a suitable arrow in on the diagram above at the point where it acts.

b) Write suitable equations of motion and find the **magnitude** of the acceleration of the wheel, assuming that it ***rolls without slipping***.

c) Find the **magnitude** of the force exerted by static friction, assuming that it rolls without slipping. **Indicate its actual direction in case your choice in a) above was backwards!**

d) What is the largest force $F_{\max}$ that can be exerted before the wheel starts to slip?

Problem 231. problems-1/rotation-pr-sliding-rolling-bowling-ball.tex

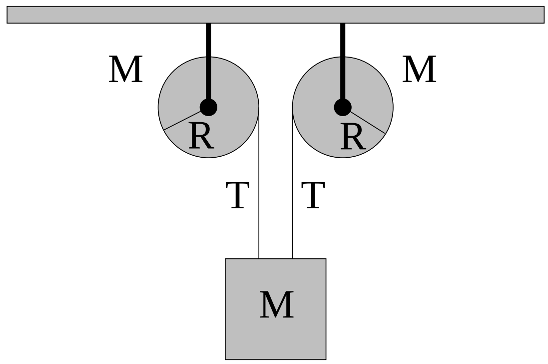
A uniform bowling ball of radius R, mass M, and moment of inertia I about its center of mass is initially launched so that it is sliding with speed v_0 without rolling on an alley with a coefficient of friction μ_k.

a) Analyze the forces acting on the bowling ball to find the acceleration of the center of mass and angular acceleration of the bowling ball about its CM;

b) Find the CM velocity as a function of time (t) and angular velocity of the ball as a function of time (t).

c) Find the CM velocity of the bowling ball when it starts rolling without slipping.

Problem 232. problems-1/rotation-pr-two-spools-one-mass.tex



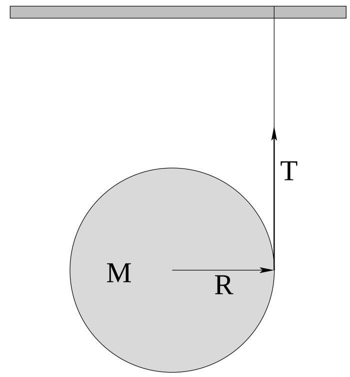
In the figure above, a mass M is connected to two independent massive spools of radius R, also of mass M (each), wrapped with massless unstretchable string. You may consider the spools to be ***disks*** as far as their moment of inertia is concerned. At $t = 0$, the mass M and spools are released from rest and the mass M falls. Find:

a) The magnitude of the acceleration a of the mass M.

b) The tension T in either string (they are the same from symmetry).

c) When the mass M has fallen a distance H, what is its speed?

Problem 233. problems-1/rotation-pr-unrolling-a-falling-spool-algebraic.tex

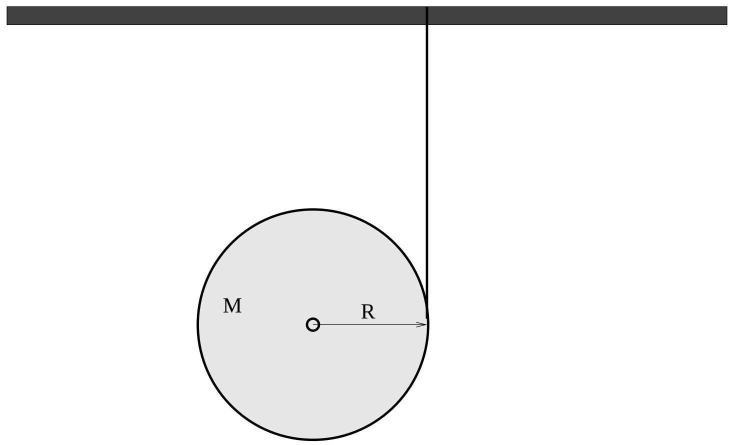


A spool of fishing line is tied to a beam and released from rest in the position shown at time $t = 0$. The spool has a mass M, a radius of R, and a moment of inertial $I = \beta M R^2$. The line itself has negligible mass per unit length. Once released, the disk falls as the taut line unrolls.

a) What is the tension in the line as the disk falls (unrolling the line)?

b) After the disk has fallen a height H, what is its angular velocity ω?

Problem 234. problems-1/rotation-pr-unrolling-a-falling-spool-numeric.tex

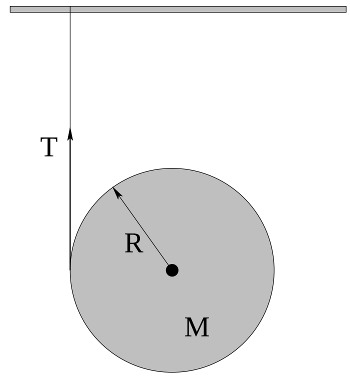


A spool of fishing line is tied to a beam and released from rest in the position shown at time $t = 0$. The spool is a disk and has a mass of 50 grams and a radius of 5 cm. The line itself has negligible mass per unit length. Once released, the disk falls as the taut line unrolls.

a) What is the tension in the line as the disk falls (unrolling the line)?

b) After the disk has fallen 2m, what is its speed?

Problem 235. problems-1/rotation-pr-unrolling-a-falling-spool-reversed.tex



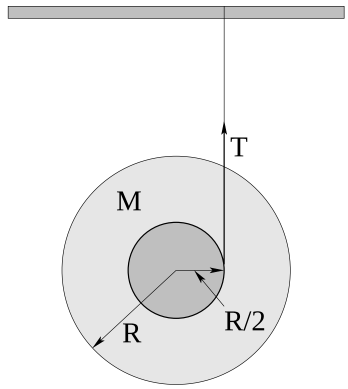
A spool of fishing line is tied to a beam and released from rest in the position shown at time $t = 0$. The spool has a mass M, a radius of R, and a moment of inertial $I = \beta M R^2$. The line itself has negligible mass per unit length. Once released, the spool falls as the taut line unrolls.

a) What is the tension in the line as the spool falls (unrolling the line)?

b) What is the magnitude of the angular acceleration of the spool α about its center of mass as it falls?

c) After the spool has fallen a height H, what is the ***direction*** of its angular velocity, $\vec{\omega}$? Indicate this direction with a labelled arrow symbol on a suitable diagram.

Problem 236. problems-1/rotation-pr-unrolling-a-falling-yo-yo.tex



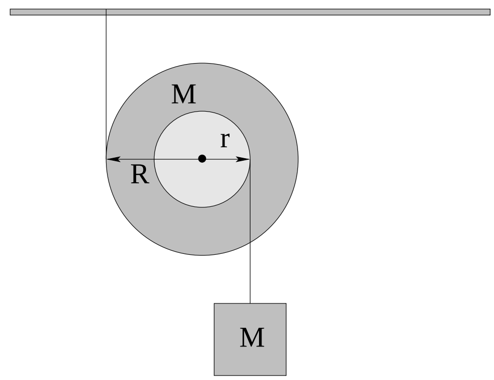
A yo-yo is tied to a beam and released ***from rest*** in the position shown at time $t = 0$. The yo-yo has a mass M, a radius of R, and a moment of inertia $I = \beta M R^2$. The unstretchable line itself has negligible mass per unit length and is wrapped around an inner spindle with radius $R/2$ as shown. Once released, the yo-yo falls as the taut line unrolls.

a) What is the ***angular acceleration*** $\vec{\alpha}$ of the yo-yo as it falls (unrolling the line)? Note that this is a ***vector*** quantity, so please indicate its direction in your answer and/or on the figure.

b) What is the tension T in the line as the yo-yo falls (unrolling the line)?

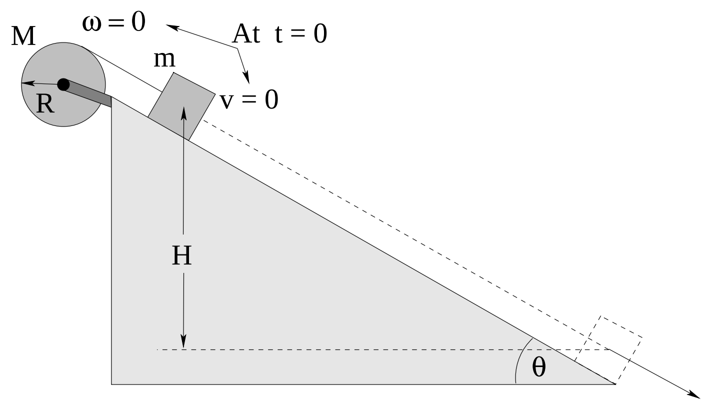
c) After the yo-yo has fallen a height H, what is its angular velocity ω?

Problem 237. problems-1/rotation-pr-unrolling-disk-and-block.tex



Challenge Problem (difficult!): In the figure above, a spool with moment of inertia $\beta M R^2$ is hanging from a rod by a (massless, unstretchable) string that is wrapped around it at a radius R, while a block of equal mass M is hung on a second string that is wrapped around it at a radius r as shown. Find the magnitude of the acceleration of the the central pulley.

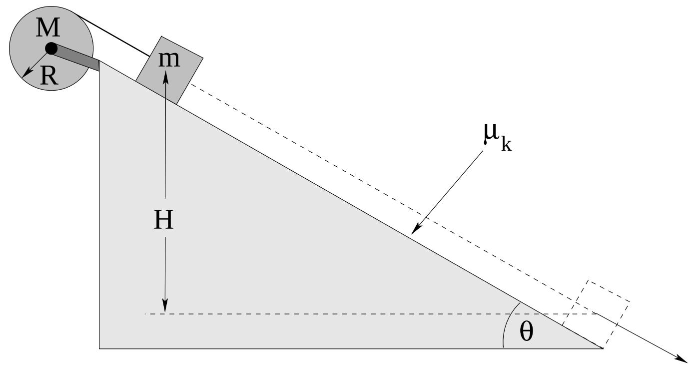
Problem 238. problems-1/rotation-pr-unrolling-spool-down-inclined-plane-1.tex



In the figure above, a pulley at rest of mass M and radius R with frictionless bearings and moment of inertia $I = \beta M R^2$ is fixed at the top of a fixed, frictionless inclined plane that makes an angle θ with respect to the horizontal. The pulley is wrapped with many turns of (approximately massless and unstretchable) fishing line. The line is also attached to a block of mass m. At time $t = 0$ the block and pulley are released ***from rest***, $v = 0$ (block) and $\omega = 0$ (pulley).

- Find the ***magnitude of the acceleration a*** of the block as it slides down the incline.
- Find the ***tension T*** in the string as it slides.
- Find the ***speed v*** with which the block reaches the bottom of the incline.

Problem 239. problems-1/rotation-pr-unrolling-spool-down-inclined-plane-2.tex



In the figure above, a pulley of mass M and radius R with frictionless bearings and moment of inertia $I = \beta M R^2$ is fixed at the top of a ***rough*** inclined plane that makes an angle θ with respect to the horizontal that is large enough that the block will definitely overcome static friction and slide. The coefficient of kinetic friction between the block and the plane is μ_k. The pulley is wrapped with many turns of (approximately massless and unstretchable) fishing line. The line is also attached to a block of mass m. At time $t = 0$ the block and pulley are released ***from rest***.

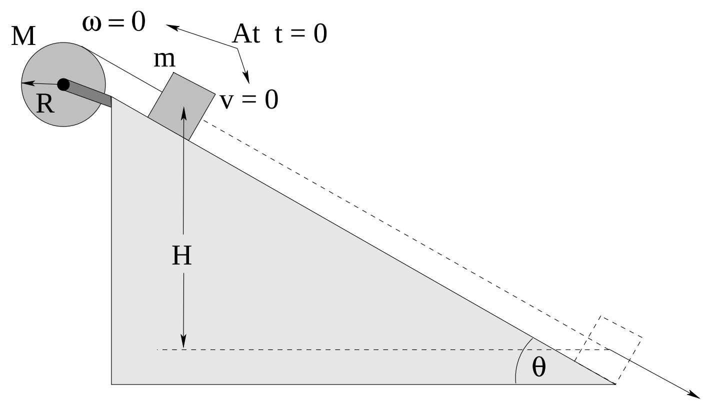
a) ***Draw a force diagram*** for both the block and the pulley separately. You do not have to represent the forces acting at the pivot of the pulley that keep it stationary, only the one(s) relevant to the solution of the problem. Represent *all* the forces on the block.

b) Find both the magnitude of the acceleration a of the block and the tension T in the string as the block slides down the incline in **terms of the givens**.

c) Find the kinetic energy of the block when it reaches the bottom of the incline.

d) Find the kinetic energy of the pulley when the block reaches the bottom of the incline.

Problem 240. problems-1/rotation-pr-unrolling-spool-down-inclined-plane-friction.tex



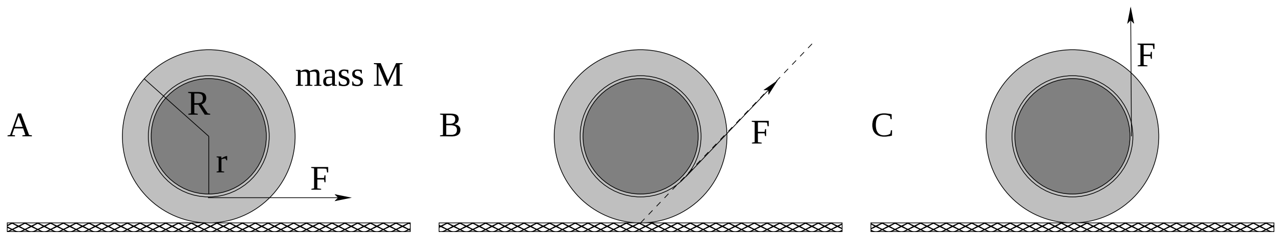
A pulley with frictionless bearings and moment of inertia $I = \beta M R^2$ is at the top of a fixed inclined plane that makes an angle θ with respect to the horizontal. The pulley is wrapped with many turns of (approximately massless and unstretchable) fishing line that is attached to a block of mass m resting on the incline a height H above the bottom. The coefficient of kinetic friction between the block and the incline is μ_k. At time $t = 0$ the block and pulley are released ***from rest*** at an angle θ that is large enough that the block ***will definitely overcome static friction and begin to slide***.

a) On the figure above or in a free body diagram to the side, draw in and label all of the forces acting ***on the block only***.

b) Find the ***magnitude*** of the acceleration a of the block as it slides down the incline.

c) Find the ***speed*** v with which the block reaches the bottom of the incline.

Problem 241. problems-1/rotation-pr-walking-the-spool.tex



In the figure above, a spool of mass M is wrapped with string around the inner spool. The spool is placed on a rough surface and the string is pulled with force F in the three directions shown. The spool, if it rolls at all, ***rolls without slipping***. (Note that if pulled too hard, the spool can both slip and/or roll.)

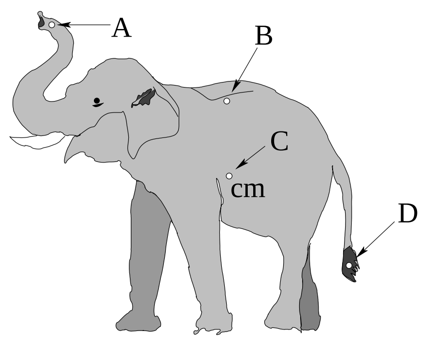
Find the acceleration and frictional force ***vectors*** (magnitude ***and*** direction) for all three figures. Use $I_{\rm cm} = \beta M R^2$.

Note Well: You can use ***either*** the center of mass ***or*** the point of contact with the ground (with the parallel axis theorem) as a pivot, the latter being ***slightly easier***.

8.2 Moment of Inertia

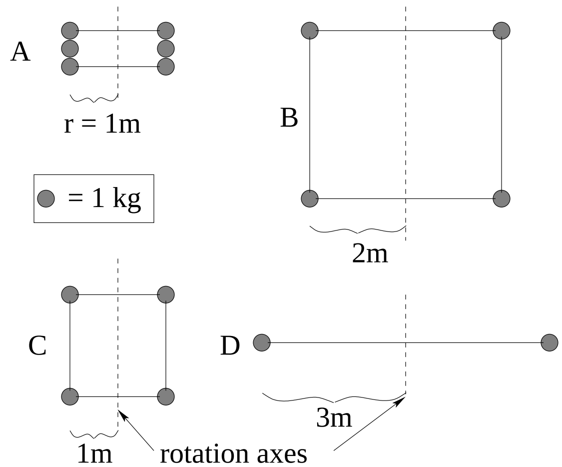
8.2.1 Ranking/Scaling

Problem 242. problems-1/moment-of-inertia-ra-parallel-axes.tex



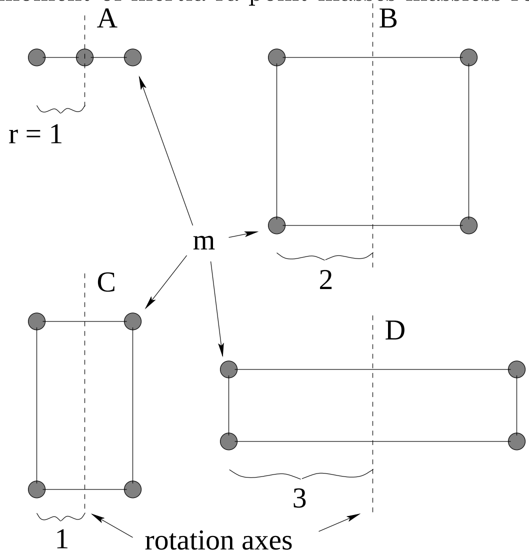
A two-dimensional cardboard cut-out of an elephant is drawn above. Small holes are drilled through it at the points A, B, C and D indicated. Hole C is at the center of mass of the figure. Rank the moment of inertia of the elephant about axes through each of the holes (with equality permitted) so that a possible (but unlikely) answer is $I_A < I_B = I_C < I_D$.

Problem 243. problems-1/moment-of-inertia-ra-point-masses-massless-rods-2.tex

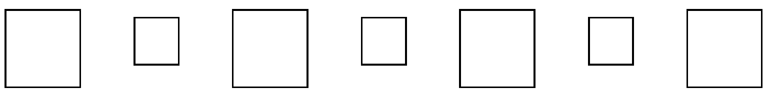


In the figure above, all of the masses m are identical and are connected by ***rigid massless*** rods as drawn. ***Rank the moments of inertia*** of the four objects about the ***rotation axes drawn as dashed lines.*** Equality is permitted, so a possible answer might be $A > C = D > B$.

Problem 244. problems-1/moment-of-inertia-ra-point-masses-massless-rods.tex

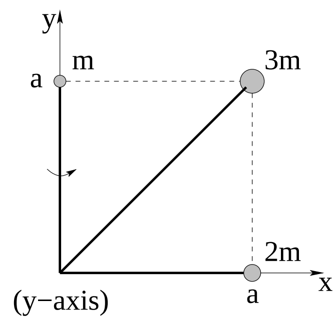


In the figure above, all of the masses m are identical and are connected by rigid ***massless*** rods as drawn. Rank the moments of inertia of the four objects about the rotation axes drawn as dashed lines. A possible answer could look like $A < C = D < B$ (but probably isn't).



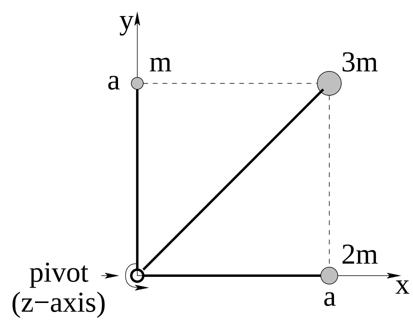
8.2.2 Short Answer

Problem 245. problems-1/moment-of-inertia-sa-3-masses-1.tex



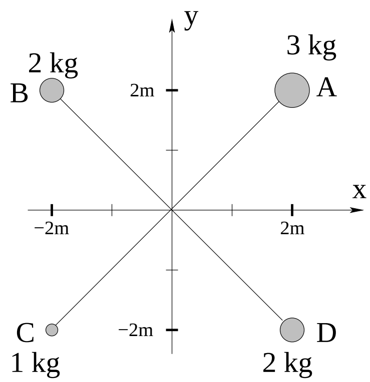
In the figure above, massless rigid rods connect three masses at the origin so that they can freely rotate around the y-axis (rotating initially into the page as shown). The masses are fixed so that they are at three corners of a square of side a. Find the ***moment of inertia*** about the y-axis of this arrangement in terms of m and a.

Problem 246. problems-1/moment-of-inertia-sa-3-masses.tex



In the figure above, massless rigid rods connect three masses to a pivot at the origin so that they can freely rotate around the z-axis (perpendicular to the page). The masses are fixed so that they are at three corners of a square of side a, with the pivot at the fourth corner as shown. Find the ***moment of inertia*** about the z-axis of this arrangement in terms of m and a.

Problem 247. problems-1/moment-of-inertia-sa-4-masses-1.tex



In the figure, massless rigid rods connect four point-like masses centered at points A, B, C, D to form a rigid body. The rigid body can rotate about any axis perpendicular to the plane of the figure.

[*Showing your work is recommended, but not mandatory.*]

a) Find the center of mass (CM) location of the rigid body (Note: proper units should be included in your answer).

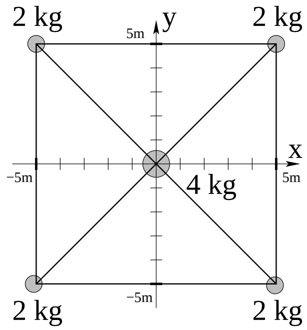
$x_{\rm cm} = \boxed{}$; $y_{\rm cm} = \boxed{}$

b) ***Mark the CM's location*** in the figure.

c) The moment of inertia about an axis perpendicular to the plane of the figure depends on the location of the axis. Answer the following by filling the box using A, B, C, D, or CM.

- The smallest moment of inertia is about an axis going through point $\boxed{}$.
- The next smallest moment of inertia is about an axis going through point $\boxed{}$.

Problem 248. problems-1/moment-of-inertia-sa-4-masses-2.tex



In the figure above, massless rigid rods connect five masses to form a rigid body. The rigid body can be rotated about any axis perpendicular to the plane of the figure. Find a point with coordinates (x_0, y_0) on the provided coordinate frame (units of meters) so that the moment of inertia of the system is ***smallest*** if the axis goes through this point. Then, enter the moment of inertia of the system about this axis.

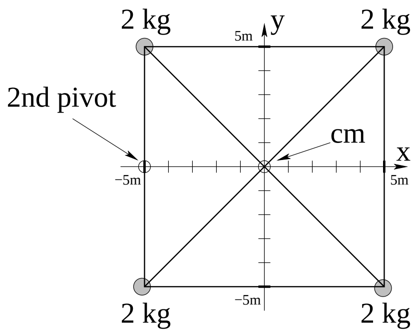
$x_0 =$ ☐ meters

$y_0 =$ ☐ meters

$\boldsymbol{I}_{\mathbf{min}} =$ ☐ kg-meter^2

Problem 249. problems-1/moment-of-inertia-sa-4-masses.tex

In the figure below, four 2 kilogram masses are held at the corners of a rigid square by massless rods as shown. The center of mass of the system is located at the origin of the provided $x - y$ coordinate frame (units in meters). The z-axis points out of the page.



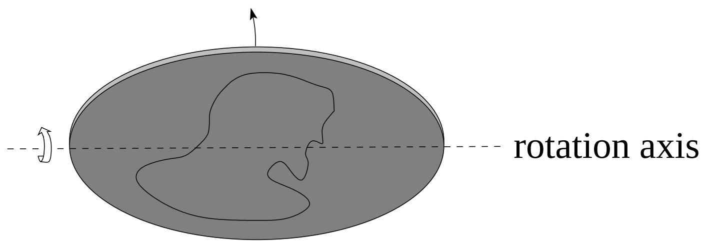
Find the moment of inertia of this system around the z-axis through the center of mass:

$\boldsymbol{I}_{\rm cm} = \boxed{}$

Now find the moment of inertia of this system around an axis ***parallel*** to the z-axis but passing through a new pivot point at $(-5, 0, 0)$ meters.

$\boldsymbol{I}_{\rm new} = \boxed{}$

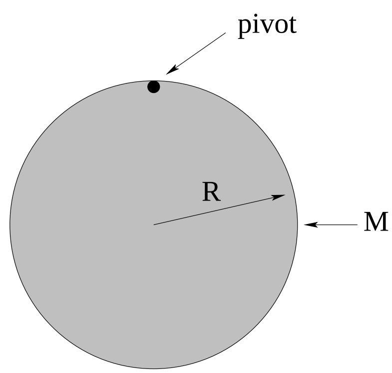
Problem 250. problems-1/moment-of-inertia-sa-disk-axis-in-plane.tex



You flip a coin with a friend to see who pays for lunch. The flipped coin spins rapidly around an axis *in the plane of the coin* as shown. Assuming the coin to be a uniform disk of mass M and radius R, find the moment of inertia of the coin ***about this axis***.

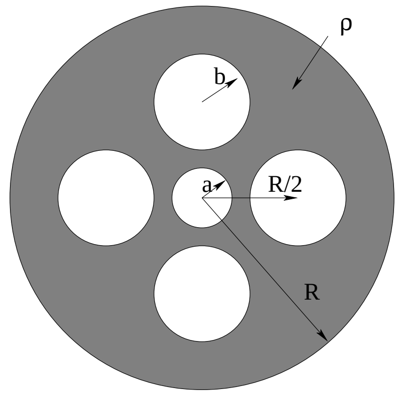
8.2.3 Long Problems

Problem 251. problems-1/moment-of-inertia-pr-disk-pivoted-at-rim.tex



In the figure above, a disk of mass M and radius R is pivoted about a point on the rim as shown. What is the moment of inertia of the disk about this pivot?

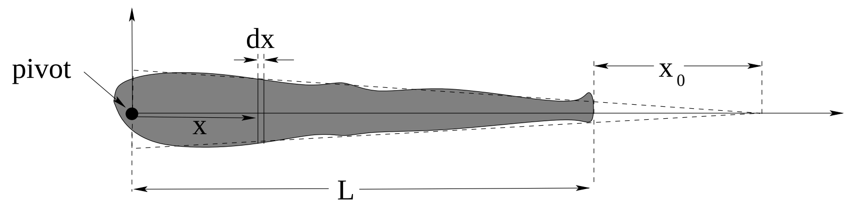
Problem 252. problems-1/moment-of-inertia-pr-disk-with-holes.tex



You are employed by a company that makes cogs, pulleys, and other widgets for lawnmower engines. They have designed a new pulley that is basically an annular disk of thickness t, outer radius R, and inner radius a, with approximately uniform density otherwise, as shown. To save on material costs (and to be able to deliver more torque to the real payload, instead of the pulley itself) they have removed all the material in four large circular holes of radius b through the solid part of the disk, centered on a circle of radius $R/2$ as shown. Your job is to compute the new moment of inertia as a function of ρ, t, R and $a, b < R/2$.

Hints: Note that you SHOULDN'T have to actually do any integrals in this problem if you remember that the moment of inertia of a disk is $\frac{1}{2}MR^2$. You are also welcome to introduce quantities like $M = \rho\pi R^2 t$, $m_a = \rho\pi a^2 t$ and $m_b = \rho\pi b^2 t$ into the problem if it would make the final answer simpler. Explain/show your reasoning regardless.

Problem 253. problems-1/moment-of-inertia-pr-moments-of-a-leg.tex



This problem will help you learn required concepts such as:

- Finding the Center of Mass using Integration
- Finding the Moment of Inertia using Integration

so please review them before you begin.

A *simple* model for the one-dimensional mass distribution of a human leg of length L and mass M is:

$$\lambda(x) = C \cdot (L + x_0 - x)$$

Note that this quantity is maximum at $x = 0$, varies linearly with x, and vanishes smoothly at $x = L + x_0$. That means that it doesn't *reach* $\lambda = 0$ when $x = L$, just as the mass per unit length of your leg doesn't reach zero at your ankles.

a) Find the constant C in terms of M, L, and x_0 by evaluating:

$$M = \int_0^L \lambda(x)\ dx$$

and solving for C.

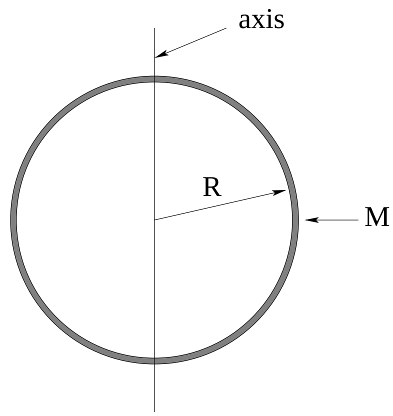
b) Find the center of mass of the leg (as a distance down the leg from the hip/pivot at the origin). You may leave your answer in terms of C (now that you know it) or you can express it in terms of L and x_0 only as you prefer.

c) Find the moment of inertia of the leg about the hip/pivot at the origin. Again, you may leave it in terms of C if you wish or express it in terms of M, L and x_0. Do your answers all have the right units?

d) How might one improve the estimate of the moment of inertia to take into account the foot (as a lump of "extra mass" m_f out there at $x = L$ that doesn't quite fit our linear model)?

This is, as you can see, something that an orthopedic specialist might well need to actually do with a much better model in order to e.g. outfit a patient with an artificial hip. True, they might use a computer to do the actual computations required, but is it plausible that they could possibly do what they need to do without knowing the physics involved in some detail?

Problem 254. problems-1/moment-of-inertia-pr-ring-axis-in-plane.tex

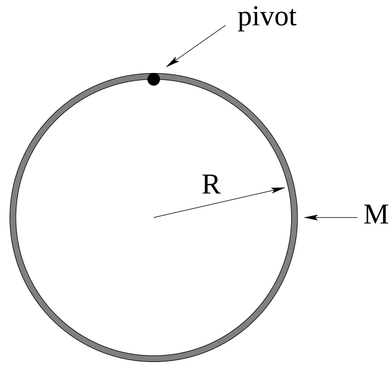


In the figure above, a ring of mass M and radius R is rotated around an axis through the middle in the plane of the ring as shown.

a) Find the moment of inertia of the ring about this axis through direct integration.

b) Find the moment of inertia of the ring about this axis using the perpendicular axis theorem. Which is easier?

Problem 255. problems-1/moment-of-inertia-pr-ring-pivoted-at-rim.tex



In the figure above, a ring of mass M and radius R is pivoted about a point on the rim as shown. What is the moment of inertia of the ring about this pivot?

Chapter 9

Vector Torque and Angular Momentum

Well, if we can rotate around an axis in the x, y, or z direction, we can rotate around an axis in ***any*** direction. So I guess torque has to be a vector quantity! Since it already has the magnitude of the cross product $\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$, we might as well define that to be the vector torque.

That, in turn, is related to the ***new*** quantity $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$, the ***angular momentum*** (also a vector quantity) and suddenly we can do ***rotational collisions*** that conserve ***angular momentum*** where before we did linear collisions that conserved regular vector momentum. Or we can even do collisions that conserve both!

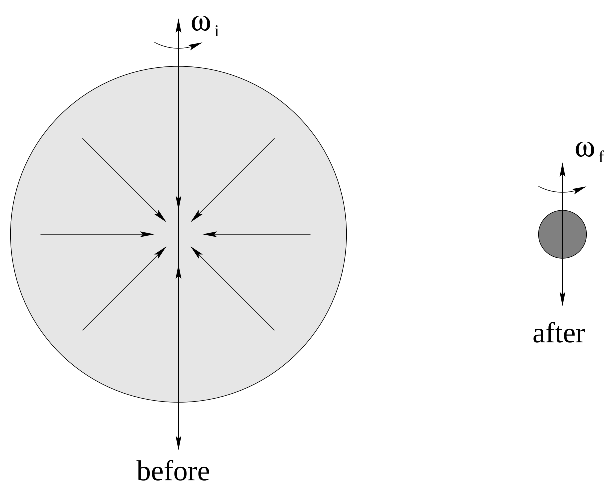
The most startling thing about vector torque, however, is when we observe a spinning object ***precess*** under the application of a vector torque. Our definitions above ***work perfectly to describe an insanely complicated motion*** (if you think about it) in *simple* terms. Ones we can compute. Ones that I, at least, ***require*** students to be able to solve for and understand.

Too bad Obi-wan didn't say ***Use the Torque, Luke!"***...

9.1 Angular Momentum

9.1.1 Multiple Choice

Problem 256. problems-1/angular-momentum-mc-collapsing-star.tex



When a star rotating with an angular speed ω_i (eventually) exhausts its fuel, escaping light energy can no longer oppose gravity throughout the star's volume and it ***suddenly*** shrinks, with most of its outer mass falling in towards the center all at the same time.

As this happens, does the magnitude of the angular speed of rotation ω_f:

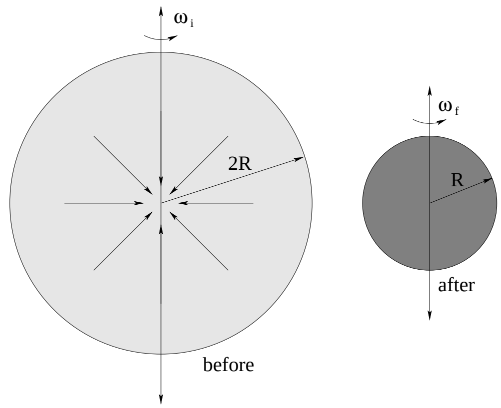
a) increase

b) decrease

c) remain about the same

Why (state the principle used to answer the question)?

Problem 257. problems-1/angular-momentum-mc-forming-star.tex

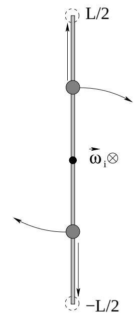


Gravity gradually assembles a star by pulling a cloud of rotating gas together into a rotating ball that then gradually shrinks. The figure above represents a star at two different stages in its formation, the first where a gas of total mass M has formed a ball of radius $2R$ rotating at angular speed ω_i, the second where the ball has collapsed to a radius R (compressing the nuclear fuel inside closer to the point of fusion and ignition), rotating at a possibly new angular speed ω_f.

Assuming that the mass is uniformly distributed in both cases, what is the best estimate for ω_f in terms of ω_i?

a) $\omega_f = \omega_i$ b) $\omega_f = 2\omega_i$ c) $\omega_f = 4\omega_i$ d) $\omega_f = \omega_i/2$ e) $\omega_f = \omega_i/4$

Problem 258. problems-1/angular-momentum-mc-rotating-rod-sliding-beads.tex



In the figure above, a massless rod of length L is rotating around a frictionless pivot through its center at angular speed ω_i. Two beads, each with mass m, are stuck a distance $L/4$ from the center. The rotating system initially has a total kinetic energy K_i (which you could actually calculate if you needed to). At a certain time, the beads are released and slide smoothly to the ends of the rod where again, they stick. Which statement about the final angular speed and rotational kinetic energy of the rotating system is true:

a) $\omega_f = \omega_i/2$ and $K_f = K_i$.

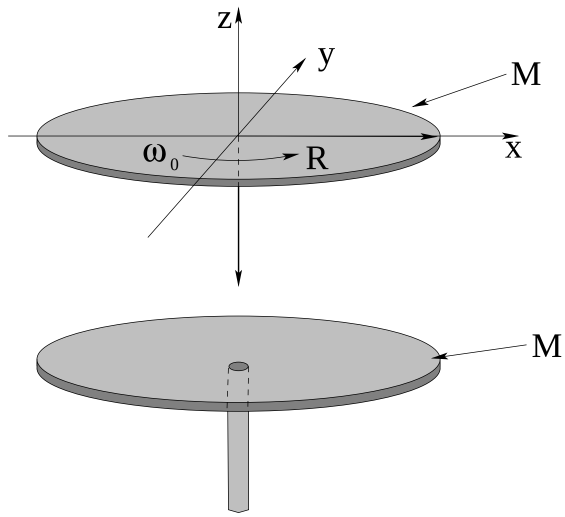
b) $\omega_f = \omega_i/4$ and $K_f = K_i/4$.

c) $\omega_f = \omega_i/4$ and $K_f = K_i/2$.

d) $\omega_f = \omega_i/2$ and $K_f = K_i/4$.

e) $\omega_f = \omega_i/2$ and $K_f = K_i/2$.

Problem 259. problems-1/angular-momentum-mc-two-circular-plates-collide.tex



A **disk** of uniformly distributed mass M and radius R sits at rest on a turntable that permits it to rotate freely. A second uniform disk of mass M with the same radius, centered on the same axis of rotation, is rotating at an (initial) angular speed ω_i and is dropped gently onto it so that (after sliding for an instant) they stick together and rotate together as one.

How do the final angular velocity and final kinetic energy relate to the initial angular velocity and initial kinetic energy?

a) $\omega_f = \omega_i, \quad K_f = K_i$

b) $\omega_f = 2\omega_i, \quad K_f = K_i/2$

c) $\omega_f = \omega_i/2, \quad K_f = K_i/2$

d) $\omega_f = \omega_i/4, \quad K_f = K_i/4$

e) We cannot tell from the information given.

9.1.2 Short Answer

Problem 260. problems-1/angular-momentum-sa-bug-on-rotating-disk.tex

A disk of mass M and radius R is rotating about its axis with initial angular velocity ω_0. A rhinoceros beetle with mass m is standing on its outer rim as it does so. The beetle decides to walk in to the very center of the disk and stand on the axis as it feels less pseudoforce there and it is easier to hold on. What is the angular velocity of the disk when it gets there?

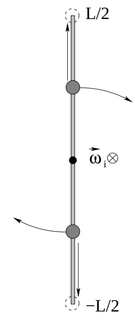
(Ignore friction and drag forces).

Problem 261. problems-1/angular-momentum-sa-conserved-quantities.tex

For each of the collisions described below, say whether the total mechanical energy, total momentum, and total angular momentum of the system consisting of the two colliding objects are conserved or not. Indicate your answer by writing "**C**" (for "is ***definitely*** conserved") or "**N**" (for "not ***necessarily*** conserved") in each box. You may write a brief word of explanation if you think there is any ambiguity in the answer.

	Total Energy	Linear Momentum	Angular Momentum
A hard ball (point particle) bounces off of a rigid wall that cannot move, returning at the same speed it had before the collision.	☐	☐	☐
A piece of space junk strikes the orbiting space shuttle and sticks to it.	☐	☐	☐

Problem 262. problems-1/angular-momentum-sa-rotating-rod-sliding-beads.tex



In the figure above, a massless rod of length L is rotating around a frictionless pivot through its center at angular speed ω_i. Two beads, each with mass m, are stuck a distance $L/4$ from the center. The rotating system initially has a total kinetic energy K_i (which you could actually calculate if you needed to). At a certain time, the beads are released and slide smoothly to the ends of the rod where again, they stick.

A) What quantities of the system (rod plus two beads) are conserved by this process? (Place a Y or N in the provided answer boxes.)

☐ Total Kinetic Energy

☐ Total Linear Momentum

☐ Total Angular Momentum

B) Determine the ratio of the following quantities:

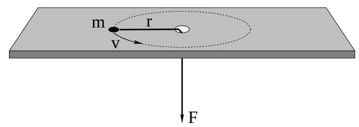
$$\frac{I_f}{I_i} =$$

$$\frac{\omega_f}{\omega_i} =$$

$$\frac{K_f}{K_i} =$$

9.1.3 Long Problems

Problem 263. problems-1/angular-momentum-pr-circular-orbit-on-table.tex



This problem will help you learn required concepts such as:

- Torque Due to Radial Forces
- Angular Momentum Conservation
- Centripetal Acceleration
- Work and Kinetic Energy

so please review them before you begin.

A particle of mass M is tied to a string that passes through a hole in a frictionless table and held. The mass is given a push so that it moves in a circle of radius r at speed v. We will now analyze the physics of its motion in two stages.

a) What is the torque exerted on the particle by the string? Will angular momentum be conserved if the string pulls the particle into "orbits" with different radii?

b) What is the magnitude of the angular momentum L of the particle in the direction of the axis of rotation (as a function of m, r and v)?

c) Show that the magnitude of the force (the tension in the string) that must be exerted to keep the particle moving in a circle is:
$$F = \frac{L^2}{mr^3}$$
This is a *general* result for a particle moving in a circle and in no way depends on the fact that the force is being exerted by a string in particular.

d) Show that the kinetic energy of the particle in terms of its angular momentum is:
$$K = \frac{L^2}{2mr^2}$$

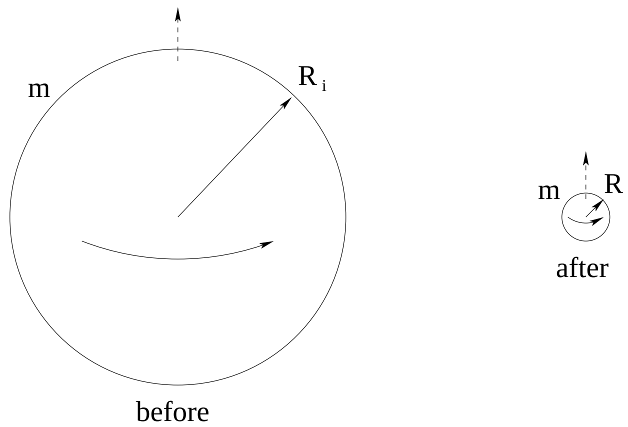
Now, suppose that the radius of the orbit and initial speed are r_i and v_i, respectively. From under the table, the string is *slowly* pulled down (so that the puck is always moving in an approximately circular trajectory and the tension in the string remains radial) to where the particle is moving in a circle of radius r_2.

e) Find its velocity v_2 using angular momentum conservation. This should be very easy.

f) Compute the work done by the force from part c) above and identify the answer *as* the work-kinetic energy theorem. Use this to to find the velocity v_2. You should get the same answer!

Note that the last two results are pretty amazing – they show that our torque and angular momentum theory so far is remarkably *consistent* since two *very* different approaches give the same answer. Solving this problem now will make it easy later to understand the *angular momentum barrier*, the angular kinetic energy term that appears in the radial part of conservation of mechanical energy in problems involving a central force (such as gravitation and Coulomb's Law). This in turn will make it easy for us to understand certain properties of orbits from their potential energy curves.

Problem 264. problems-1/angular-momentum-pr-collapse-of-sun.tex



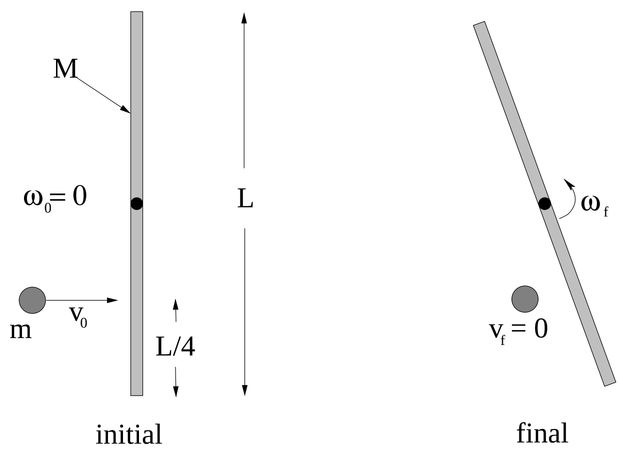
The sun reaches the end of its life and gravitationally collapses quite suddenly, forming a white dwarf. Before it collapses, it has a mass m, a radius R_i, and a period of rotation T_i. After it collapses, its radius is $R_f \ll R_i$ and we will assume that its mass is unchanged. We will also assume that before and after the moment of inertia of the sun is given by $I = \beta m R^2$ where R is the appropriate radius.

a) What is its final period of rotation T_f after the collapse?

b) Evaluate the escape velocity from the surface of the sun before and after its collapse.

For 2 points of extra credit, evaluate the numbers associated with these expressions given $\beta = 0.25$, $m = 2 \times 10^{30}$ kg, $R_i = 5 \times 10^5$ km, $R_f = 100$ km, and $T_i = 108,000$ seconds. These numbers are actually quite interesting in cosmology, as the escape velocity from the surface of the white dwarf approaches the speed of light...

Problem 265. problems-1/angular-momentum-pr-disk-collides-with-pivoted-rod.tex



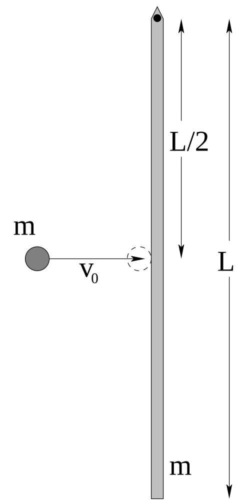
A steel rod of mass M and length L with a frictionless pivot in the center and moment of inertia $\frac{1}{12}ML^2$ sits on a frictionless table at rest. The pivot is attached to the table. A steel disk of mass m approaches with velocity v_0 from the left and strikes the rod a distance $L/4$ from the lower end as shown. This ***elastic*** collision instantly brings the disk to rest and causes the rod to rotate with angular velocity ω_f.

a) What quantities are conserved in this collision?

b) Find the angular velocity ω_f of the rod about the pivot after the collision.

c) Find the ratio m/M such that the collision occurs ***elastically***, as described.

Problem 266. problems-1/angular-momentum-pr-marble-and-rod.tex



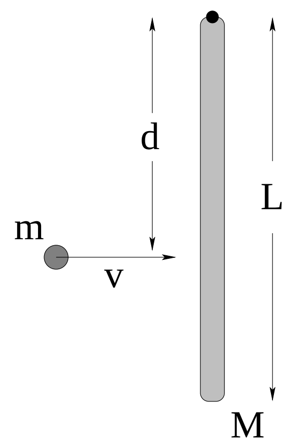
In the figure above, a marble with mass m travelling to the right at speed v_0 collides with a rigid rod of length L pivoted about one end, also of mass m, . The marble strikes the rod $L/2$ down from the pivot and comes **precisely to rest** in the collision. Ignore gravity, drag forces, and any friction in the pivot.

a) What is the ***rotational velocity*** $\boldsymbol{\omega_f}$ of the rod after the collision?

b) What is the change in ***linear momentum in the x direction*** $\boldsymbol{\Delta p_x}$ (to the right) during this collision?

c) What is the ***change in kinetic energy*** $\boldsymbol{\Delta K}$ in this collision? The sign of your answer should indicate whether energy was gained or lost.

Problem 267. problems-1/angular-momentum-pr-putty-sticks-to-pivoted-rod-gravity.tex



A rod of mass M and length L is hanging vertically from a frictionless pivot (where gravity is "down"). A blob of putty of mass m approaches with velocity v from the left and strikes the rod a distance d from its center of mass as shown, sticking to the rod.

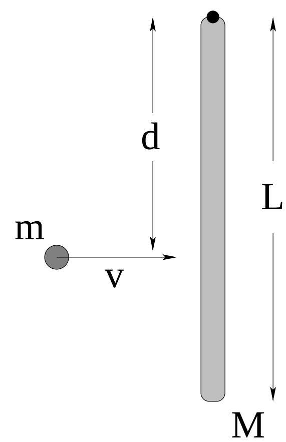
a) Find the angular velocity ω_f of the system about the pivot (at the top of the rod) after the collision.

b) Find the distance $x_{\rm cm}$ from the pivot of the center of mass of the rod-putty system immediately after the collision.

c) After the collision, the rod swings up to a maximum angle $\theta_{\rm max}$ and then comes momentarily to rest. Find $\theta_{\rm max}$.

All answers should be in terms of M, m, L, v, g and d as needed. The moment of inertia of a rod pivoted about one end is $I = \frac{1}{3}ML^2$, in case you need it.

Problem 268. problems-1/angular-momentum-pr-putty-sticks-to-pivoted-rod.tex



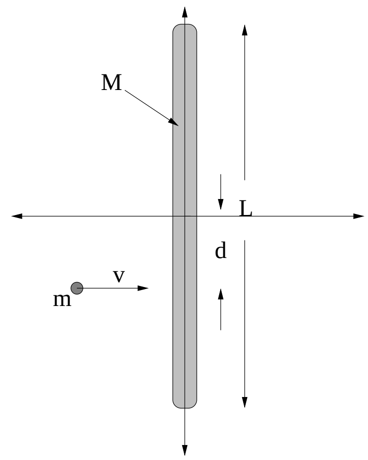
A rod of mass M and length L rests on a frictionless table and is pivoted on a frictionless nail at one end as shown. A blob of putty of mass m approaches with velocity v from the left and strikes the rod a distance d from the end as shown, sticking to the rod.

a) Find the angular velocity ω of the system about the nail after the collision.

b) Is the linear momentum of the rod/blob system conserved in this collision for a general value of d? If not, why not?

c) Is there a value of d for which it *is* conserved? If there were such a value, it would be called the *center of percussion* for the rod for this sort of collision.

Problem 269. problems-1/angular-momentum-pr-putty-sticks-to-unpivoted-rod.tex



This problem will help you learn required concepts such as:

- Angular Momentum Conservation
- Momentum Conservation
- Inelastic Collisions
- Impulse

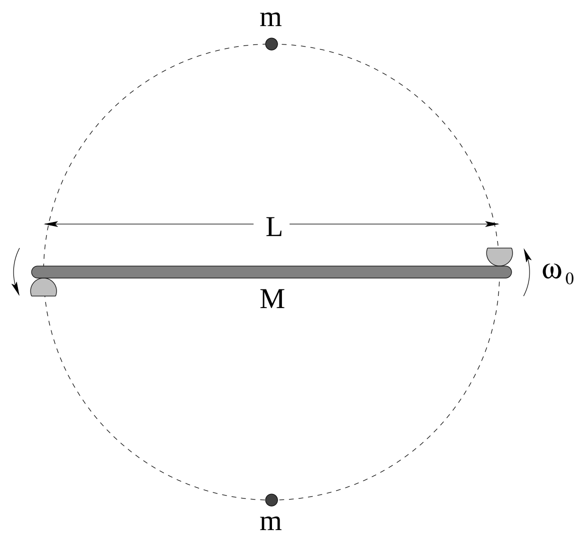
so please review them before you begin.

A rod of mass M and length L rests on a frictionless table. A blob of putty of mass m approaches with velocity v from the left and strikes the rod a distance d from the end as shown, sticking to the rod.

- Find the angular velocity ω of the system about the center of mass *of the system* after the collision. Note that the rod and putty will not be rotating about the center of mass of the rod!
- Is the linear momentum of the rod/blob system conserved in this collision for a general value of d? If not, why not?

All answers should be in terms of M, m, L, v and d as needed. Note well that you should clearly indicate what physical principles you are using to solve this problem at the *beginning* of the work.

Problem 270. problems-1/angular-momentum-pr-spinning-cups-catch-balls.tex

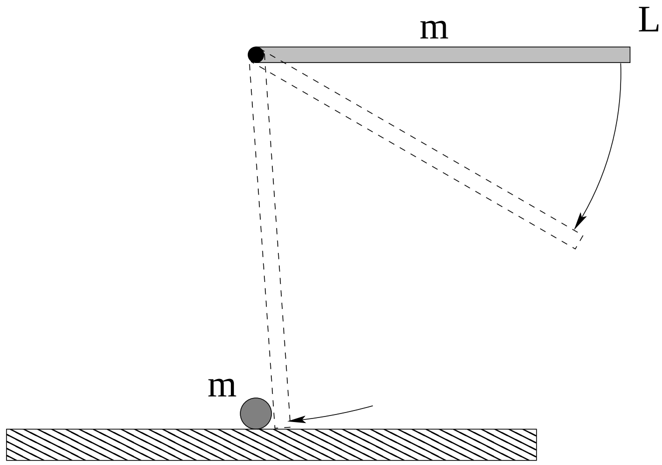


In the figure above, a bar of length L with two cups at the ends is freely rotating (in space – ignore gravity and friction or drag forces) about its center of mass with angular velocity ω_0. The bar and cups together have a mass M and a moment of intertia of $I = \beta M L^2$. When the bar reaches the vertical position, the cups catch two small balls of mass m that are at rest, which stick in the cups. The balls have a negligible moment of inertia about their own center of mass – you may think of them as particles.

a) What is the velocity of the center of mass of the system after the collision?

b) What is the angular velocity of the bar after it has caught the two balls in its cups? Is kinetic energy gained or lost in this process?

Problem 271. problems-1/angular-momentum-pr-swinging-rod-strikes-putty.tex



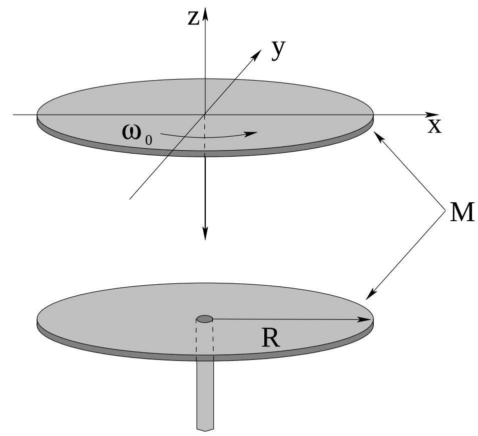
A uniform rod of mass m and length L swings about a frictionless peg through its ***end***. The rod is held horizontally and released from rest as shown in the figure. At the bottom of its swing the rod strikes a ball of putty of mass m that sits at rest on a frictionless table. In answering the questions take the magnitude of acceleration due to gravity to be g and assume that gravity acts downward (in the usual way). The questions below should be answered in terms of the given quantities.

a) What is the angular speed ω_i of the rod just before it hits the putty?

b) If the putty ***sticks to the rod***, what is the angular speed ω_f of the rod-putty system immediately after the collision?

c) What is ΔE, the mechanical energy change of the system in this collision (be sure to specify its sign).

Problem 272. problems-1/angular-momentum-pr-two-circular-plates-collide.tex



A **disk** of mass M and radius R sits at rest on a turntable that permits it to rotate freely. A second identical disk, this one rotating around their mutual axis at an angular speed ω_0, is dropped gently onto it so that (after sliding for an instant) they rotate together. In terms of the givens M, R, ω_0 and known constants:

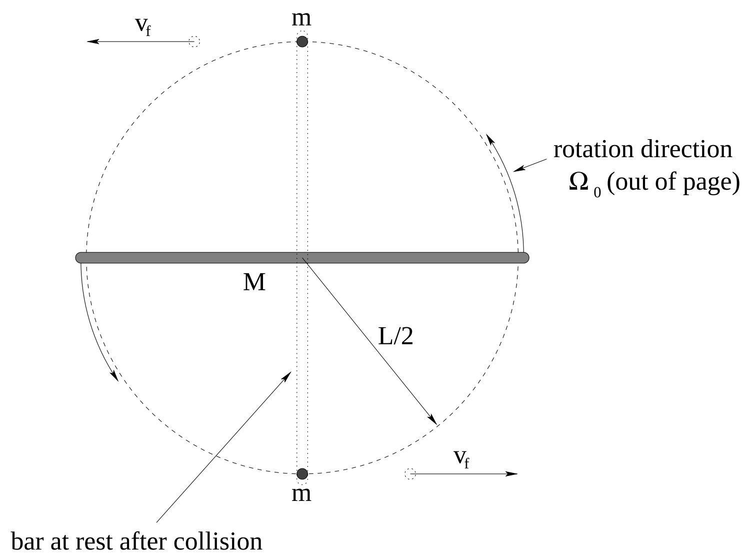
a) Find the ***final*** angular speed ω_f of the two disks moving together ***after*** the collision:

$$\omega_f = \boxed{}$$

b) What ***fraction*** of the original kinetic energy of the system K_0 is gained (+) or lost (-) in this rotational collision?

$$\Delta K = \boxed{} \times K_0$$

Problem 273. problems-1/angular-momentum-pr-rotating-bar-elastic-collision-balls.tex



In the figure above, a unpivoted solid rod of length L and mass M is rotating around its center of mass with an angular velocity Ω_0 out of the page. It simultaneously strikes two hard balls of mass m sitting at rest a distance $L/2$ from the center of rotation as shown, causing them to recoil to the left and right respectively. After the collision ***the rod is at rest***.

a) Is momentum conserved in this collision?

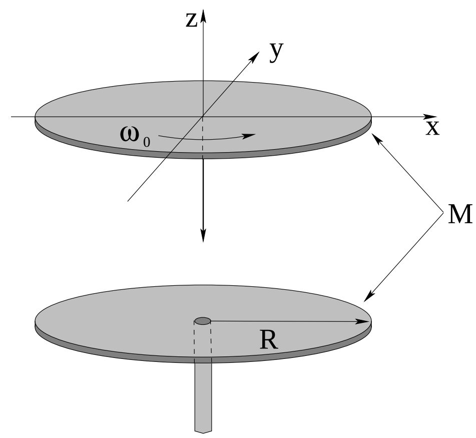
b) Find the final ***speed*** of either ball, v_f.

c) Find the ratio of masses m/M such that the collision as described is ***elastic***.

9.2 Vector Torque

9.2.1 Multiple Choice

Problem 274. problems-1/torque-vector-mc-rotational-collision-two-disks.tex



Two identical disks with mass M and radius R have a common axis and frictionless bearing. Initially, one disk is spinning with some angular velocity ω_0 and the other is rest. The two disks are brought together quickly so that they stick and rotate as one without the application of any external torque. Circle the true statement below:

a) The total kinetic energy and the total angular momentum are unchanged.

b) The total kinetic energy and total angular momentum are both reduced to half their original values.

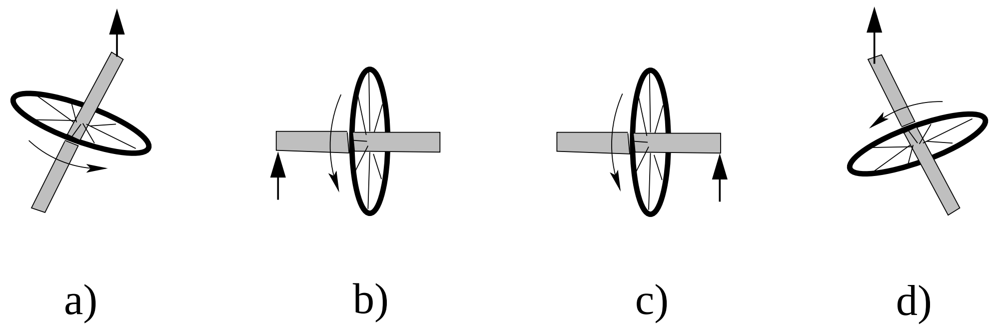
c) The total kinetic energy is unchanged, but the total angular momentum is reduced to half of its original value.

d) The total angular momentum is unchanged, but the total kinetic energy is reduced to half of its original value.

e) We cannot tell what happens to the angular momentum and kinetic energy from the information given.

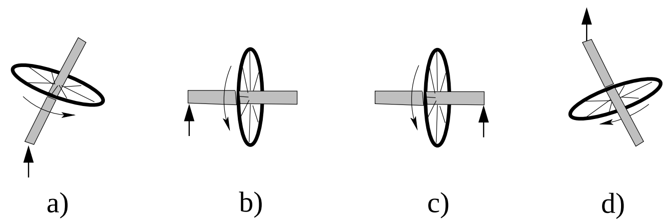
9.2.2 Short Answer

Problem 275. problems-1/torque-vector-sa-direction-of-precession-1.tex



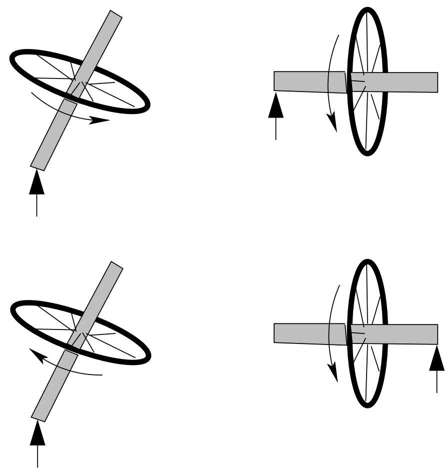
In the figure above four symmetric gyroscopes are portrayed. Each gyroscope is spinning very rapidly in the direction shown, and is suspended/pivoted from one end as shown at the big arrow (gravity points ***down***). For each figure a-d indicate whether the gyroscope will precess ***in*** or ***out*** of the page at the *other* (non-pivoted, free) *end* at the instant shown.

Problem 276. problems-1/torque-vector-sa-direction-of-precession-2.tex



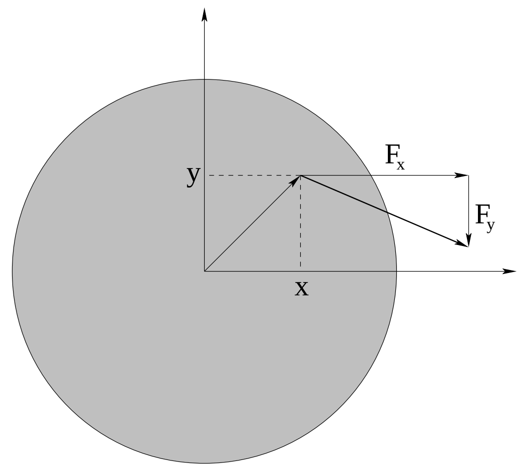
In the figure above four symmetric gyroscopes are portrayed. Each gyroscope is spinning very rapidly in the direction shown, and is suspended/pivoted from one end as shown at the big arrow (gravity points ***down***). For each figure a-d indicate whether the gyroscope will precess ***in*** or ***out*** of the page at the *other* (non-pivoted, free) *end* at the instant shown.

Problem 277. problems-1/torque-vector-sa-direction-of-precession.tex



In the figure above four symmetric gyroscopes are portrayed. Each gyroscope is spinning very rapidly in the direction shown, and is suspended from one end as shown (at the big arrow). For each figure indicate whether the gyroscope will precess **in** or **out** of the page at the *other* (free) *end* at the instant shown.

Problem 278. problems-1/torque-vector-sa-evaluate-the-torque-1.tex



In the figure above, a force

$$\vec{\boldsymbol{F}} = 2\hat{\boldsymbol{x}} - 1\hat{\boldsymbol{y}}$$

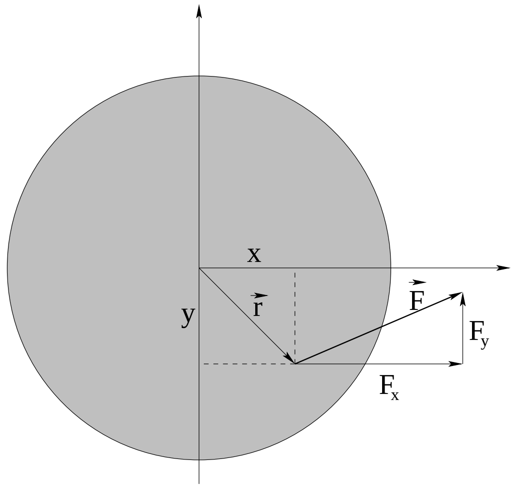
Newtons is applied to a disk at the point

$$\vec{\boldsymbol{r}} = 2\hat{\boldsymbol{x}} + 2\hat{\boldsymbol{y}}$$

as shown. (That is, $F_x = 2$ N, $F_y = -1$ N, $x = 2$ m, $y = 2$ m). Find the *total torque* about a pivot *at the origin*.

Don't forget that torque is a vector, so either give the answer in cartesian coordinates or otherwise specify its direction!

Problem 279. problems-1/torque-vector-sa-evaluate-the-torque-2.tex



In the figure above, a force

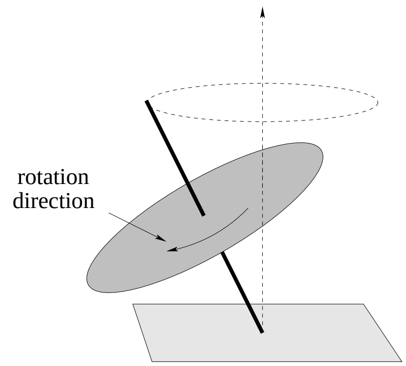
$$\vec{\boldsymbol{F}} = 2\hat{\boldsymbol{x}} + 1\hat{\boldsymbol{y}}$$

Newtons is applied to a disk at the point

$$\vec{\boldsymbol{r}} = 2\hat{\boldsymbol{x}} - 2\hat{\boldsymbol{y}}$$

as shown. (That is, $F_x = 2$ N, $F_y = 1$ N, $x = 2$ m, $y = -2$ m). Find the *total torque* about a pivot *at the origin*. Don't forget that torque is a ***vector***, so specify its direction as well as its magnitude (or give the answer as a cartesian vector)! Show your work!

Problem 280. problems-1/torque-vector-sa-precession-of-top-1.tex

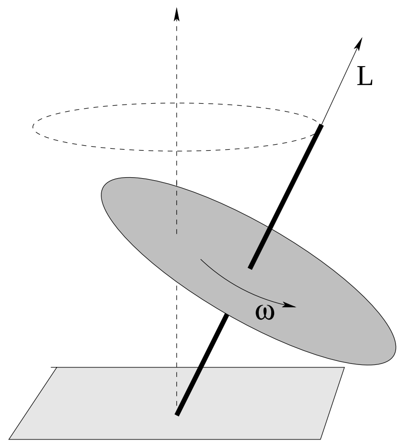


Draw the direction of $\vec{L}$ onto the spinning top in the figure above, and ***circle*** the direction that the upper tip of the top will precess:

in **out**

of the page. **Draw this direction onto the figure as well.**

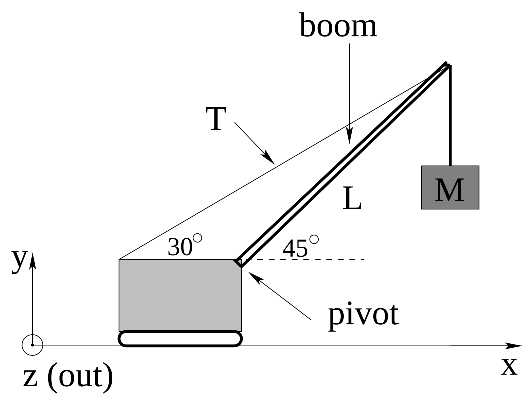
Problem 281. problems-1/torque-vector-sa-precession-of-top-2.tex



Clearly show the direction that the spinning top will precess on the figure above, given the direction of its angular momentum as indicated.

9.2.3 Long Problems

Problem 282. problems-1/torque-vector-pr-crane-boom.tex



$$\sin(30^\circ) = \cos(60^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$$

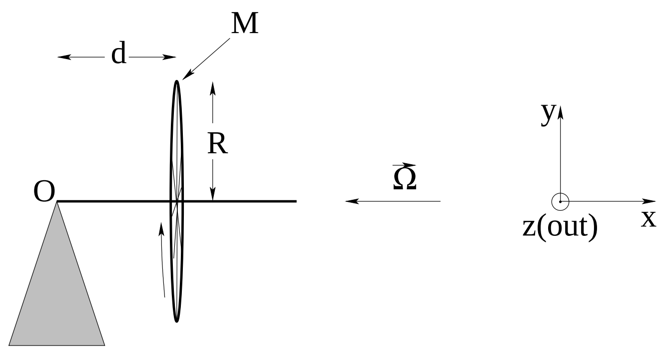
$$\sin(45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$$

A crane with a "massless" boom (the long support between the body and the load) of length L holds a mass M suspended as shown. Note that the wire with the tension T is **fixed** to the top of the boom, not run over a pulley to the mass M.

a) Find the torque (magnitude and direction) exerted by the tension in the wire on the boom, relative to a pivot at the base of the boom.

b) Find the torque (magnitude and direction) exerted by the hanging mass, relative to a pivot at the base of the boom.

Problem 283. problems-1/torque-vector-pr-precessing-bicycle-wheel.tex



A bicycle wheel (basically a ring) of mass M and radius R has massless spokes and a massless axle of length d. The other end O of the axle rests on a conical support as shown. The axle is held in a horizontal position and the wheel is spun with a large angular velocity $\vec{\Omega}$ that points ***towards O***, and then released so that the wheel precesses about O.

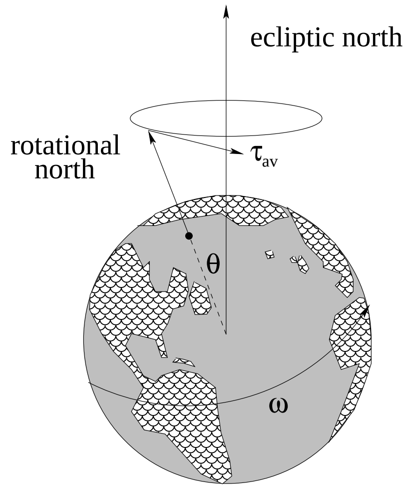
(Note: To specify the direction of vectors you may use *up*, *down*, *towards O*, *away from O*, *into the page*, *out of the page* as shown.)

a) What is the angular momentum $\vec{L}$ of the wheel about its center of mass?

b) What is (find, with any argument) the ***angular frequency of precession*** $\vec{\omega}_p$ of the wheel? Don't forget to give the direction!

c) What is the kinetic energy K of the wheel in the frame of O (i.e., the lab frame) ***including the contribution from the motion of the center of mass as it precesses!*** This is one of the factors we ignored in our elementary treatment in class.

Problem 284. problems-1/torque-vector-pr-precession-of-equinoxes.tex



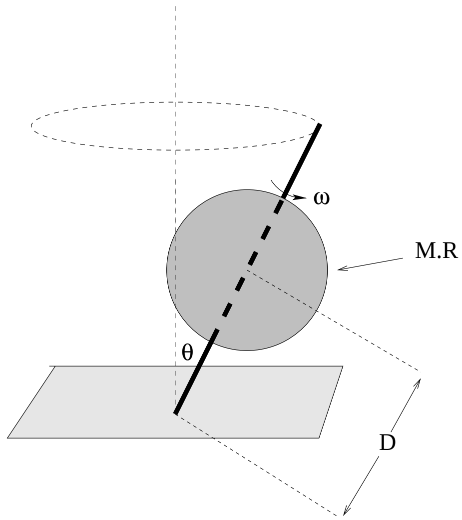
The Earth revolves on its axis. Its north (right handed) axis is significantly tipped relative to the "ecliptic" pole of the Earth's revolution around the Sun. It is currently aligned with Polaris, the pole star, but because the Sun exerts a small *torque* on it due to tides acting on its slightly oblate spheroidal shape, it also *precesses* around the ecliptic north once every (approximately) 26,000 years!

a) Assuming that the average torque on the earth over the course of any given year remains perpendicular to its angular momentum in the direction/handedness shown, derive an algebraic expression for the angular frequency of precession in terms of the magnitude of the torque. You may use I as the moment of inertia for the earth about its rotational axis as that quantity is given below.

b) Given the data that the moment of inertia of the Earth about its axis of rotation is roughly 8×10^{37} kg-m^2, that its axis is tipped at roughly 20 degrees relative to the ecliptic and that its period of revolution about its own axis is one day, estimate the approximate magnitude of the average torque exerted by the Sun on the Earth over the course of a year.

(You may find it useful to know that 1 day = 86400 seconds, and 1 year = 3.15×10^7 seconds – you can remember the latter as approximately $\pi \times 10^7$ seconds.)

Problem 285. problems-1/torque-vector-pr-precession-of-spherical-top.tex



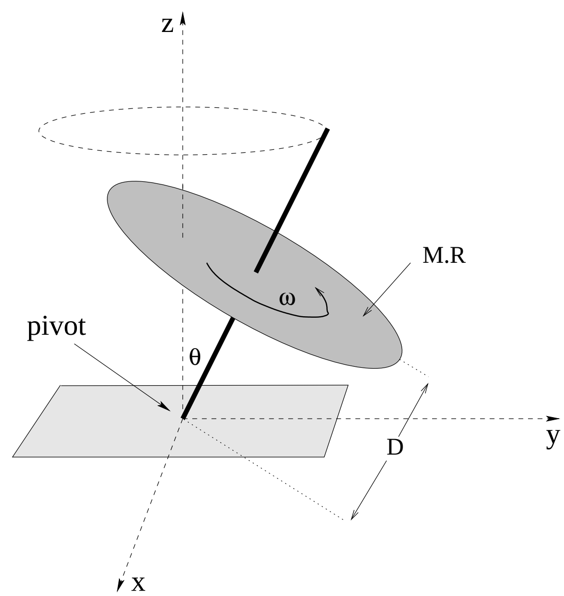
A top is made from a ball of radius R and mass M with a very thin, light nail ($r \ll R$ and $m \ll M$) for a spindle so that the center of tha ball is a distance D from the tip. The top is spun with a large angular velocity ω, and has a moment of inertia $I = \frac{2}{5}MR^2$.

a) What is the angular momentum of the spinning ball? Indicate its (vector) direction with an arrow on the figure.

b) When the top is spinning at a small angle θ with the vertical (as shown) what is the angular frequency ω_p of the top's precession?

c) Does the top precess ***into*** or ***out of*** the page at the instant shown?

Problem 286. problems-1/torque-vector-pr-precession-of-top-3-parts.tex



A top is made of a uniform ***disk*** of radius R and mass M with a very thin, light (assume massless) nail for a spindle so that the center of the disk is a distance D from the tip. The top is spun with a large angular velocity ω with the nail vertically above the y-axis as shown above.

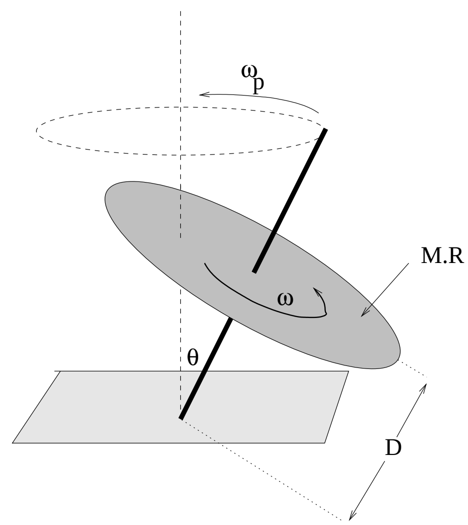
a) Find the ***vector*** torque $\vec{\tau}$ exerted about the pivot at the instant shown in the figure. You may express the vector however you wish (e.g. magnitude and direction, cartesian components).

b) What is the axis of precession?

c) Derive the precession frequency ω_p. Any of the derivations used in class or discussed in the textbook are acceptable.

Express all answers in terms of M, R, g, D, and θ as needed.

Problem 287. problems-1/torque-vector-pr-precession-of-top.tex



This problem will help you learn required concepts such as:

- Vector Torque
- Vector Angular Momentum
- Geometry of Precession

so please review them before you begin.

A top is made of a disk of radius R and mass M with a very thin, light nail ($r \ll R$ and $m \ll M$) for a spindle so that the disk is a distance D from the tip. The top is spun with a large angular velocity ω. When the top is spinning at a small angle θ with the vertical (as shown) what is the angular frequency ω_p of the top's precession?

Chapter 10

Statics

Ah, time for a rest. We've spent a lot of energy (now that we know what it is) learning how to solve problems where lots of stuff moves and accelerates and does all kinds of dynamical things. Let's think about systems where the interesting thing is that ***nothing happens***.

We actually really need a lot of these systems, and yeah, they involve a fair bit of physics. Engineers do better if they build bridges and buildings that *don't fall down*. Physicians like for their patients ***not to fall over***. Human beings like to hang pictures, see-saw with their kids, arm wrestle, put things on tables, carry around glasses full of beer, balance things on their heads, build houses of cards and so much more where they idea is that these things should *not move*, or tip over, or fall down, or snap a suspending wire.

In order to sit still, an object has to *start out* sitting still and not accelerate (linearly or angularly). So a necessary condition for static equilibrium is that ***the total vector force and torque must vanish*** on the object(s) in question. Sounds simple!

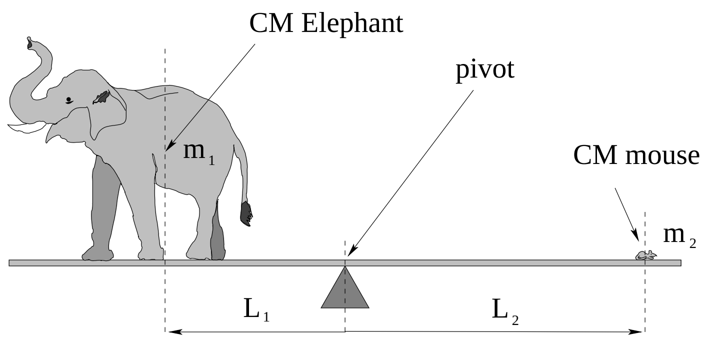
But of course this is as many as ***six conditions***, one for each of the possible vector components of force and torque. All of which have to be zero at the same time. Which means as many as six simultaneous equations per object have to be satisfied. Urp.

Maybe not so simple?

10.1 Statics

10.1.1 Multiple Choice

Problem 288. problems-1/statics-mc-elephant-mouse.tex



An elephant and a mouse sit at either end of a really long, really strong see-saw. The elephant, whose mass is m_1, sits so that its center of mass is a distance L_1 from the pivot. The mouse, whose mass is m_2, sits at L_2. The see-saw is ***balanced*** so the mouse and elephant are not moving up or down. Which is the following must be true:

a) $m_2 = m_1$

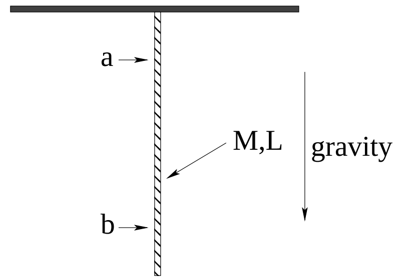
b) $m_2 = m_1(L_2/L_1)$

c) $m_1 = m_2(L_2/L_1)$

d) $m_1 = m_2(L_2/L_1)^2$

e) The mouse can never balance the elephant!

Problem 289. problems-1/statics-mc-hanging-rope.tex



In the figure above, a rope of mass M, length L is hanging from the ceiling in static equilibrium. Select the correct rank order of the tension in the rope at the points a and b:

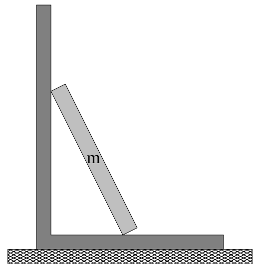
a) $T_a < T_b$

b) $T_b < T_a$

c) $T_a = T_b$

d) Insufficient information given to determine the answer.

Problem 290. problems-1/statics-mc-leaning-bar-reaction-pairs.tex



In the figure above, a board is sitting on a rough floor and leaning against a wall. Circle **three** action-reaction pairs in the list below:

a) The ladder top pushes against the wall; the wall pushes back against the ladder top.

b) The floor pushes up on the ladder base; gravity pulls the ladder base down towards the floor.

c) Static friction from the floor pushes the ladder base towards the wall; the wall pushes back on the ladder.

d) The floor pushes down on the ground; the ground pushes back on the floor.

e) The Earth pulls down on the ladder via gravity; the ladder pulls up on the Earth via gravity.

Problem 291. problems-1/statics-mc-pick-reaction-pairs-2.tex

Which of the following list are *not* action-reaction force pairs? (More than one answer is possible.)

a) A hydraulic piston pushes on the fluid in its cylinder; the fluid pushes back on the hydraulic piston.

b) The earth's gravity pulls a pendulum bob at rest down; the string pulls it up.

c) My finger pushes down against a grape I'm squeezing; my thumb pushes up against the grape.

d) My hammer pushes on a nail as it hits it; the nail pushes back on the hammer.

e) A bathroom scale pushes up on my feet as I stand on it; my feet push down on the scale.

Problem 292. problems-1/statics-mc-pick-reaction-pairs.tex

(3 points) Which of the following list are *not* action-reaction force pairs? (More than one answer is possible.)

a) The earth's gravity pulls down on an apple; the stem of the apple holds it up.

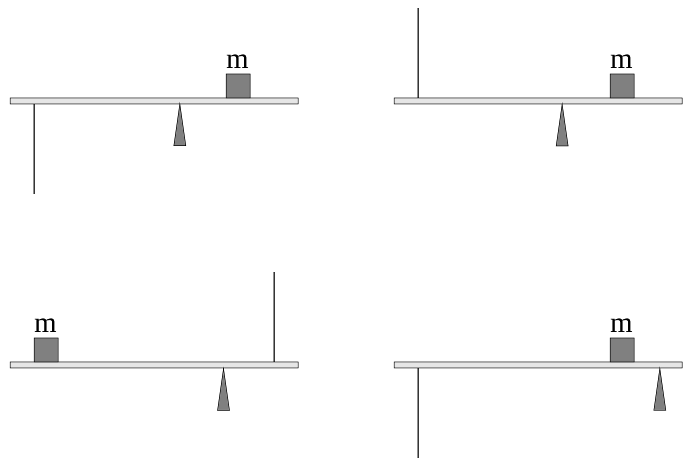
b) Water pressure pushes out against a glass, the glass holds in the water.

c) I push forward on a bow; the bowstring pulls forward on me (as I draw an arrow).

d) I lean my head on the wall; the wall pushes back on my head.

e) I pull down on the rope with my hand; the rope pulls up on my hand.

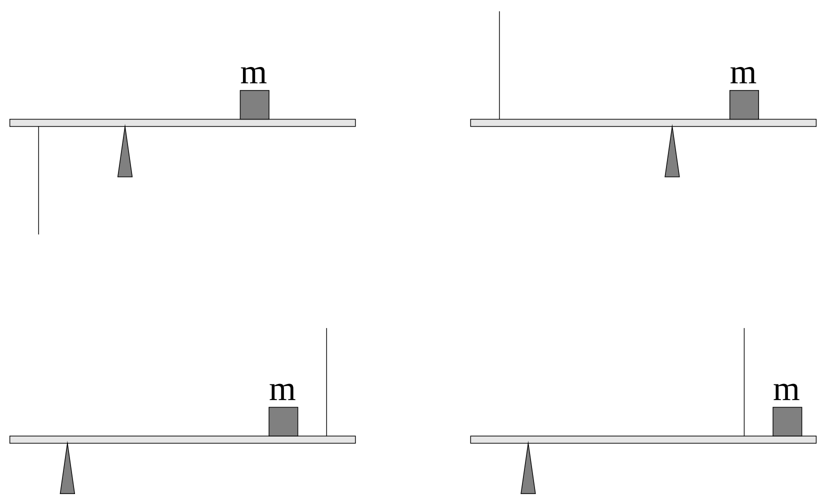
Problem 293. problems-1/statics-mc-plank-mass-rod.tex



(3 points) In the figure above, a very light (approximately massless) plank supports a mass m. The plank is resting on (not attached to) a sawhorse that can support as much weight as you like, and a rod is attached to the plank as shown (where the other end is firmly attached to the ceiling or floor as the case may be). The rod, however, will *break* if it is compressed or stretched with a force $F_b = mg$, the weight of the mass.

Circle all of the configurations where the plank and mass will not move **and** the rod will not break.

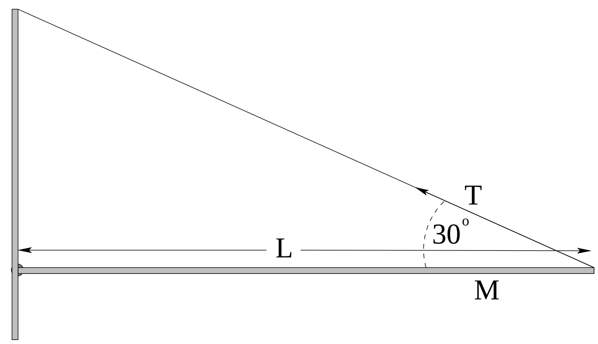
Problem 294. problems-1/statics-mc-plank-mass-string.tex



In the figure above, a very light (approximately massless) plank supports a mass m. The plank is resting on (not attached to) a sawhorse/pivot that can support as much weight as you like, and a massless ***string*** is attached to the plank as shown (the other end is tied to the ceiling or floor as the case may be). The string, however, will ***break*** at a force $F_b = mg$, the weight of the mass.

Circle all of the configurations where the plank and mass will not move ***and*** the string will not break.

Problem 295. problems-1/statics-mc-string-and-bar.tex



A bar of mass M and length L is pivoted by a hinge on the left and is supported on the right by a string attached to the wall and the right hand end of the bar. The angle made by the string with the bar is $\theta = 30°$. Select the true statement from the list below.

a) $T = Mg/2$

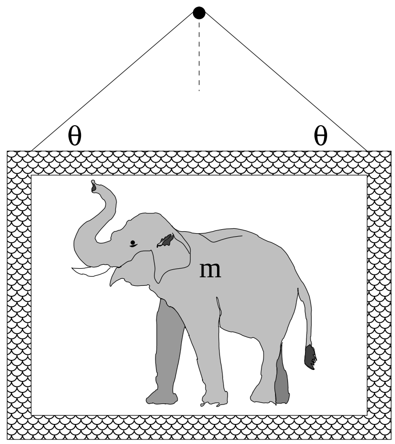
b) $T = Mg$

c) $T = \frac{\sqrt{3}}{2}Mg$

d) $T = 2Mg$

e) There is not enough information to determine T.

Problem 296. problems-1/statics-mc-support-the-picture.tex



A picture of mass m has been hung by a piece of thread as shown. The thread will break at a tension of mg. Find the smallest angle theta such that the thread will not break. FYI: $\sin(30°) = \cos(60°) = 1/2$, $\cos(30°) = \sin(60°) = \sqrt{3}/2$, $\sin(45°) = \cos(45°) = \sqrt{2}/2$, $\sin(90°) = \cos(0°) = 1$.

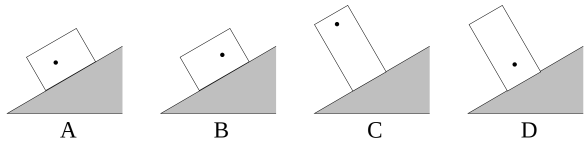
a) 30°

b) 45°

c) 60°

d) 90°

Problem 297. problems-1/statics-mc-tipping-blocks.tex



In the figure, four blocks are placed on an inclined plane that has sufficient static friction that the blocks will not slip. The dots in the figures indicate the center of mass of each block. Which of the following is/are true?

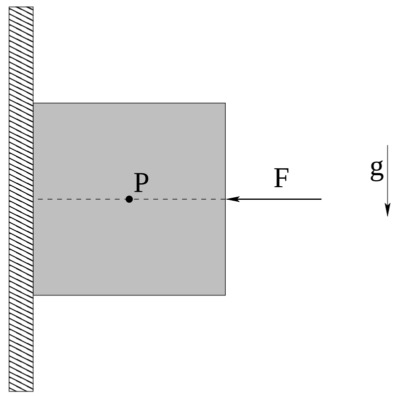
a) A and D will tip.

b) A B and D will not tip.

c) B and C will tip.

d) C and D will tip.

Problem 298. problems-1/statics-mc-torque-direction-block-held-to-wall-friction.tex



A cube of mass M is held at rest against a vertical ***rough wall*** by applying a perfectly horizontal force $\vec{F}$ as shown. Gravity is down as usual as shown. What is the ***direction*** of the torque ***about the point P*** due to the ***force of friction*** exerted by the wall on the block?

a) Left.

b) Right.

c) Up.

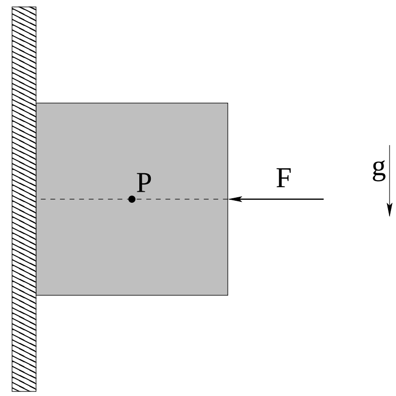
d) Down.

e) Into the plane of the figure.

f) Out of the plane of the figure.

g) The torque is zero, so the direction is undefined.

Problem 299. problems-1/statics-mc-torque-direction-block-held-to-wall-normal.tex



A cube of mass M is held at rest against a vertical ***rough wall*** by applying a perfectly horizontal force $\vec{\boldsymbol{F}}$ as shown. Gravity is down as usual as shown. What is the ***direction*** of the torque ***about the point P*** due to the ***normal force*** exerted by the wall on the block?

a) Left.

b) Right.

c) Up.

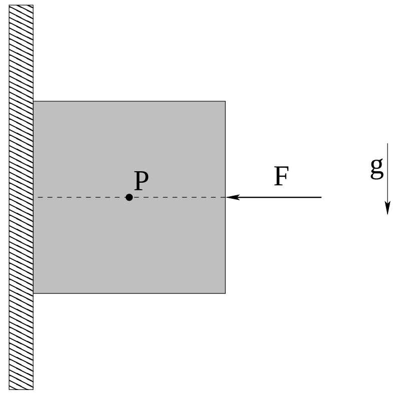
d) Down.

e) Into the plane of the figure.

f) Out of the plane of the figure.

g) The torque is zero, so the direction is undefined.

Problem 300. problems-1/statics-mc-torque-direction-block-held-to-wall.tex



A cube of mass M is held at rest against a vertical ***rough wall*** by applying a perfectly horizontal force $\vec{F}$ as shown. Gravity is down as usual as shown. What is the ***direction*** of the torque ***about the point P*** due to the ***force of friction*** exerted by the wall on the block?

a) Left.

b) Right.

c) Up.

d) Down.

e) Into the plane of the figure.

f) Out of the plane of the figure.

g) The torque is zero, so the direction is undefined.

Now, what is the ***direction*** of the torque about the point P due to the ***normal force*** exerted by the wall on the block?

a) Left.

b) Right.

c) Up.

d) Down.

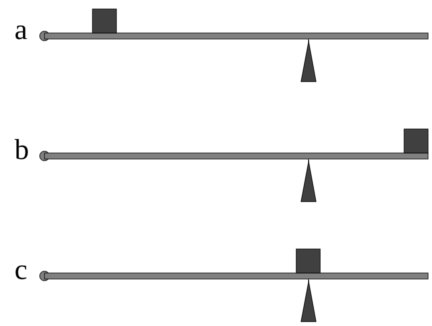
e) Into the plane of the figure.

f) Out of the plane of the figure.

g) The torque is zero, so the direction is undefined.

10.1.2 Ranking/Scaling

Problem 301. problems-1/statics-ra-board-and-pivot-force.tex

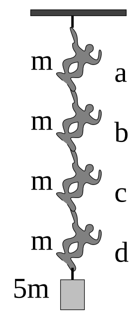


In the three figures above, a ***massless*** board is held in ***static equilibrium*** by a hinge at the left end and a trestle. A mass M is placed on the board at the three places shown. For each figure:

a) Draw an arrow at the ***hinge*** indicating the ***direction*** of the force (if any) exerted by the hinge for all three figures. If the force is zero please indicate this.

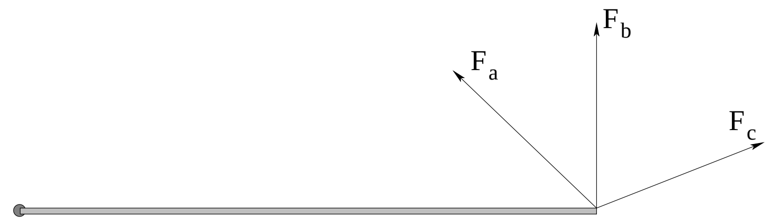
b) Rank the three figures in the order of the ***magnitude*** of the force exerted on the board ***by the trestle***, from least to greatest.

Problem 302. problems-1/statics-ra-chain-of-hanging-monkeys.tex



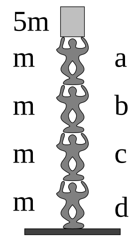
In the figure above four monkeys, each of mass m, are shown holding very still as they hang from a pole at the top of a circus tent. The top monkey (a) is holding a strap attached to the pole above, and the bottom monkey (d) is holding a mass $5m$ above with his foot. Which monkey (a-d) is pulling ***up with the largest force*** with its feet?

Problem 303. problems-1/statics-ra-holding-up-the-bar.tex



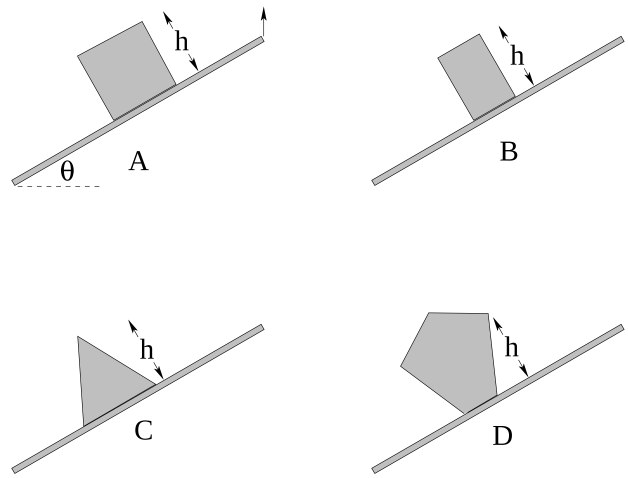
A bar of mass M is pivoted by a hinge on the left and has a wire attached to the right as shown. The wire can be attached to the ceiling on eyebolts on any one of the three angles shown to suspend the rod so that it is in static equilibrium. Rank the force F exerted on the rod ***by the wire*** when the wire comes off in the a, b, c directions (where equality is a possibility). That is, your answer might look like $F_a < F_b = F_c$ (but don't count on this being the answer). Note well: The arrows in the figure above are **not proportional** to the forces, they indicate **only** the directions.

Problem 304. problems-1/statics-ra-stack-of-standing-monkeys.tex



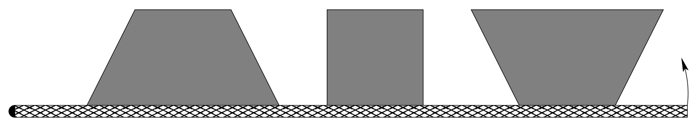
(3 points) In the figure above four monkeys, each of mass m, are shown holding very still in a tower they've made at the circus. The bottom monkey (d) is standing on the floor, the top monkey (a) is holding a mass $5m$ above his head. Which monkey is pushing *up* with the largest force with its arms?

Problem 305. problems-1/statics-ra-tipping-shapes-2.tex



In the four figures above, the coefficient of static friction is high enough that the uniform objects shown will not slip before they tip. Rank the angles at which each mass will tip over as the right end of the plank they sit on is raised, from ***smallest*** (tipping angle θ) ***to the largest*** (for example, A,B,C,D):

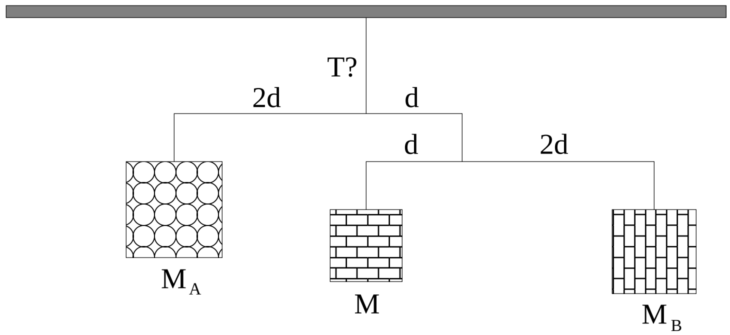
Problem 306. problems-1/statics-ra-tipping-shapes.tex



In the figure above, three shapes (with uniform mass distribution and thickness) are drawn sitting on a plane that can be tipped up gradually. Assuming that static friction is great enough that all of these shapes will tip over before they slide, rank them in the order they will tip over as the angle of the board they are sitting on is increased. Be sure to indicate any ties.

10.1.3 Short Answer

Problem 307. problems-1/statics-sa-balance-the-mobile-1.tex



A static mobile suspends three beautifully patterned blocks over a baby's bed. The lengths of the supporting rigid rods (of negligible mass) are given in the figure above, as is the mass of the central block, M. You must find M_A and M_B (in terms of/units of M as shown) so that the mobile perfectly balances, and you must also make sure that the string you are using to hang the mobile is strong enough to support its weight. Note well that the unknown blocks are ***not necessarily drawn to scale!***

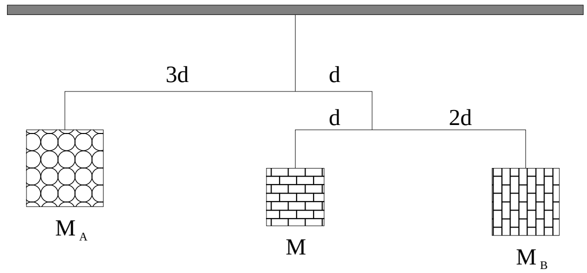
a) $\dfrac{M_A}{M} =$ $\boxed{}$

b) $\dfrac{M_B}{M} =$ $\boxed{}$

c) What is the total tension T in the top supporting string when the mobile perfectly balances?

$T =$ $\boxed{}$

Problem 308. problems-1/statics-sa-balance-the-mobile-2.tex

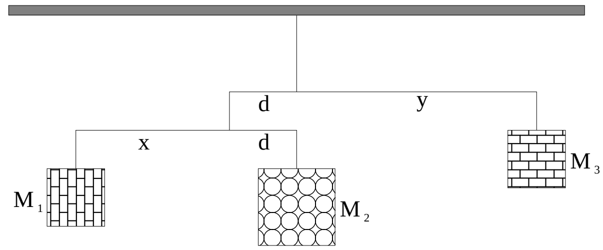


A static mobile suspends three patterned blocks over a baby's bed. The lengths of the supporting rigid rods (of negligible mass) are given in the figure above, as is the mass of the central block, M. Find M_A and M_B in terms of M so that the mobile perfectly balances. Note well that the unknown blocks are ***not necessarily drawn to scale!***

a) $M_A =$

b) $M_B =$

Problem 309. problems-1/statics-sa-balance-the-mobile-reversed.tex



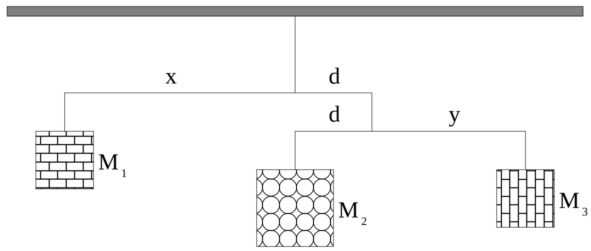
A static mobile suspends three patterned blocks over a baby's bed. The masses of the blocks and the lengths of the supporting rigid rods (of negligible mass) are given in the figure above (although the relative distances may not be correctly to scale). Find x and y in terms of d so that the mobile perfectly balances when:

$M_1 = 1$ kg, $M_2 = 3$ kg, $M_3 = 1$ kg

x =

y =

Problem 310. problems-1/statics-sa-balance-the-mobile.tex

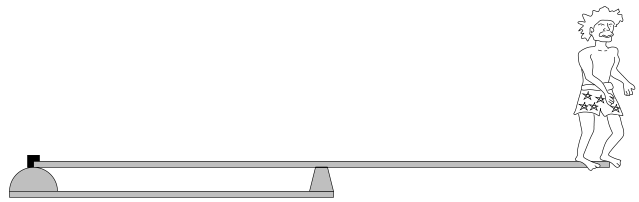


A static mobile suspends three patterned blocks over a baby's bed. The masses of the blocks and the lengths of the supporting rigid rods (of negligible mass) are given in the figure above. Find x and y in terms of d so that the mobile perfectly balances when $\boldsymbol{M_1 = 1}$ kg, $\boldsymbol{M_2 = 4}$ kg, $\boldsymbol{M_3 = 1}$ kg.

$x =$ $\boxed{}$

$y =$ $\boxed{}$

Problem 311. problems-1/statics-sa-diving-board.tex



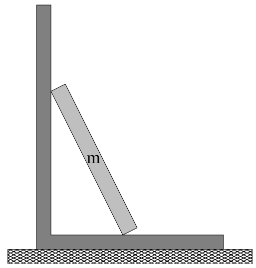
Albert tries to make a diving board for his backyard swimming pool by attaching the board firmly to two vertical supports. The perfectly rigid uniform board has a length of 6 m and a mass of 40 kg. The left hand support is attached to the left end, and the right hand support is attached 3 m to the right of the left support (at the center of the board).

a) What force (magnitude and direction) does the ***middle*** support exert on the board when Albert (whose mass is 80 kg) stands on the right hand end of the board as shown?

b) What force (magnitude and direction) does the ***left*** support exert on the board at this time?

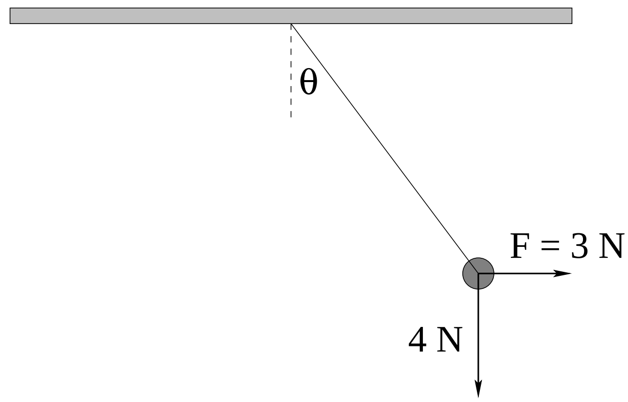
c) Which support needs to be bolted down?

Problem 312. problems-1/statics-sa-leaning-bar-reaction-pairs.tex



In the figure above, a board is sitting on a rough floor and leaning against a wall. Identify *three* action-reaction force pairs in the figure.

Problem 313. problems-1/statics-sa-pendulum-bob.tex



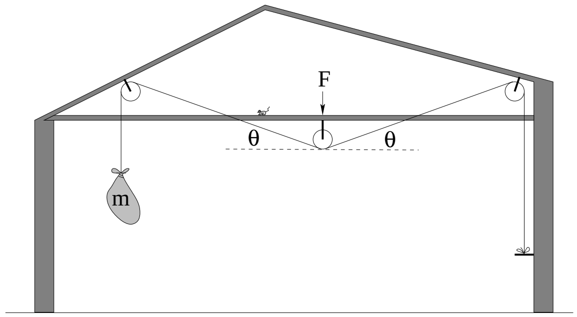
A 4 N pendulum bob supported by a massless string is held motionless at an angle θ from the vertical by a horizontal force $F = 3$ N as shown. The string used to hang the mass will break at any tension $T > T_c = 4\sqrt{2}$ N.

a) What is the angle θ (expression OK).

b) The force F is slowly increased (while keeping the force horizontal). At what value will the string break?

c) What is the angle θ at which the string breaks?

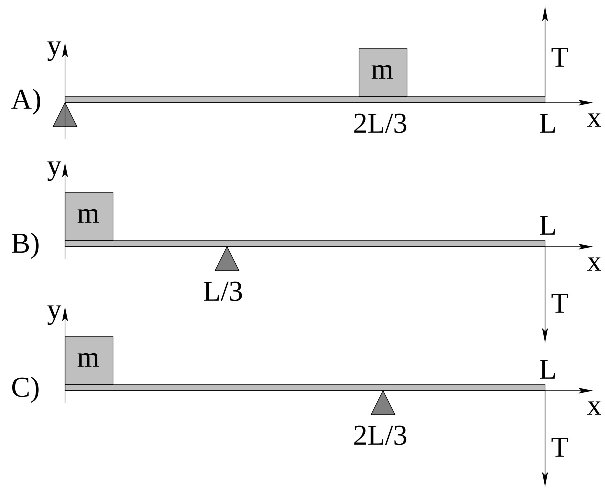
Problem 314. problems-1/statics-sa-suspended-food-bag.tex



A gold prospector living in a rustic cabin mounts a sturdy wooden peg and three (approximately massless and frictionless) pulleys in fixed positions on the wall and rafters as shown in the diagram so he can suspend his food bag up off the floor and away from mice. He hangs a bag of food of mass m so that the rope makes an angle θ with the central pulley as shown.

Help him find the magnitude of the force F that his rafter must exert downward on the pulley when he has hung his bag of food.

Problem 315. problems-1/statics-sa-which-mass-breaks-string.tex



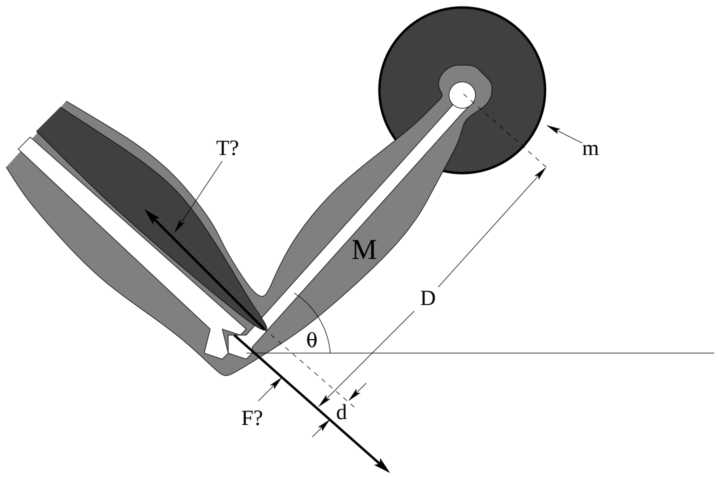
In the figure above, a massless plank supports a massive block m placed at the locations shown. The plank is supported by a wedge shaped support and a string that will break at the same tension $T_{\max}$ (in all three cases) positioned as shown.

a) Suppose the mass m is gradually increased (in all three figures). In which configuration (**A**, **B**, or **C**) will the string break ***first***?

b) For ***that configuration*** (that you picked in part a), what is the value of the upward support force F_s exerted by the wedge right as (just before) the string breaks?

10.1.4 Long Problems

Problem 316. problems-1/statics-pr-arm-with-barbell.tex

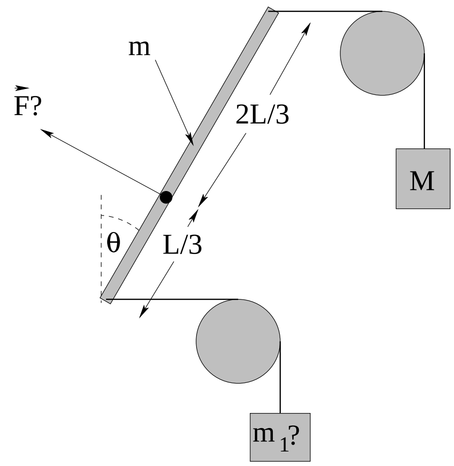


An exercising human person holds their arm of mass M and a barbell of mass m at rest at an angle θ with respect to the horizontal in an isometric curl as shown. The muscle that supports the suspended weight is connected a short distance d up from the elbow joint. The bone that supports the weight has length D.

a) Find the tension T in the muscle, assuming for the moment that the center of mass of the forearm is in the middle at $D/2$. Note that it is *much larger* than the weight of the arm and barbell combined, assuming a reasonable ratio of $D/d \approx 25$ or thereabouts.

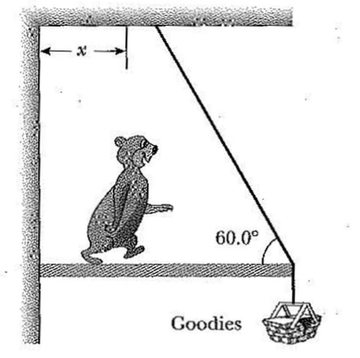
b) Find the force $\vec{F}$ (magnitude *and* direction) exerted on the supporting bone by the elbow joint. Again, note that it is much larger than "just" the weight being supported.

Problem 317. problems-1/statics-pr-bar-and-pulleys.tex



Find the components of the pivot force $\vec{\boldsymbol{F}} = (F_x, F_y)$ and find m_1 **in terms of** M **and** m **as givens** in the figure above, if the bar of mass m is in static equilibrium.

Problem 318. problems-1/statics-pr-bear-seeking-goodies.tex



A bear of mass M_B walks out on a beam of mass m_b to get a basket of food of mass of mass m_f. The beam has length L, and is supported by a wire at an angle of 60 degrees, as in the sketch.

a) Draw a free-body diagram for the beam that shows all forces. Include an indication of a coordinate system and also indicate the origin of that coordinate system.

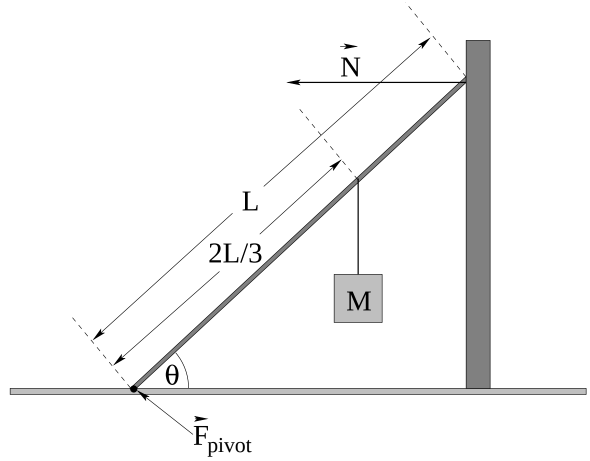
b) Using that origin (as a pivot), write down all the force and torque balance equations, assuming that the bear is located a distance x from the left end of the beam.

c) Solve these equations to find the (vector) force that the wall exerts on the left end of the beam.

d) Find the tension in the wire.

e) Suppose that the bear is too heavy to reach the basket without breaking the wire. If the maximum tension that the wire can support without breaking is $T_{\max}$, find an expression for the largest distance from the wall $x_{\max}$ that the bear can walk without breaking the wire.

Problem 319. problems-1/statics-pr-brace-against-house.tex

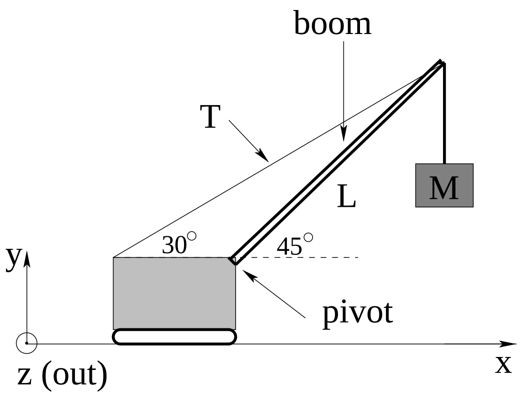


In the figure above, a "massless" rigid beam of length L that makes an angle of θ with the ground is leaned against a frictionless wall at the upper end, which exerts a ***normal force only*** N as shown on the beam. A mass M is suspended vertically from a point 2/3 of the way from the pivot attached to the ground. Find:

a) The *magnitude* of the normal force N exerted ***by the wall on the beam*** when the entire beam is in static equilibrium.

b) The *vector force* $\vec{\boldsymbol{F}}_p$ exerted by the pivot on the ground ***on the beam*** to hold the beam in place. It is probably easiest to express this answer as F_{px} and F_{py}.

Problem 320. problems-1/statics-pr-crane-boom.tex



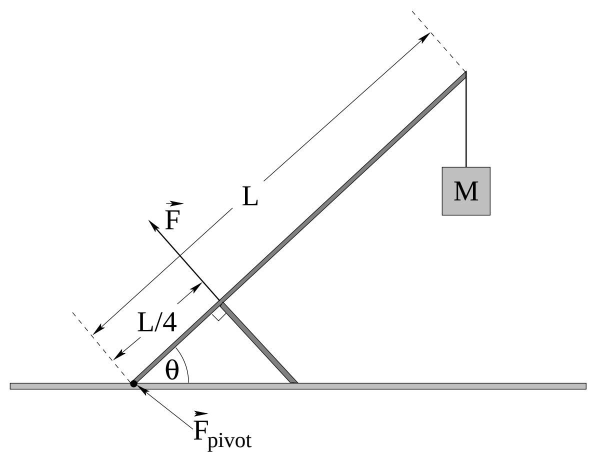
$$\sin(30°) = \cos(60°) = \frac{1}{2}$$
$$\cos(30°) = \sin(60°) = \frac{\sqrt{3}}{2}$$
$$\sin(45°) = \cos(45°) = \frac{\sqrt{2}}{2}$$

A crane with a boom (the long support between the body and the load) of mass m and length L holds a mass M suspended as shown. Assume that the center of mass of the boom is at $L/2$. Note that the wire with the tension T is **fixed** to the top of the boom, not run over a pulley to the mass M.

a) Find the tension in the wire.

b) Find the force exerted on the boom by the crane body.

Problem 321. problems-1/statics-pr-crane-vertical-support.tex

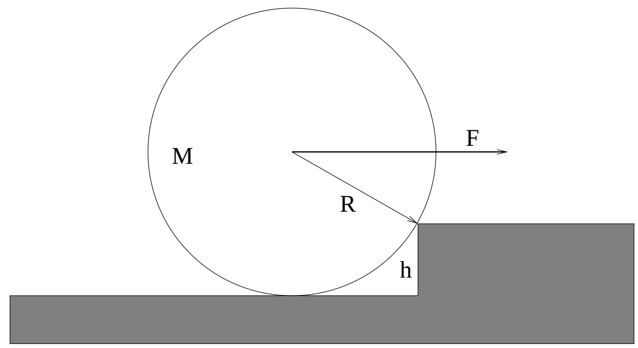


In the figure above, a "massless" rigid beam of length L that makes an angle of θ with the ground is braced with a piece of wood a distance $L/4$ from the end on the ground. This piece of wood is attached at right angles to the beam as shown. At the upper end of the beam a mass M is suspended. Find:

a) The *magnitude* F of the force exerted ***by the support bar*** when the entire beam is in static equilibrium.

b) The *vector force* $\vec{\boldsymbol{F}}_p$ exerted by the pivot on the ground ***on the beam*** (**not** the support bar) to hold the beam in place. It is probably easiest to express this answer as F_{px} and F_{py}.

Problem 322. problems-1/statics-pr-cylinder-and-corner-2.tex

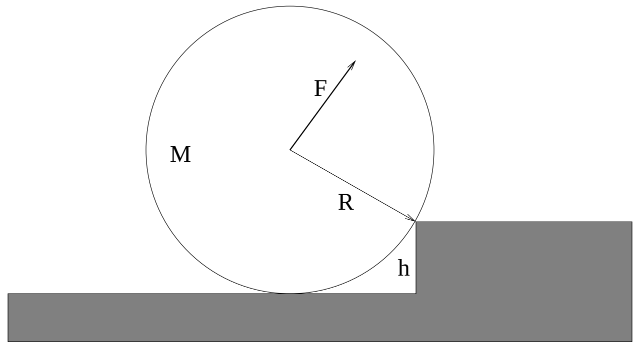


A cylinder of mass M and radius R sits against a step of height $h = R/2$ as shown above. A force $\vec{F}$ is applied parallel to the ground as shown. All answers should be in terms of M, R, g.

a) Find the minimum value of $|\vec{F}|$ that will roll the cylinder over the step if the cylinder does not slide on the corner.

b) What is the force exerted by the corner (magnitude and direction) when that force $\vec{F}$ is being exerted on the center?

Problem 323. problems-1/statics-pr-cylinder-and-corner.tex

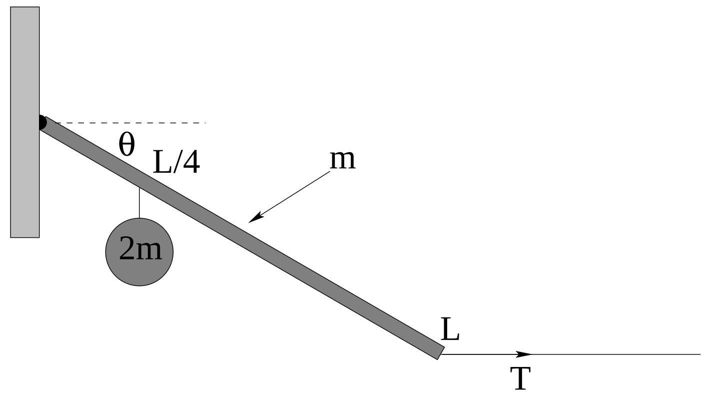


A cylinder of mass M and radius R sits against a step of height $h = R/2$ as shown above. A force $\vec{F}$ is applied at right angles to the line connecting the corner of the step and the center of the cylinder. All answers should be in terms of M, R, g.

a) Find the minimum value of $|\vec{F}|$ that will roll the cylinder over the step if the cylinder does not slide on the corner.

b) What is the force exerted by the corner (magnitude and direction) when that force $\vec{F}$ is being exerted on the center?

Problem 324. problems-1/statics-pr-dangling-bar.tex

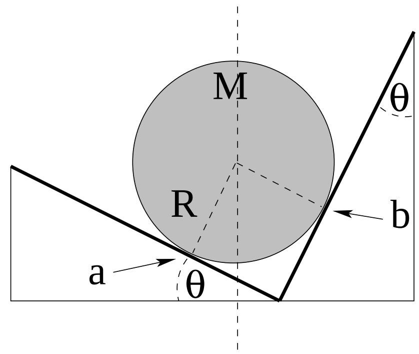


In the figure above, a rod of length L with mass m is suspended by a hinge on the left and a horizontal string on the right. A second mass $2m$ is suspended from the rod a distance $L/4$ from the hinge end. Find:

a) The tension T in the horizontal string.

b) The vector force $\vec{F}$ exerted by the hinge, in any of the acceptable forms we use to completely specify a vector.

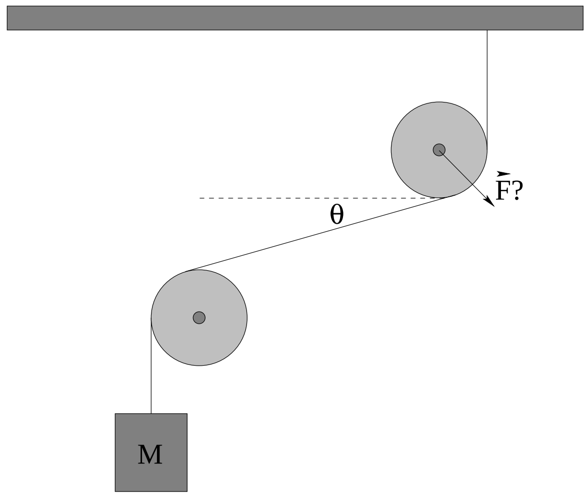
Problem 325. problems-1/statics-pr-disk-in-corner.tex



Find the magnitude of the normal forces N_a and N_b exerted by the two walls on the disk of mass M and radius R at the points a and b such that it sits in static equilibrium in the picture above:

- $N_a =$
- $N_b =$

Problem 326. problems-1/statics-pr-double-diagonal-pulleys.tex



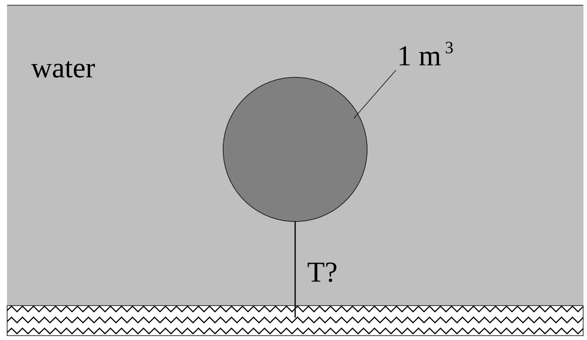
In the figure above, two massless pulleys and a massless unstretchable string support a mass M in static equilibrium as shown. The pulleys are fixed on unmoveable frictionless axles.

a) (3 points) Draw a force diagram for the mass M *and both pulleys*.

b) (5 points) Find the *vector* force $\vec{F}$ exerted by the axle of the upper pulley at equilbrium.

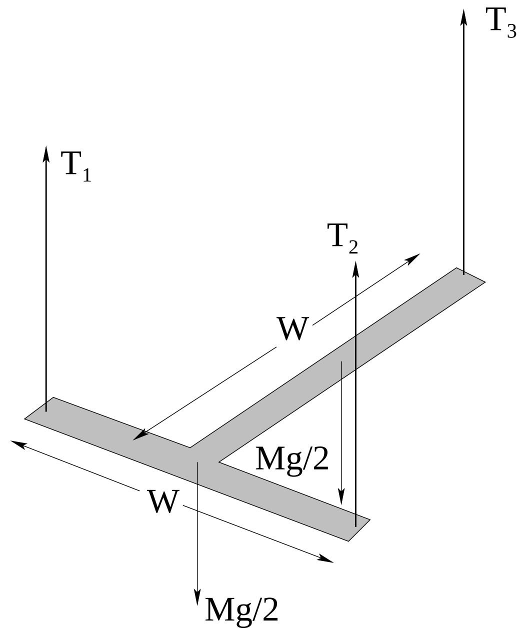
c) (1 point) If the angle θ is increased (by lowering the lower pulley, for example) is there *more* or *less* force exerted by the upper axle to keep the pulley in place?

Problem 327. problems-1/statics-pr-floating-buoy.tex



A round buoy at the beach floats in fresh water when it is exactly half submerged. Its spherical volume is 1 cubic meter. If it is pulled all the way underwater and suspended from the bottom by means of an anchored rope, what is the tension in the rope?

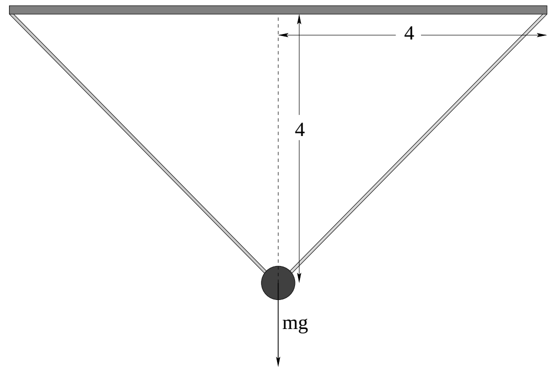
Problem 328. problems-1/statics-pr-hang-a-stable-T.tex



The "**T**" shaped object above has mass M, and has both a height and width of W. Assume that this mass is uniformly distributed in the long arm and the crossbar, that is, that the center of mass of the long arm is at $W/2$ and the center of mass of the crossbar is also at $W/2$ and that the long arm and crossbar each has mass $M/2$ (and hence gravity exerts a downward force at their centers of mass of $Mg/2$ as shown).

Find the tension $T_{1,2,3}$ in ***each of the three ropes*** that support the **T** above. Note that the ropes all pull straight up (they are vertical) and the **T** is completely horizontal.

Problem 329. problems-1/statics-pr-hanging-ball-2.tex



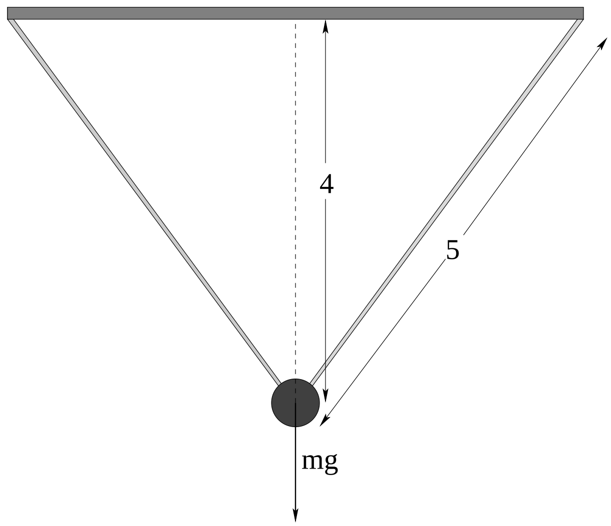
(9 points total) In the figure above, a mass m is hanging from two massless, unstretchable ropes. Gravity pulls straight down on the mass with a force of magnitude mg. Assume that the tension in both ropes has the equal magnitude T. The mass is hanging 4 meters beneath the ceiling, and each rope is fastened to the ceiling offset by 4 meters from where the mass hangs as shown.

a) (3 points) Draw a coordinate system and free body diagram representing all the forces acting on the hanging mass. Label any angles that might be of use to you.

b) (3 points) Write the *algebraic* equations for the total force in the x and y directions that are the conditions for static equilibrium.

c) (3 points) Find the tension T in terms of mg.

Problem 330. problems-1/statics-pr-hanging-ball.tex



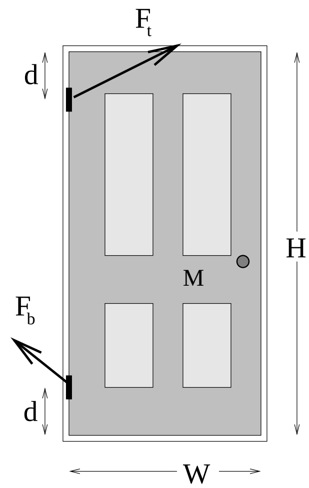
In the figure above, a mass m is hanging from two massless, unstretchable ropes. Gravity pulls straight down on the mass with a force of magnitude mg. Assume that the tension in both ropes has the equal magnitude T. The length of the each rope is 5 meters, and the mass is hanging 4 meters beneath the ceiling as shown

a) Draw a coordinate system and free body diagram representing all the forces acting on the hanging mass. Label any angles that might be of use to you.

b) Write the *algebraic* equations for the total force in the x and y directions that are the conditions for static equilibrium.

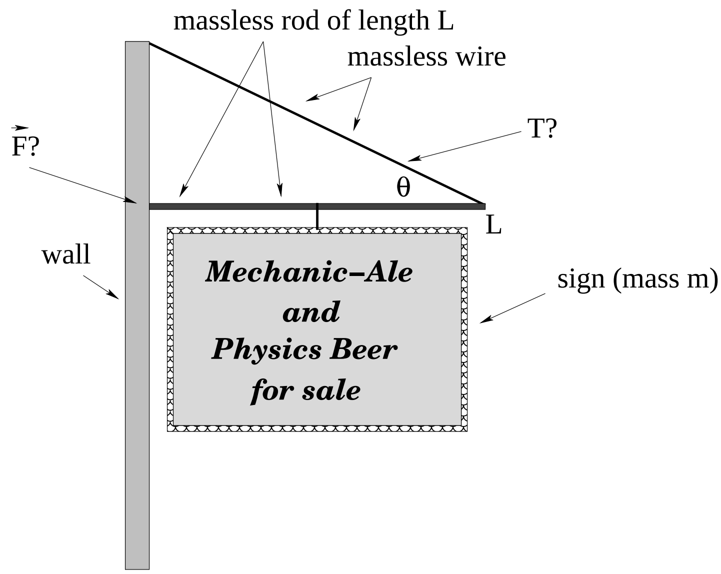
c) Find the tension T in terms of mg.

Problem 331. problems-1/statics-pr-hanging-door.tex



A door of mass M that has height H and width W is hung from two hinges located a distance d from the top and bottom, respectively. Assuming that the weight of the door is equally distributed between the two hinges, find the total force (magnitude and direction) exerted by each hinge. (Neglect the mass of the doorknob. The force directions drawn for you are **NOT** likely to be correct or even close.)

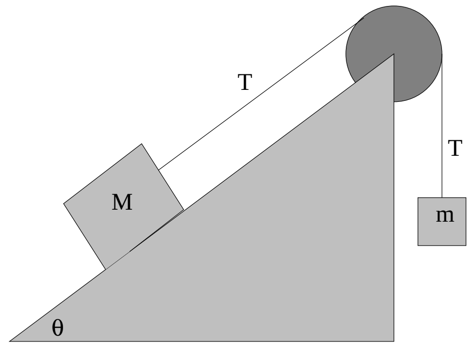
Problem 332. problems-1/statics-pr-hanging-tavern-sign.tex



In the figure above, a tavern sign belonging to a certain home-brewing physics professor is shown suspended from the ***middle*** of a ***massless*** supporting rod of length L (at $L/2$). Find the ***tension*** in the (massless) wire, $\boldsymbol{T}$, and the ***total force*** exerted on the suspending rod by ***the wall***, $\vec{\boldsymbol{F}}$, in terms of m, g, L, and θ.

Please indicate the coordinate system you are using on the figure and the location of the pivot point used, if any.

Problem 333. problems-1/statics-pr-inclined-plane.tex



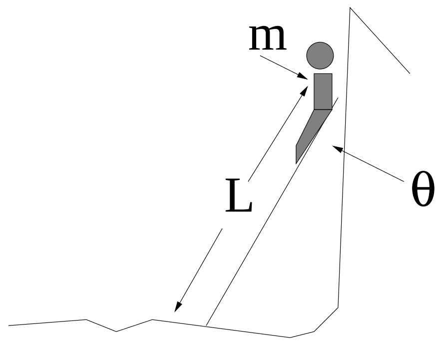
This problem will help you learn required concepts such as:

- Newton's Third Law
- Momentum Conservation
- Fully Inelastic Collisions

so please review them before you begin.

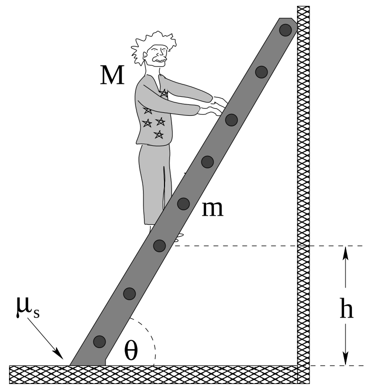
In the inclined plane problem above all masses are at rest and the pulley and string are both massless. Find the normal force exerted by the inclined plane on the mass M and the mass m required to keep the system in static balance in terms of M and θ.

Problem 334. problems-1/statics-pr-ladder-on-glacier.tex



An ultralight (assume massless) ladder of length L rests against a vertical block of (frictionless) ice during a hazardous ascent of a glacier at an angle $\theta = 30^\circ$ as drawn. A mountaineer of mass m climbs the ladder. When the mountaineer is standing *at rest* at the very top of the ladder and about to reach over the cliff edge, what is the net force exerted on the *base* of the ladder by the glacier?

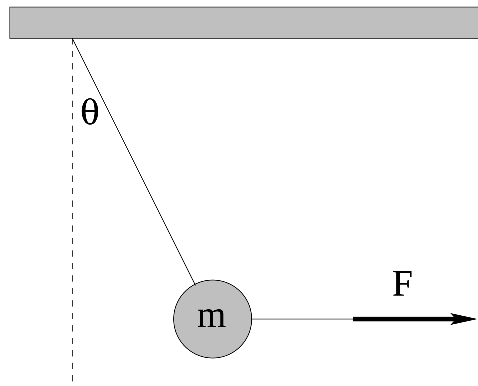
Problem 335. problems-1/statics-pr-ladder-on-wall.tex



In the figure above, a ladder of mass m and length L is leaning against a wall at an angle θ. A person of mass M begins to climb the ladder. The ladder sits on the ground with a coefficient of static friction μ_s between the ground and the ladder. The wall is frictionless – it exerts only a normal force on the ladder.

If the person climbs the ladder, find the height h where the ladder slips.

Problem 336. problems-1/statics-pr-mass-two-strings.tex



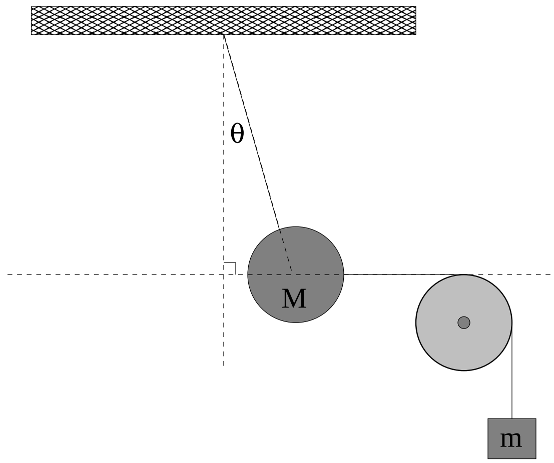
A ball of mass m hangs from the ceiling on a massless string. A second massless string is attached to the ball and a force $\vec{F}$ is applied to it in the horizontal direction so that the system remains in static equilibrium in the position shown, where θ is the angle between the first string and the vertical. Gravity acts down as usual. Each string can support a ***maximum*** tension $T_{\max} = 2mg$ without breaking.

a) If $\vec{F}$ is slowly increased while keeping its direction horizontal, which string will break first? Explain your reasoning.

b) Find the maximum value $\theta_{\max}$ that the hanging string can have when the system is in static equilibrium with both strings unbroken. (You may express this angle as an inverse sine, cosine, or tangent if you wish – you do not need a calculator.)

c) Find the force magnitude $F_{\max}$ that produces the maximum angle $\theta_{\max}$ in static equilibrium. Express this answer in terms of m and g.

Problem 337. problems-1/statics-pr-pendulum-bob-pulley.tex



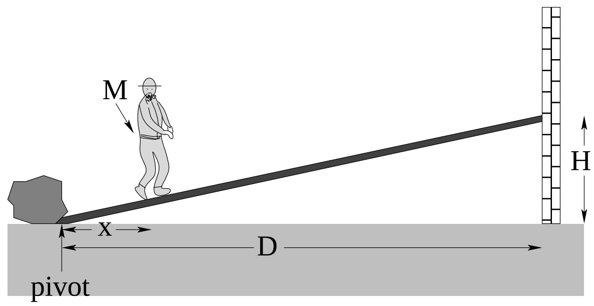
A pendulum bob of mass m is attached both to the ceiling and to a mass M hanging over a pulley by unstretchable massless strings as shown. The pulley is fixed on an unmoveable frictionless axle.

a) (3 points) Draw free body diagrams for *both* mass m *and* mass M.

b) (3 points) Find an expression for the angle θ at which the system is in static equilibrium.

c) (3 points) Find the total tension T in the string connecting the pendulum bob to the ceiling.

Problem 338. problems-1/statics-pr-pushing-down-wall.tex



Tom is a hefty construction worker (mass $M = 100$ kilograms) with a good sense of balance who wants to push down a brick wall. The wall, however, is strong enough to withstand any horizontal push up to 2000 N and Tom can only exert a sideways equal to his weight with his muscles.

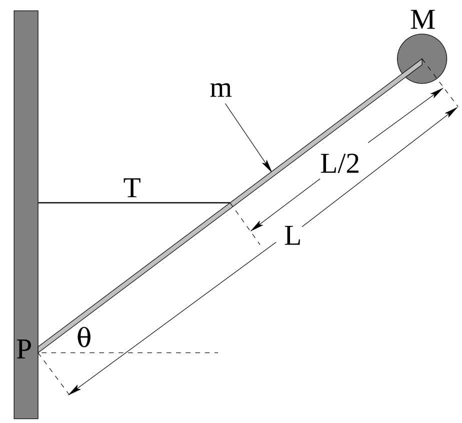
Fortunately, Tom has a perfectly rigid 4×4 beam (of negligible mass), and there is a solid rock (that can withstand essentially any push) a distance $D = 5$ meters from the wall to brace it on. Even more fortuitously, Tom has taken introductory physics! He therefore cuts the beam to lean against the wall a height H as shown and proceeds to walk up the beam towards the wall..

a) Assuming that the beam is frictionless where it presses against the wall what is the largest value of H that will permit him to knock down the wall if he walks to the end of the beam so that his horizontal distance $x = D$?

b) Suppose that he has cut the beam so that it rests a height $H = 1$ meter above the ground against the wall. What is his horizontal position x when the beam knocks down the wall (if it does at all)?

c) Of course the beam is ***not*** frictionless where it rests against the wall. Does this fact mean that, for any given value of H, the wall is easier to knock down (happens when he has walked a smaller horizontal distance x toward the wall), harder to knock down (happens when he has walked a greater horizontal distance x), or just the same (it falls at the same horizontal distance x) as it is without friction?

Problem 339. problems-1/statics-pr-rod-mass-on-hinge.tex

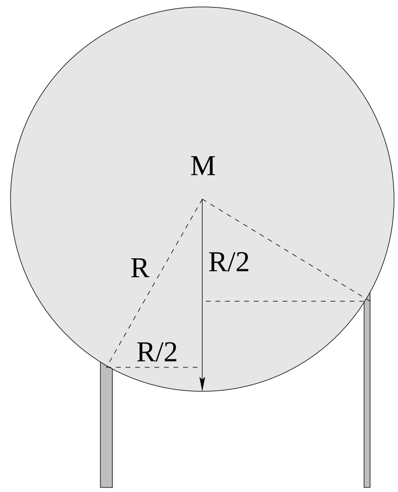


A small round mass M sits on the end of a rod of length L and mass m that is attached to a wall with a hinge at point P. The rod is kept from falling by a thin (massless) string attached horizontally between the midpoint of the rod ($L/2$ from either end) and the wall. The rod makes an angle θ with the ground. Find:

a) the tension T in the string;

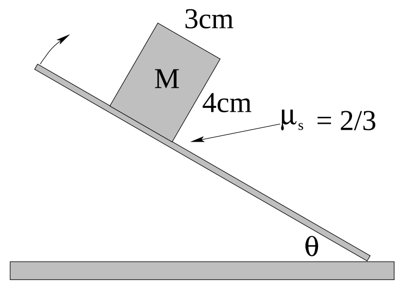
b) the ***vector*** force $\vec{\boldsymbol{F}}$ exerted by the hinge on the rod.

Problem 340. problems-1/statics-pr-supporting-a-disk.tex



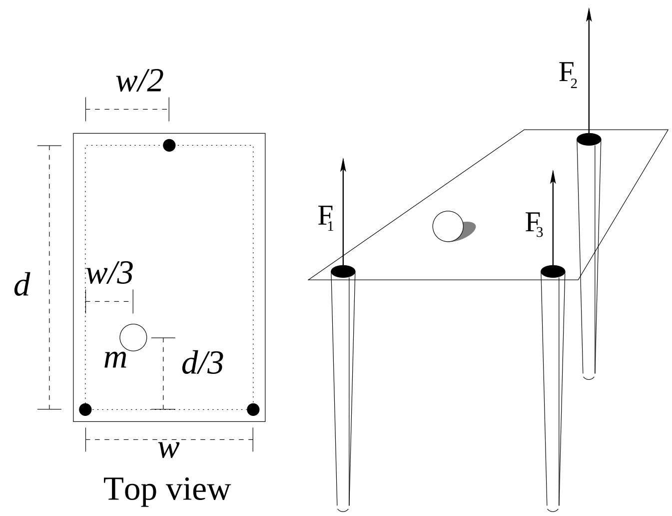
Find the force exerted by each of the two rods supporting the disk of mass M and radius R as shown. Note that the two triangles shown are both 30-60-90 triangles with side opposite the small angle of $R/2$.

Problem 341. problems-1/statics-pr-tipping-vs-slipping.tex



A block of mass M with width 3 cm and height 4 cm sits on a rough plank. The coefficient of static friction between the plank and the block is $\mu_s = 2/3$. The plank is slowly tipped up. Does the block slip first, or tip first?

Problem 342. problems-1/statics-pr-vector-torque-plexiglass-table.tex



The figure below shows a mass m placed on a table consisting of three narrow cylindrical legs at the positions shown with a light (presume massless) sheet of Plexiglas placed on top. Find the vertical forces F_1, F_2, F_3 exerted on the Plexiglas by each leg when the mass is at rest in the position shown.

Chapter 11

Fluids

Fluids have statics too! Water can *sit still* in a drinking glass, held in by normal forces exerted by the glass, held down by gravity, and internally held in place by – water. Even air is static (when their is no wind.

But the really interesting things are what happens when fluids move. We barely scratch the surface in this course – fluid dynamics is arguably one of the most difficult theories in all of physics, especially in the general, nonlinear, chaotic, turbulent regime. Which is where we, and a whole lot of everyday stuff, live.

Fluid statics and dynamics is once again useful to everybody from physicians to engineers to physicists to physicians to boat captains to airline pilots to physicians, to...

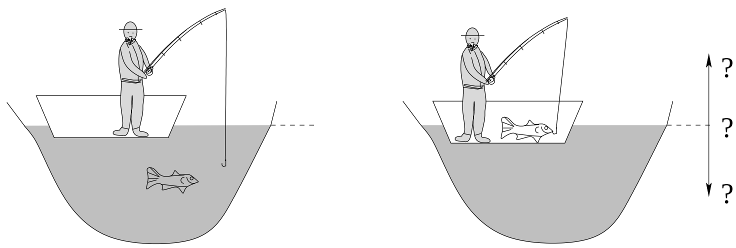
Did I mention that the human body, from one point of view is a big, walking, talking, thinking bag of water (complete with pumps and plumbing) with a few very important contaminants in it?

No?

11.1 Fluids

11.1.1 Multiple Choice

Problem 343. problems-1/fluids-mc-boat-catches-fish.tex



I go fishing in a pond where there is a big, fat fish perfectly suspended by buoyant forces in the water under the boat. I catch him and reel him in up into the boat. As I do so, the level of the water in the pond will:

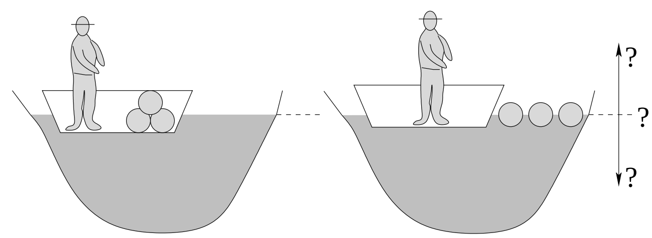
a) Rise a bit.

b) Fall a bit.

c) Remain unchanged.

d) Can't tell from the information given (it depends, for example, on the kind of fish...).

Problem 344. problems-1/fluids-mc-boat-floats-wood.tex



A person stands in a boat floating on a pond and containing several pieces of wood. He throws the wood out of the boat so that it floats on the surface of the pond. The water level of the pond will:

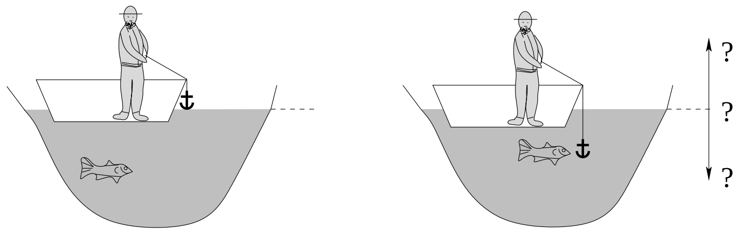
a) Rise a bit.

b) Fall a bit.

c) Remain unchanged.

d) Can't tell from the information given (it depends on, for example, the shape of the boat, the mass of the person, whether the pond is located on the Earth or on Mars...).

Problem 345. problems-1/fluids-mc-boat-lowers-anchor.tex



I go fishing in a pond and spot a big, fat fish in the water under the boat and decide to anchor for a bit to try to catch it. As I lower the anchor into the water (so that it ***hangs suspended*** under the boat as shown) level of the water in the pond will:

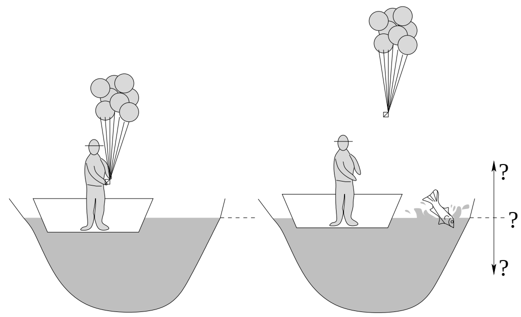
a) Rise a bit.

b) Fall a bit.

c) Remain unchanged.

d) Can't tell from the information given (it depends, for example, on whether the anchor is made of iron or lead...).

Problem 346. problems-1/fluids-mc-boat-releases-balloons.tex



The fish aren't biting, so a person standing in a boat floating on a pond and inflates a bunch of helium balloons instead. Then an enormous fish jumps nearby and he is so startled that he accidentally releases the balloons. As he does so, the water level of the pond will:

a) Rise a bit.

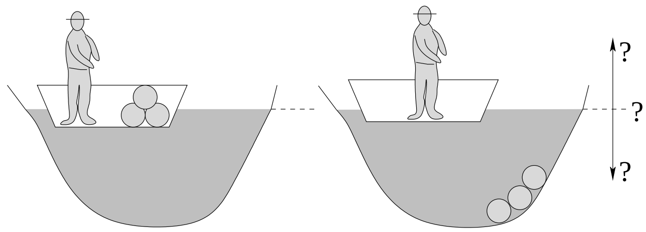
b) Fall a bit.

c) Remain unchanged.

d) Can't tell from the information given.

(Ignore the fish!)

Problem 347. problems-1/fluids-mc-boat-sinks-rocks.tex



A person stands in a boat floating on a pond and containing several large, round, rocks. He throws the rocks out of the boat so that they sink to the bottom of the pond. The water level of the pond will:

a) Rise a bit.

b) Fall a bit.

c) Remain unchanged.

d) Can't tell from the information given (it depends on, for example, the shape of the boat, the mass of the person, whether the pond is located on the Earth or on Mars...).

Problem 348. problems-1/fluids-mc-buoyant-boxes.tex

Two wooden boxes with the same shape but different density are held in the same orientation beneath the surface of a large container of water. Box A has a smaller average density than box B. When the boxes are released, they accelerate up towards the surface. Which box has the greater acceleration when they are initially released?

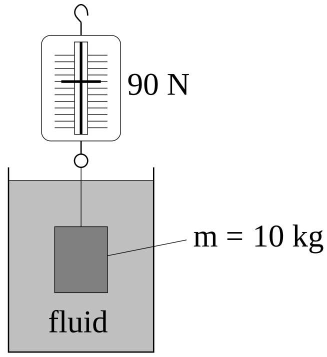
a) Box A.

b) Box B.

c) They are the same.

d) We cannot tell from the information given.

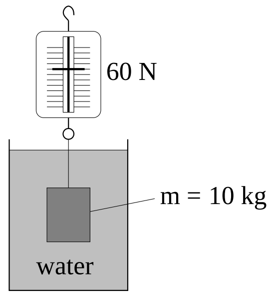
Problem 349. problems-1/fluids-mc-density-of-fluid-from-stone.tex



A block of lead has a mass of $m = 10$ kg (that weighs 100 Newtons in air) and a density of $\rho = 1.1 \times 10^4$ kg/m^3 is hung from a scale and immersed in an unknown fluid. The scale then reads 90 Newtons. What is the approximate ***density of the fluid***? (Use $g = 10.$ m/sec^2)

$\rho_f = \boxed{}$ kg/m^3

Problem 350. problems-1/fluids-mc-density-of-stone-underwater.tex



A stone of mass $m = 10$ kg (that weighs 100 Newtons in air) is hung from a scale and immersed in water. The scale reads 60 Newtons. What is the ***density*** of the stone? (Use $g = 10$ m/sec^2)

a) $\rho = 1000$ kg/m^3

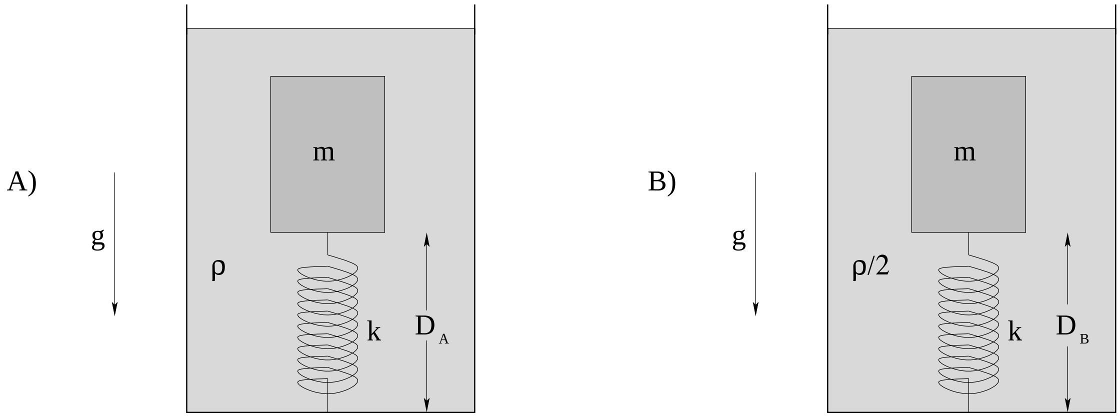
b) $\rho = 4000$ kg/m^3

c) $\rho = 6000$ kg/m^3

d) $\rho = 1667$ kg/m^3

e) $\rho = 2500$ kg/m^3

Problem 351. problems-1/fluids-mc-floating-mass-on-spring-different-rho.tex

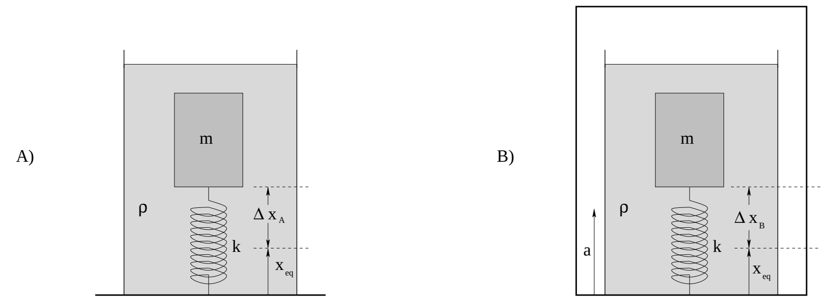


In the figures above, two identical springs (with spring constant k) are attached to the bottoms of two identical containers filled with two ***different*** fluids with densities (A) ρ and (B) $\rho/2$ respectively. Wooden blocks that would ordinarily float are attached to these springs, which stretch out to total lengths D_A and D_B and suspend the blocks so that they are fully immersed as shown.

Circle the true statement:

$\boldsymbol{D_A > D_B}$ $\qquad$ $\boldsymbol{D_A < D_B}$ $\qquad$ $\boldsymbol{D_A = D_B}$

Problem 352. problems-1/fluids-mc-floating-mass-on-spring-elevator.tex



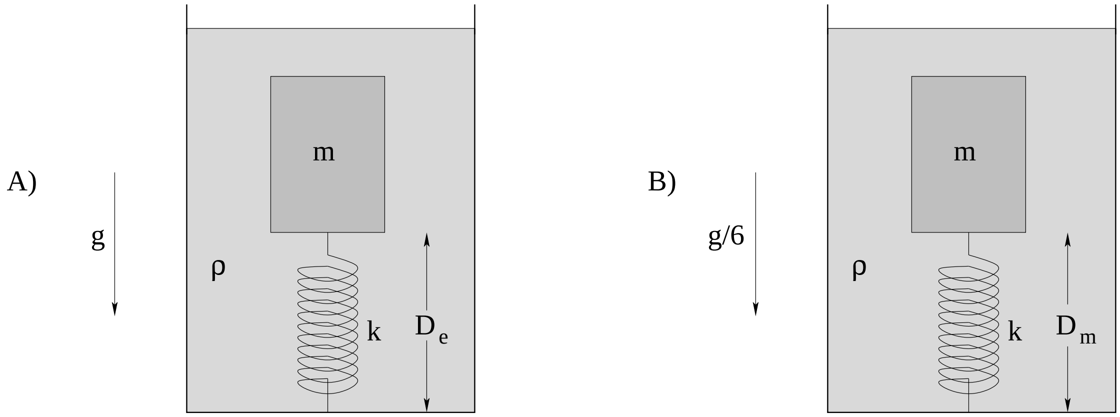
In the figures above, two identical springs (with spring constant k) are attached to the bottoms of two identical containers filled with water (density ρ). At the other end, the springs are attached to identical wooden blocks that would ordinarily ***float*** on the water so that they are completely submerged.

The container on the left (A) is located at rest on the ground, and Δx_A is the total distance that its spring is stretched from its equilibrium length when the block is stationary relative to container A. The container on the right (B) is located on the floor of an elevator accelerating upwards with an acceleration a, and Δx_B is the total length that its spring is stretched from its equilibrium length when the block is stationary relative to container B (accelerating upwards with the elevator).

Circle the true statement:

$\boldsymbol{\Delta x_A > \Delta x_B}$ $\qquad$ $\boldsymbol{\Delta x_A < \Delta x_b}$ $\qquad$ $\boldsymbol{\Delta x_A = \Delta x_B}$

Problem 353. problems-1/fluids-mc-floating-mass-on-spring-moon.tex



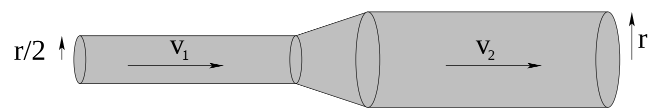
In the figures above, two identical springs (with spring constant k) are attached to the bottoms of two identical containers filled with water (density ρ). At the other end, the springs are attached to identical wooden blocks that would ordinarily ***float*** on the water so that they are completely submerged.

The apparatus on the left (A) is located on the Earth's surface, where the acceleration due to gravity is g. The apparatus on the right (B) is located on the moon, where the acceleration due to gravity is $g/6$. D_e is the total length of the stretched spring on the Earth, D_m is the total length of the stretched spring on the moon.

Circle the true statement:

$\boldsymbol{D_e > D_m}$ $\boldsymbol{D_e < D_m}$ $\boldsymbol{D_e = D_m}$

Problem 354. problems-1/fluids-mc-flow-constricted-pipe-reverse.tex



Water flows at speed v_1 in a pipe with radius $r/2$ and passes into a pipe with radius r through a smooth constriction as shown. Neglect viscosity. Select the statements that correctly describes v_2, the speed in the wider section of pipe, and the relative pressure in the narrower and wider segments of pipe

a) $v_2 = 4v_1$

b) $v_2 = 2v_1$

c) $v_2 = v_1$

d) $v_2 = \frac{1}{2}v_1$

e) $v_2 = \frac{1}{4}v_1$

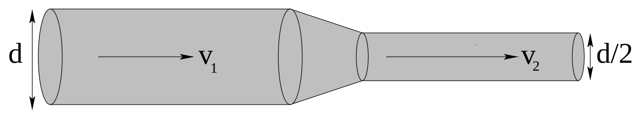
a) $P_1 > P_2$

b) $P_1 = P_2$

c) $P_1 < P_2$

d) We cannot tell from Bernoulli's equation without knowing the fluid's density ρ.

Problem 355. problems-1/fluids-mc-flow-constricted-pipe.tex



Water flows at speed v_1 in a pipe with diameter d and passes into a pipe with diameter $d/2$ through a smooth constriction as shown. Select the statement that correctly describes v_2, the speed in the narrower pipe.

a) $v_2 = 4v_1$

b) $v_2 = 2v_1$

c) $v_2 = v_1$

d) $v_2 = \frac{1}{2}v_1$

e) $v_2 = \frac{1}{4}v_1$

11.1.2 Ranking/Scaling

Problem 356. problems-1/fluids-ra-archimedes-objects-2.tex

Four large identical beakers are filled with water and also contain objects in static equilibrium with the water and beaker (they are not attached to or supported by anything outside of the beaker. The objects are, ***listed in the order of strictly decreasing density***:

a) A solid gold coin that has a mass of 100 grams;

b) A cast aluminum frog that has a mass of 100 grams;

c) An ice cube that has a mass of 100 grams;

d) A wooden carved monkey that has a mass of 100 grams.

You remove each object from the water in its beaker and measure the ***drop in water depth*** Δd_i, $i = a, b, c, d$.

Rank the Δd_i you expect to observe in this experiment from ***smallest to largest***. As always, in the case that some of the Δd_i are equal to neighbors, indicate that explicitly.

Problem 357. problems-1/fluids-ra-archimedes-objects.tex

A large beaker is filled to a marked line with water. You have the following objects (in order of ***decreasing density***):

a) A solid gold coin that has a mass of 100 grams.

b) A cast aluminum frog that has a mass of 100 grams.

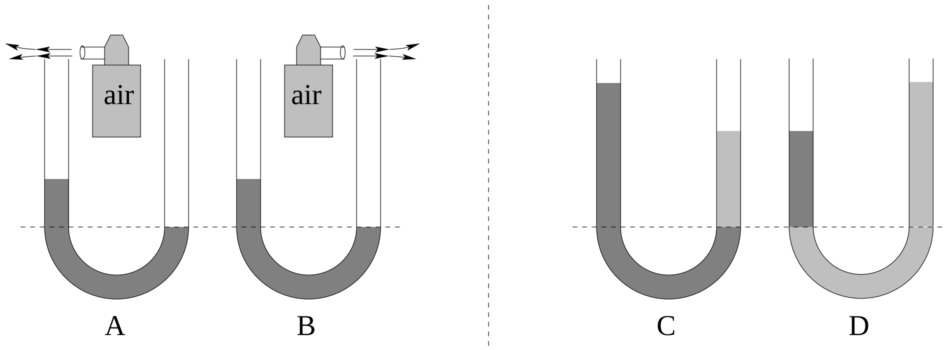
c) An ice cube that has a mass of 100 grams

d) A 100 gram chunk of shipping styrofoam.

You drop each item, one at a time, into the beaker in the water and record d_i, the change in water depth, and then remove it.

Rank the expected results for d_i for $i = a, b, c, d$. ***Indicate*** whether d_i is positive (so that the water in the beaker rises) or negative (falls). As always, in the case that some of the d_i are equal to neighbors, ***indicate that explicitly***.

Problem 358. problems-1/fluids-ra-four-utubes-venturi.tex

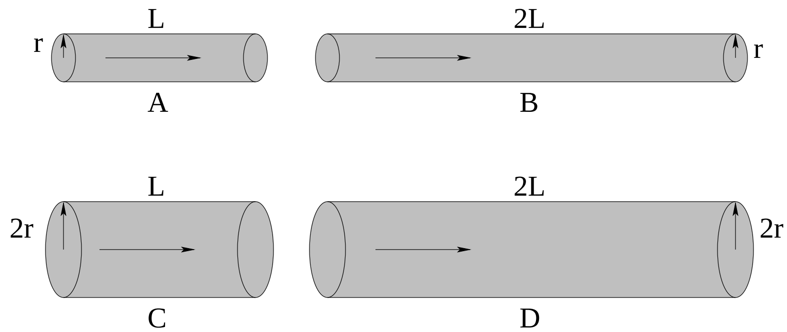


In the four u-tubes pictured above, only one of the two cases in each pair (A vs B and C vs D) make sense. In A vs B, a can of compressed air is blowing air ***across the top*** of one of the tube tops and the tube contains only a single fluid. In C vs D, the density of the immiscible fluids is indicated by the shading where the ***darker fluid has the greater density***.

Which two u-tubes DO make physical sense? (Circle one of each pair.)

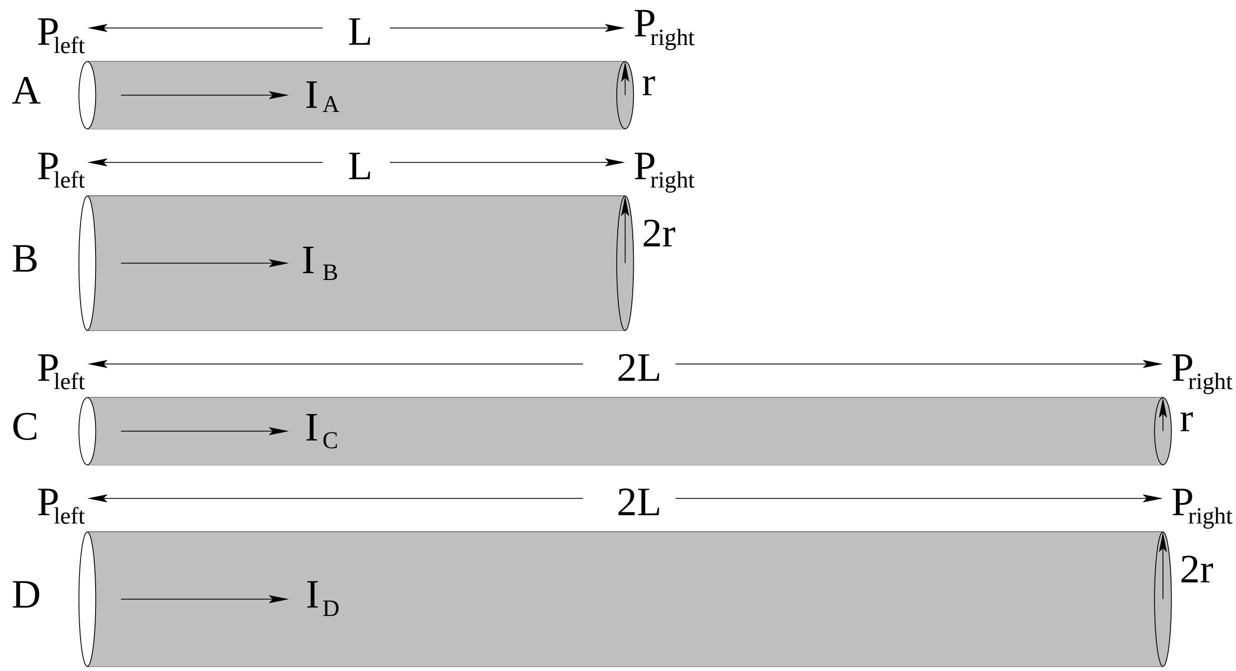
A **B** **C** **D**

Problem 359. problems-1/fluids-ra-poiseiulles-law-2.tex

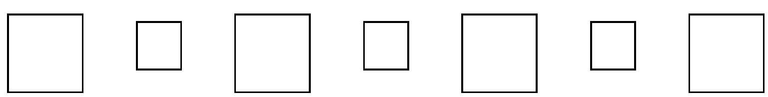


In the figure above, several circular pipes carry fluids with the same viscosity. Rank the pipes in the order of their **resistance** to laminar flow, from least to greatest. Equality is a possible answer. Think carefully about the dependence on r in Poiseuille's Law! This is why obstructions in arteries increase the resistance so dramatically!

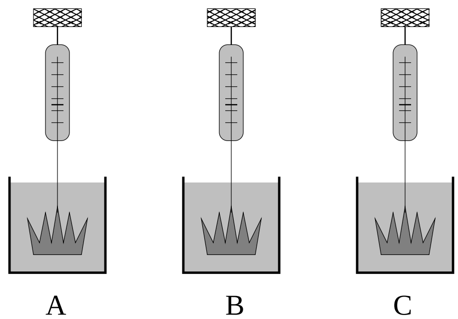
Problem 360. problems-1/fluids-ra-poiseiulles-law.tex



Rank the volume flow I from the lowest to highest in the boxes below by filling A, B, C, and D into the large boxes and putting "<" or "=" signs into the small boxes in between for the four circular pipes illustrated in the figure above, assuming that in all cases that the flow, from left to right, is maintained by the *same* $\Delta P = P_{\text{left}} - P_{\text{right}} > 0$ and that the same fluid (with the same viscosity μ) is flowing through the pipes.

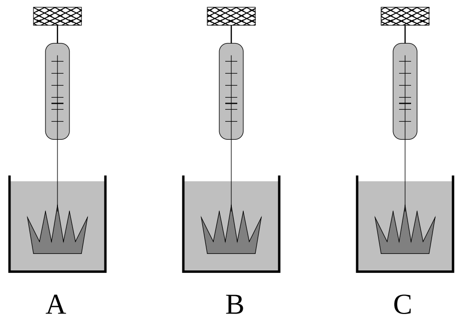


Problem 361. problems-1/fluids-ra-three-crowns-density.tex



Three crowns are shown above. Crown A is made of solid lead (specific gravity 11.3) covered with a thin veneer of gold leaf. Crown B is made of platinum (specific gravity 21.5), also covered with a thin veneer of gold leaf. Crown C is made of pure gold (specific gravity 19.3). The scale suspending the three crowns in water ***all read a weight of 5 newtons***. Rank the crowns in order of their ***true weight*** as measured in air from lowest to highest.

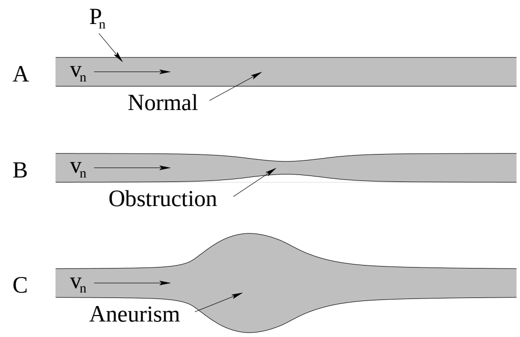
Problem 362. problems-1/fluids-ra-three-crowns.tex



Three crowns are shown above. Crown A is made of solid lead (specific gravity 11.3) covered with a thin veneer of gold leaf. Crown B is made of platinum (specific gravity 21.5), also covered with a thin veneer of gold leaf. Crown C is made of pure gold (specific gravity 19.3). All three crowns weigh **exactly 500 grams in air**. Rank the crowns in the order of effective weight while immersed in the water (what the three scales will read) lowest to highest.

11.1.3 Short Answer

Problem 363. problems-1/fluids-sa-aneurism-pressure-flow.tex



Consider the models above of a normal blood vessel (A), an obstructed blood vessel (B) and an aneurism (C). In case (A) blood is flowing from left to right at a "normal" fluid velocity v_n and pressure P_n. Assume that the blood pressure right before and after the obstruction or aneurism is also P_n. Neglect viscosity while answering the following questions:

a) Is the blood pressure in the obstructed region P_o in (B) higher or lower than P_n?

b) Is the blood pressure in the aneurism P_a in (C) higher or lower than P_n?

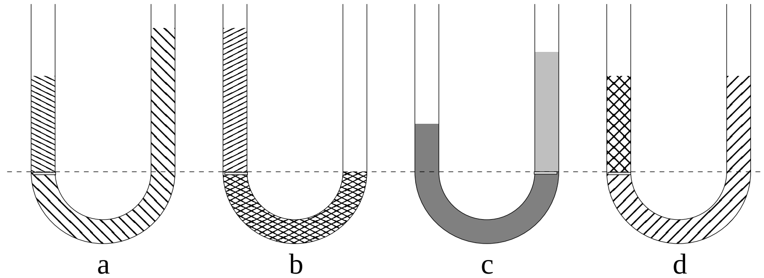
Problem 364. problems-1/fluids-sa-balloon-in-car.tex

A small boy is riding in a minivan with the windows closed, holding a helium balloon. The van goes around a corner to the left. Does the balloon swing to the left, still pull straight up, or swing to the right as the van swings around the corner?

Problem 365. problems-1/fluids-sa-breathing-underwater-through-a-tube.tex

In adventure movies, the hero is often being chased by the bad guys and escapes by hiding deep underwater and breathing through a tube of some sort. Assuming that you can barely manage to breathe if a 500 Newton person is standing directly on your chest while you are lying on the floor, ***estimate*** the maximum depth (of your chest) where one is likely to have the muscular strength to be able to breathe through a rigid tube extending to the surface. Your estimate should be quantitative and you should support it with both a very short piece of algebra and a picture clearly showing the forces you must work against to breathe underwater.

Problem 366. problems-1/fluids-sa-four-utubes-1.tex

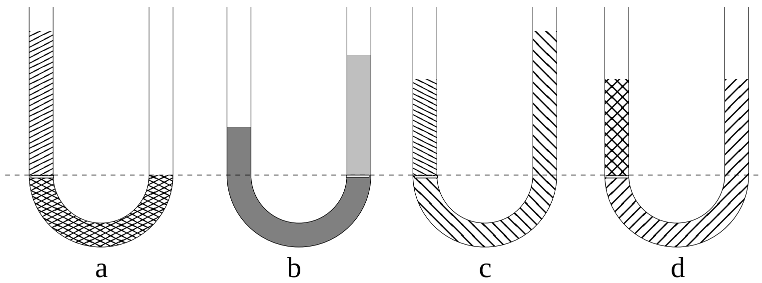


Two different incompressible fluids separated by a thin (massless, frictionless) piston so that they cannot mix are open to the atmosphere and are in static equilibrium in each of the four U-tubes pictured above.

a) One of the four U-tubes ***makes no sense*** (cannot be in equilibrium). Circle it and label it "impossible".

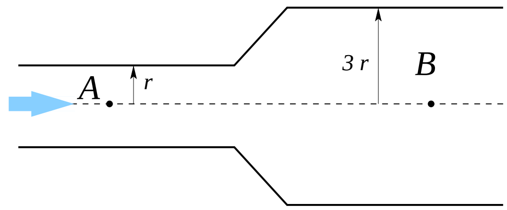
b) Underneath each u-tube that ***does*** make sense indicate whether the fluid at the top of the ***left-hand side*** of the "U" is ***denser than, less dense than, or the same density as*** the fluid at the top of the ***right***-hand side.

Problem 367. problems-1/fluids-sa-four-utubes-2.tex



(6 points) Two different incompressible fluids separated by a thin (massless, frictionless) piston so that they cannot mix are open to the atmosphere and presumably in static equilibrium in each of the four u-tubes pictured above. One of the four u-tubes makes no sense (cannot be in equilibrium). Circle it. Underneath each u-tube that *does* make sense indicate whether the fluid at the top of the **left**-hand side of the "u" is denser than, less dense than, or the same density as the fluid at the top of the **right**-hand side. Briefly indicate your reasons.

Problem 368. problems-1/fluids-sa-horizontal-necked-pipe.tex



An incompressible, non-viscous fluid flows from the left to the right through a pipe of varying radius as shown in the figure. Let us compare the fluid at point A and the fluid at point B.

Answer questions (a) to (c) below by entering the letters “A”, “B”, or if the same magnitude, “=” in the provided boxes.

[*Showing your work is recommended, but not mandatory.*]

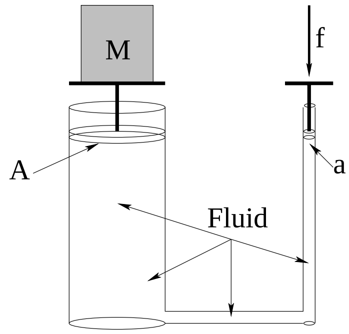
a) The speed of the fluid is larger at point $\boxed{}$;

b) Volume flow rate (Q) is higher at point $\boxed{}$;

c) Pressure is higher at point $\boxed{}$;

d) Write down the ratio of the flow speed at two points: $\dfrac{v_B}{v_A} = \boxed{}$.

Problem 369. problems-1/fluids-sa-hydraulic-lift-2.tex



A piston of small cross sectional area a is used in a hydraulic press to exert a force f on the enclosed liquid. A connecting pipe leads to the larger piston of cross sectional area A, so that $A > a$. The two pistons are at the same height. The weight $w = Mg$ that can be supported by the larger piston is

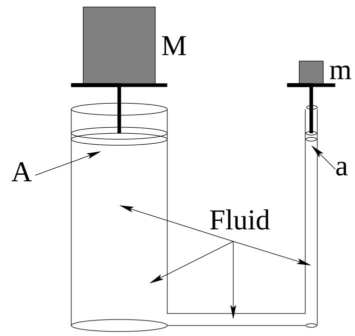
(a) $w > f$

(b) $w < f$

(c) $w = f$

(d) depends on whether the liquid is compressible or not.

Problem 370. problems-1/fluids-sa-hydraulic-lift.tex



The pair of coupled piston-and-cylinders shown above are sitting in air and filled with an incompressible fluid. The entire system is in static equilibrium (so nothing moves). The cross-sectional area of the large piston is A; the cross-sectional area of the small piston is a. In this case we know that:

a) $M = \frac{A}{a} m$

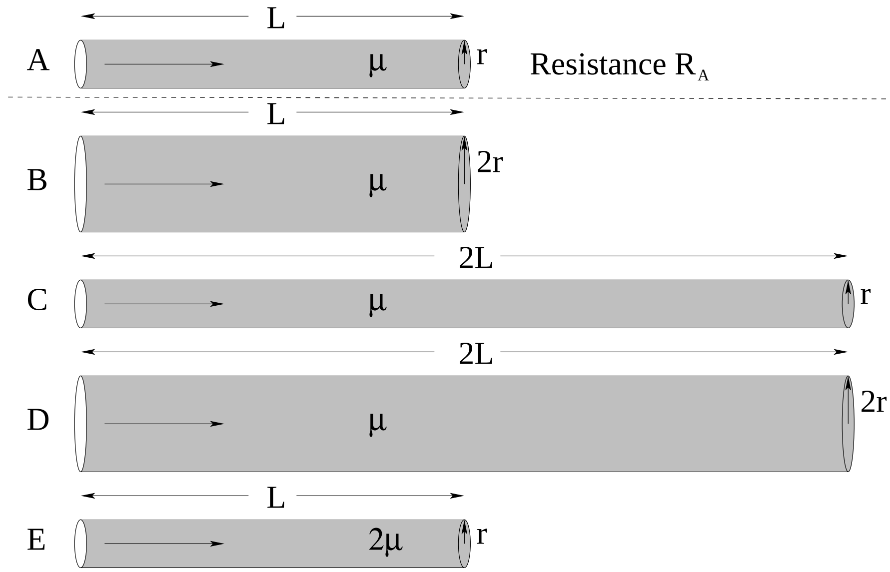
b) $M = \frac{a}{A} m$

c) $M = \sqrt{\frac{A}{a}} m$

d) $M = m$

e) We cannot tell what M is relative to m without more information.

Problem 371. problems-1/fluids-sa-poiseiulles-law-2.tex



In the figure above, fluids of the given viscosities flow through circular pipes A-E with the given dimensions. The ***resistance*** to fluid flow of circular pipe A is known to be R_A. What are the resistances of the other four pipes in terms of R_A?

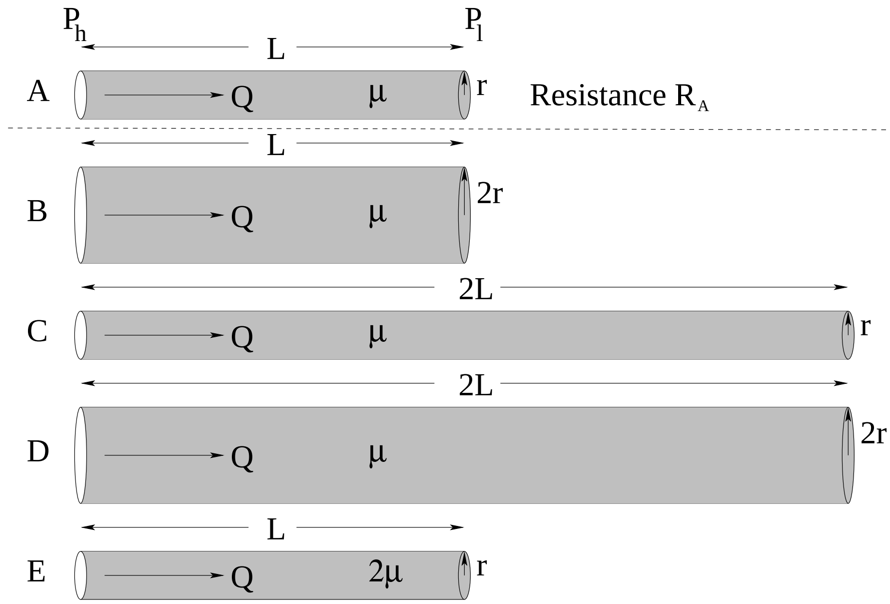
$\frac{R_B}{R_A} =$ ☐

$\frac{R_C}{R_A} =$ ☐

$\frac{R_D}{R_A} =$ ☐

$\frac{R_E}{R_A} =$ ☐

Problem 372. problems-1/fluids-sa-poiseiulles-law-3.tex



In the figure above, fluids of the given viscosities flow through circular pipes A-E with the given dimensions. In all cases the volumetric flow through the pipes is held ***constant*** at Q by varying the pressure difference $\Delta P_i = P_{\rm high} - P_{\rm low}$ across each $(i = A, B, C, D, E)$ pictured pipe segment.

The pressure difference that maintains flow Q fluid flow of circular pipe A is defined to be ΔP_A. What are the pressure differences across the other four pipes in terms of ΔP_A?

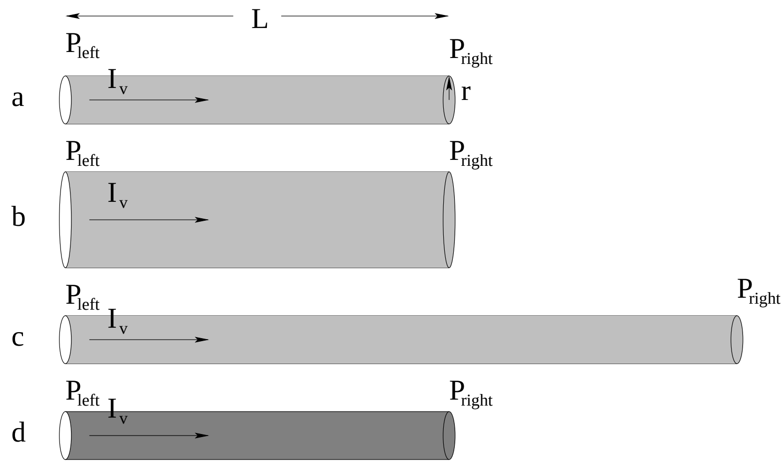
$\frac{\Delta P_B}{\Delta P_A} = \boxed{}$

$\frac{\Delta P_C}{\Delta P_A} = \boxed{}$

$\frac{\Delta P_D}{\Delta P_A} = \boxed{}$

$\frac{\Delta P_E}{\Delta P_A} = \boxed{}$

Problem 373. problems-1/fluids-sa-poiseiulles-law.tex



Use *Poiseuille's Law* to answer the following questions:

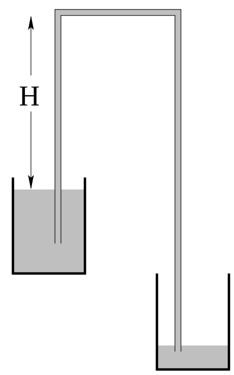
a) Is $\Delta P_a = P_{\rm left} - P_{\rm right}$ greater than, less than, or equal to zero in figure a) above, where blood flows at a rate I_v horizontally through a blood vessel with constant radius r and some length L against the **resistance** of that vessel?

b) If the radius r **increases** (while flow I_v and length L remain the same as in a), does the pressure difference ΔP_b increase, decrease, or remain the same compared to ΔP_a?

c) If the length **increases** (while flow I_v and radius r remains the same as in a), does the pressure difference ΔP_c increase, decrease, or remain the same compared to ΔP_a?

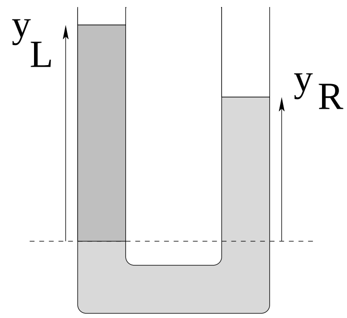
d) If the viscosity μ of the blood **increases** (where flow I_v, radius r, and length L are all unchanged compared to a) do you expect the pressure difference ΔP_d difference across a blood vessel to increase, decrease, or remain the same compared to ΔP_a?

Problem 374. problems-1/fluids-sa-siphon.tex



A **siphon** is a device for lifting water out of one (higher) reservoir and delivering it another (lower) reservoir as shown above. Estimate the probable maximum height H one can lift the water above the upper reservoir's water level before the tube descends into the lower reservoir. Explain your reasoning – how, and where, will the siphon fail?

Problem 375. problems-1/fluids-sa-utube-fluid-height.tex



A vertical U-tube open to the air at the top is filled with oil (density ρ_o) on one side and water (density ρ_w) on the other, where $\rho_o < \rho_w$.

a) Make your own diagram of the problem and clearly label the oil and the water.

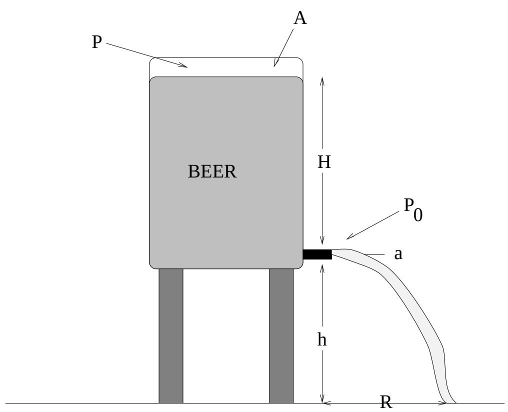
b) Find (derive) an expression for $\Delta y = y_L - y_R$, the ***difference*** in the heights of the two columns in terms of y_L.

Problem 376. problems-1/fluids-sa-walking-in-a-pool.tex

People with vascular disease or varicose veins (a disorder where the veins in one's lower extremeties become swollen and distended with fluid) are often told to walk in water 1-1.5 meters deep. Explain why.

11.1.4 Long Problems

Problem 377. problems-1/fluids-pr-bernoulli-beer-keg-horizontal.tex



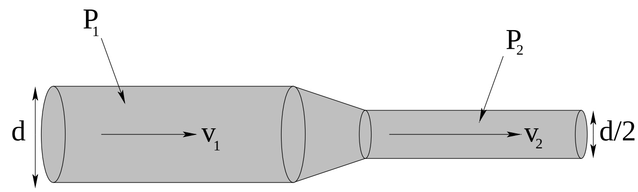
In the figure above, a CO_2 cartridge is used to maintain a pressure P on top of the beer in a beer keg, which is full up to a height H above the tap at the bottom (which is obviously open to normal air pressure) a height h above the ground. The keg has a cross-sectional area A at the top. Somebody has pulled the tube and valve off of the tap (which has a cross sectional area of a) at the bottom and it is spurting out onto the ground.

a) Find the speed with which the beer emerges from the tap. You may use the approximation $A \gg a$, but please do so only at the end of your algebra, not at the beginning. Assume laminar flow and no resistance.

b) Find the value of R at which you should place a pitcher (initially) to catch the beer.

c) Evaluate the answers to a) and b) for $A = 0.25$ m^2, $P = 2$ atmospheres, $a = 0.25$ cm^2, $H = 50$ cm, $h = 1$ meter and $\rho_{\text{beer}} = 1000$ kg/m^3 (the same as water).

Problem 378. problems-1/fluids-pr-bernoulli-constricted-pipe.tex



Water flows at a pressure P_1 and a speed v_1 in a circular storm culvert pipe of diameter d. The pipe narrows smoothly to a second pipe section where the diameter is only $d/2$.

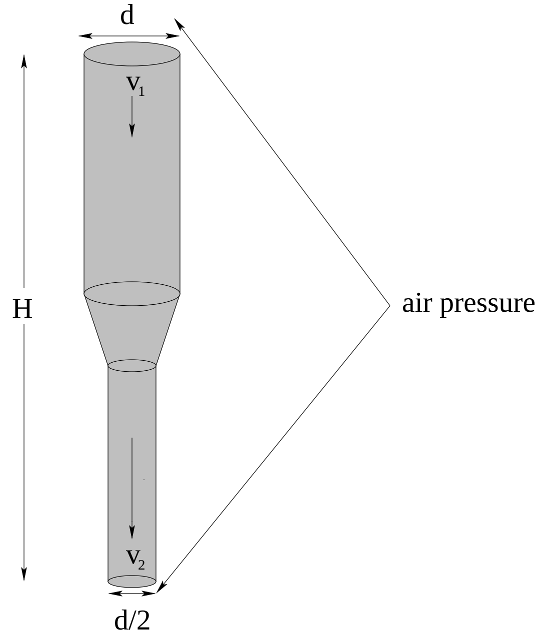
a) Find v_2, the speed in the second pipe.

b) Find P_2, the pressure in the second pipe.

c) Write an algebraic expression in terms of the givens for the current (flow) I, the volume of water per second that passes through the pipe(s).

d) Evaluate your answer(s) given the data: $P_1 = 1.075 \times 10^5$ Pa, $v_1 = 1$ m/sec, $d = 2/\sqrt{\pi}$ meters. No calculator should be needed.

Problem 379. problems-1/fluids-pr-bernoulli-constricted-pipe-vertical.tex



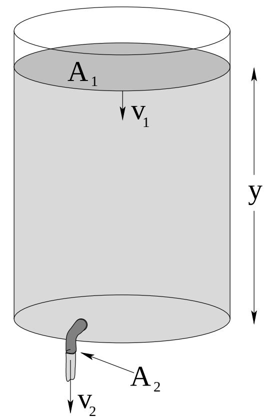
A drain pipe in a house starts out at a **diameter** of d and narrows smoothly to a second pipe section where the **diameter** is only $d/2$. It is filled with water to a height H above the exit point of the lower pipe where it empties into a storm sewer. Both ends of the pipe are ***open to the air***.

a) Find v_1 and v_2, the speed of the flowing fluid in both pipe sections.

b) Write an algebraic expression in terms of the givens for the current (flow) Q, the volume of water per second that passes through the pipe(s). Give the expression in terms of d and v_1 and/or v_2 so that your answer does not depend on your answer to a).

c) How long Δt will it take for the water level in the top pipe to drop a distance $\Delta x \ll H$?

Problem 380. problems-1/fluids-pr-bernoulli-emptying-iced-tea-time.tex



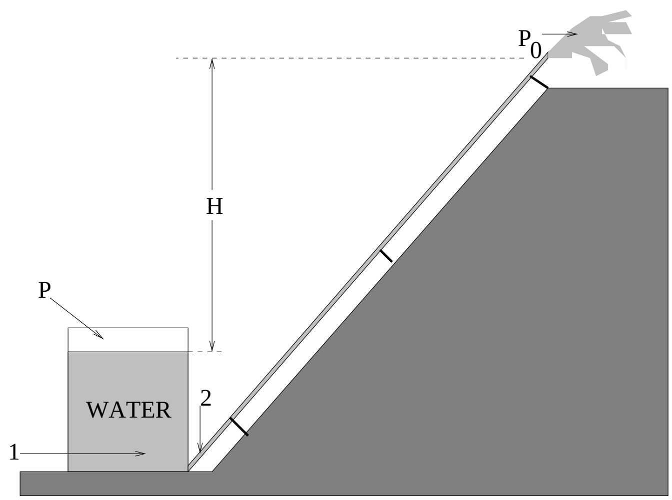
In the figure above, a big jug of iced tea with cross-sectional area A_1 is open to the air on top. A tap on the bottom has a hole with a cross-sectional area A_2. The surface of the iced tea is a height H above the tap.

a) Find the rate at which the height of the iced tea drops – dH/dt – when the tap is opened.

b) How long does it take for all the iced tea to run out?

You may assume that $A_1 \gg A_2$ and use any approximations that may suggest.

Problem 381. problems-1/fluids-pr-bernoulli-irrigation-pipe.tex



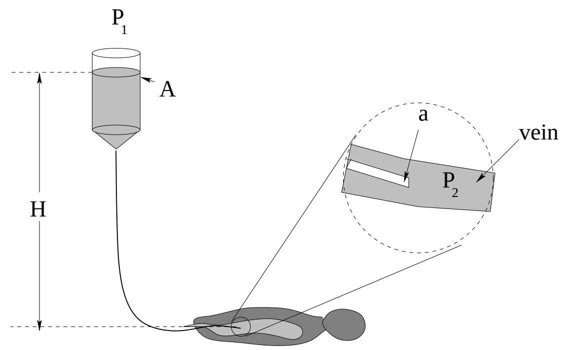
In the figure above, a pump maintains a pressure of P in the air at the top of a tank of water with a cross sectional area A. An irrigation pipe at the bottom leads up a slope to a farmer's field. The vertical distance between the top surface of water in the tank and the opening of the pipe is H. The cross-sectional area of the pipe is a. The top pipe is open to air pressure $P_0 = 1$ atm. Recall that the density of water is $\rho = 10^3$ kg/m^3.

a) What is the velocity of the water coming from the pipe? (Find this ***algebraically*** from the appropriate law(s).)

b) Is the pressure at the bottom of the tank greater inside the main vessel (point 1 on figure above) or inside the pipe (point 2)? ***Briefly*** explain.

c) ***After*** finding the answer to a) algebraically and answering b), evaluate v numerically using: $P = 2.5$ atm, $A = 10$ m^2, $H = 10$ m, and $a = 4$ cm^2. You shouldn't need a calculator for this.

Problem 382. problems-1/fluids-pr-bernoulli-IV.tex



Although mechanized and precise in modern first-world medicine, IV fluid delivery in the rest of the world is an imprecise gravity-driven system. A bag or bottle filled with a saline solution, plasma, blood, or medicine is hung above a patient's bed and a tube delivers that fluid directly into a patient's vein. A physician practicing medicine in many clinics or hospitals around the world may well need to be able to estimate things like the time of delivery of a bolus of fluid by a gravity-driven IV line for a given needle size.

Make such an estimate below, assuming that the bag of cross-sectional area A holds a fluid of density ρ, is effectively open to air pressure in the room P_1, and is suspended a height H above the level of the patient as shown. Use a as the cross-sectional area of the needle. Ignore viscosity and the fluid flow resistance of the tubing. ***Express all your answers algebraically in terms of*** A, a, ρ, P_1 ***and*** P_2 ***for full credit***.

a) What is the minimum height H_{min} such that flow is from the bag ***to*** the patient instead of from the patient ***back*** towards the bag? (We don't want the patient to inadvertently donate blood!)

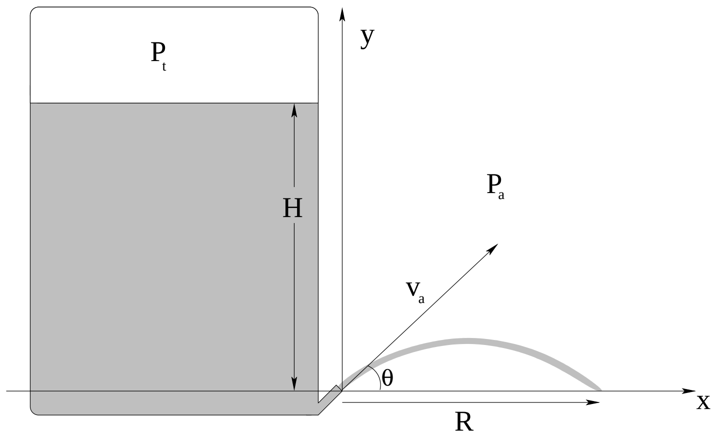
b) Suppose you raise the bag height to $H = 2H_{\text{min}}$. With what velocity does the fluid flow into the patient?

c) If the bag holds a fluid volume V ***estimate*** how long does it will take to deliver all of the fluid in the bag into the patient at this new height. Assume that H does not change (much) while the bag empties.

d) If one included viscosity and the drop in fluid height as the bag empties, would it increase or decrease the time from this rough estimate?

After finding the algebraic answers, you may estimate the numerical values of these quantities without a calculator for one point of EXTRA credit per answer for a maximum of three extra points. Assume that the fluid is water, $V = 500$ cubic centimeters, $P_1 = 1$ atm, $P_2 = 1.1$ atm, $A = 2 \times 10^{-3}$ m^2, and $a = \sqrt{5} \times 10^{-7}$ m^2. (Note that leaving radicals like $\sqrt{5}$ in your answers is OK).

Problem 383. problems-1/fluids-pr-bernoulli-range-arc.tex

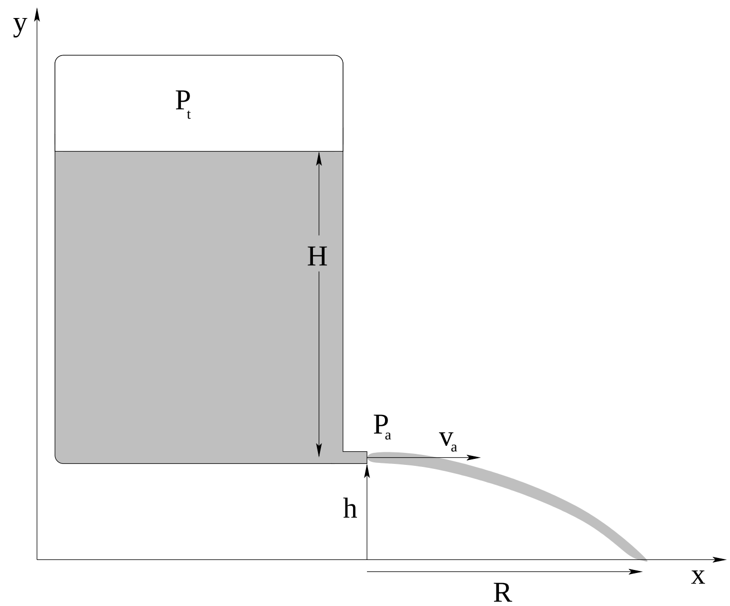


A sealed tank of water (density ρ) is shown above. Inside it is pressurized to a pressure $P_t = 3P_a$ (where P_a is the pressure outside of the tank, one atmosphere). The water escapes through a ***small pipe at the bottom*** where the stream is angled up at an angle θ with respect to the ground as shown. The cross-sectional area of the tank A is much larger than the cross-sectional area a of the small pipe at the bottom, $A \gg a$. (Picture is not necessarily to scale.)

a) What is the (approximate) speed v_a with which the water exits the small pipe? Express your answer (for this part only) in terms of ρ, g, P_t, P_a and possibly A and a.

b) What is the ***horizontal range*** of the stream of water, R, measured from the tip of the spout as shown. Express your answer (for this part only) in terms of v_a.

Problem 384. problems-1/fluids-pr-bernoulli-range-horizontal.tex

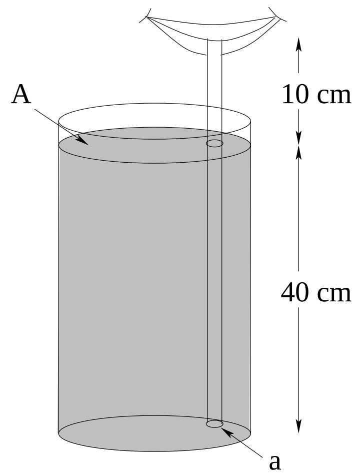


A sealed tank of water (density ρ_w) is shown above. Inside the air is pressurized above the water to a pressure $P_t = 2P_a$ (where P_a is the air pressure outside of the tank, one atmosphere). The water escapes through a ***small pipe at the bottom*** where the stream emerges parallel to the ground as shown. The cross-sectional area of the tank A is much larger than the cross-sectional area a of the small pipe at the bottom. Neglect viscosity and flow resistance. Picture is not necessarily to scale.

a) Find the (approximate) speed v_a with which the water exits the small pipe. You may assume $A \gg a$.

b) Find the ***horizontal range*** of the stream of water, R, measured from the tip of the spout as shown. Express your answer in terms of v_a, so that it needn't depend on getting a) correct.

Problem 385. problems-1/fluids-pr-bernoulli-sipping-through-straw.tex

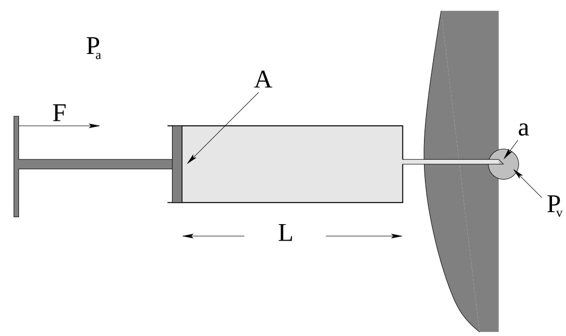


When you drink through a straw, you create a pressure P_m in your mouth that is less than atmospheric pressure. Suppose $P_m = 9 \times 10^4$ Pa, and the end of the straw in your mouth is 10cm above the surface of your 40cm high drink as shown above. You may assume that the cross-sectional area of the straw a is much less than the cross-sectional area A of the fluid at the top of your glass.

a) At what speed will the fluid in the straw be moving into your mouth? (Use $P_0 = 10^5$ Pa for the pressure of the air, $\rho = 1000$ kg/m^3, $g = 10$ m/s^2 and compute a number ***after showing how you obtained an algebraic expression for the answer.***)

b) Find an algebraic expression for how long it will take to sip a small volume ΔV of your drink through the straw. Assume that the fluid height in the container makes a negligible change during this sip.

Problem 386. problems-1/fluids-pr-bernoulli-syringe.tex



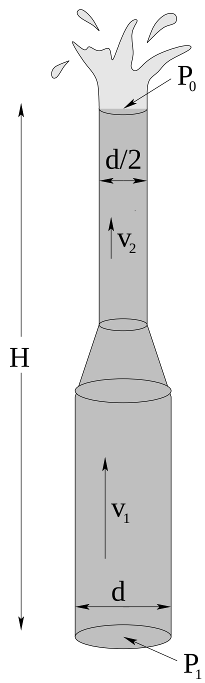
You are in a room at normal air pressure P_a and are given a hypodermic syringe full of medicine that we will treat as a zero-viscosity fluid with the density of water. The syringe tube has length L and cross-sectional area A and hence contains a volume AL of fluid. The cross-sectional area of the needle aperture is $a \ll A$. Holding the syringe ***horizontally*** as shown, you press on the (frictionless) plunger to inject the medicine into a patient's vein where the (given) blood pressure is $P_v > P_a$.

a) What force $F_{\rm min}$ (magnitude) do you have to exert on the plunger to hold the fluid in ***static equilibrium*** once the needle is in the patient?

b) Suppose you push with a force $F > F_{\rm min}$ on the plunger. Find an expression for the speed v_v with which the fluid flows *through the needle* into the vein. Don't forget the pressure of the air in the room!

c) Find an expression for the time required to empty the syringe ***in terms of v_v*** (so you do not have to use the results for b) or get b) correct to get full credit for c)).

Problem 387. problems-1/fluids-pr-bernoulli-vertical-pipe-fountain.tex

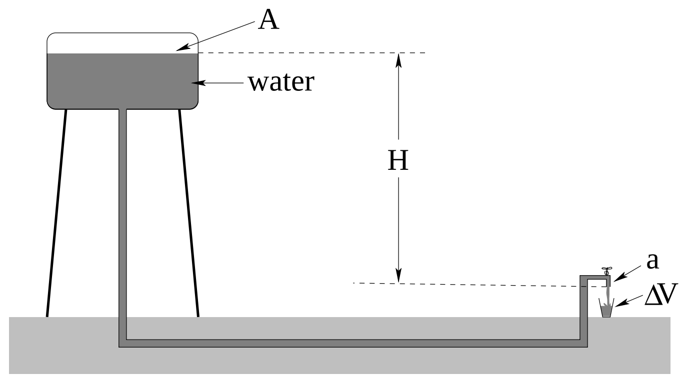


A small fountain used in a zen rock garden is pictured above. A pump (not shown) maintains a given pressure P_1 at the base of a pipe of diameter d that lifts the water and narrows to a diameter of $d/2$ at the top to speed it up. The water exits into air at pressure $P_0 = 1$ atm. The overall pipe has height H between the pump and the exit.

a) Find v_1 and v_2, the speed of the flowing fluid in both pipe sections.

b) Write an algebraic expression in terms of the givens for the current (flow) Q, the ***volume of water per second*** that passes through the pipe(s). Give the expression in terms of d and v_1 and/or v_2 so that your answer does not depend on your answer to a).

Problem 388. problems-1/fluids-pr-city-water-supply-tank.tex



The figure above represents the water distribution system of a typical city or town. An elevated tank is filled with water from a purified source. Sealed pipes descend from the tank and extend through the ground to your house, where your closed water tap holds in the pressure. When you open the tap, water flows from the tank, through the pipe, and out into your glass.

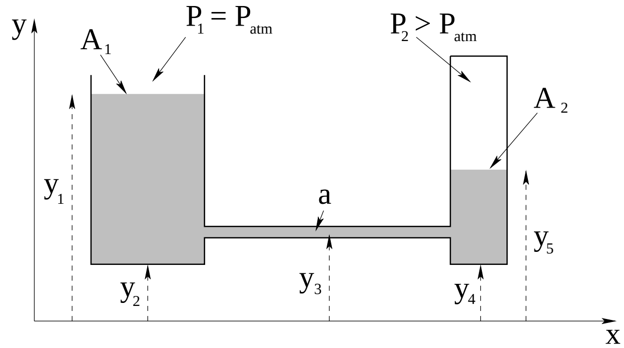
Suppose that the top of the tank has a cross-sectional area $A \gg a$, where a is the cross-sectional area of your spigot. A pump (not shown) maintains the water height in the tank so that it remains a height H above your spigot as shown whether the tap is open or closed. The tank is filled with water of density ρ, and both the top of the tank and the spigot are open to air at the same pressure (one atmosphere). Assume laminar flow and zero viscosity.

a) When your tap is closed, what is the pressure of the water just inside the tap?

b) When the tap is opened, with what speed does water flow out of the tap?

c) How long will it take to fill the cup of volume ΔV shown with water?

Problem 389. problems-1/fluids-pr-flow-between-containers.tex



In the figure above, water (density ρ) is being pushed through a pipe of cross-sectional area a from a small (sealed) tank to a larger one open to the air at normal pressure $P_1 = P_{\rm atm}$ by the pressure difference when $P_2 > P_{\rm atm}$. The cross-sectional areas of the two containers are given in terms of the cross-sectional area of the pipe by $A_1 = 400a$ and $A_2 = 100a$.

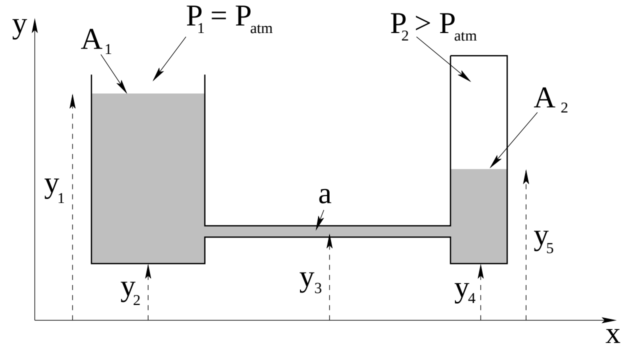
The givens are: a, ρ, P_1, and the various heights y_i labeled in the figure that ***may or may not be of interest*** . Neglect viscosity and drag/resistance in the containers or pipe.

a) Find the smallest pressure $P_{2,\rm min}$ that will cause water to flow *from* the smaller container *to* the larger one instead of the other way around.

b) Suppose the pressure P_2 is ***larger*** than this minimum pressure (so water *does* flow from the smaller to the larger container). What is the speed v_p of the fluid in the **pipe** as this happens? Hint: the velocity in *both containers* is negligible compared to the velocity in the *pipe!* What is the (nearly static) force that drives water one way or the other through the pipe?

c) What are the speeds v_1 with which the water ***rises*** in the first container and v_2 ***falls*** in the second?

Problem 390. problems-1/fluids-pr-flow-between-containers-soln.tex



First, this is a hard problem. Full credit will be given for any solution attempt that gets a) correct and indicates that you know both Bernoulli's formula and the equation for conservation of flow. A five point bonus of extra credit will be given if you have the insight that you need to neglect ***both*** tank velocities compared to the velocity in the pipe!

Indeed, this problem can only consistently be solved by applying Bernoulli's formula *across the pipe*. This is because there are *three* velocities, and $v_1, v_2 \ll v_p$, so that *only* the kinetic energy term in the pipe (also called the "dynamical pressure") is not negligible compared to that in *either* tank. Neglecting the kinetic energy terms in both tanks is equivalent to using the the *static pressure difference* at the bottom of the two tanks at the entrance and exit of the pipe as the source of the "work" that drives the water through the pipe. This makes sense! If you actually get this point, you will get a bonus of five points on the problem and the exam! If you don't, you will encounter serious difficulties (e.g. imaginary numbers) trying to apply the Bernoulli formula to tanks 1 and 2 while ignoring the pipe.

We start, then, by evaluating the (approximately!) static pressure in the bottoms of both tanks at the height of the pipe. On the left end of the pipe (tank 1) is:

$$P_{b1} = P_1 + \rho g(y_1 - y_3)$$

on the right (tank 2) it is:

$$P_{b2} = P_2 + \rho g(y_5 - y_3)$$

For part a), Fluid will not flow if this static pressure matches across the pipe! This result is exact, and everybody should be able to get it.

$$P_{b1} = P_1 + \rho g(y_1 - y_3) = P_2 + \rho g(y_5 - y_3) = P_{b2}$$

or (with $P_1 = P_{\rm atm}$):

$$P_{2,\rm min} = P_{\rm atm} + \rho g(y_1 - y_5)$$

and:

$$\boxed{P_2 > P_{\rm atm} + \rho g(y_1 - y_5)}$$

will make the fluid flow uphill into the larger container. This makes complete sense. It *also* suggests, if you think about it, that the work (per unit volume) that speeds the water up effectively from "rest" is:

$$(P_2 - P_{\rm atm}) - \rho g(y_1 - y_5) = \frac{1}{2}\rho v_p^2$$

To answer both b) and c) we need to use Bernoulli where the pipe pressure equals the static pressure at the bottom of tank 1 (the tank the water is flowing in*to*), and where we equate the formula to the static pressure on the bottom of tank 2:

$$P_{b1} + \rho g y_3 + \frac{1}{2}\rho v_p^2 = P_{b2} + \rho g y_3$$

Note that even if there is a small contribution from the motion of the fluids in tanks 1 and 2, it is a negligible correction to P_{b1} and P_{b2} respectively compared to the $\frac{1}{2}\rho v_p^2$ term! Then (cancelling the $\rho g y_3$ bits):

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_p^2 = P_2 + \rho g y_5$$

and indeed:

$$(P_2 - P_{\text{atm}}) - \rho g(y_1 - y_5) = \frac{1}{2}\rho v_p^2$$

as we guessed above. Solving for v_p:

$$v_p = \sqrt{\frac{2\left((P_2 - P_{\text{atm}}) - \rho g(y_1 - y_5)\right)}{\rho}}$$

As you can see, as long as $P_2 > P_{\text{atm}} + \rho g(y_1 - y_5)$, the pressure at the bottom of tank 2 will be higher than the pressure at the bottom of tank 1, and the pressure difference will drive water from tank 2 to tank 1 at speed v_p in the pipe. If we go the other way, the role of starting and ending height reverse (changing sign) and we'll only get a real answer if $P_2 < P_{2,min}$.

To get the speeds at the top of the tanks is now simple. From flow conservation:

$$A_1 v_1 = a v_p = A_2 v_2$$

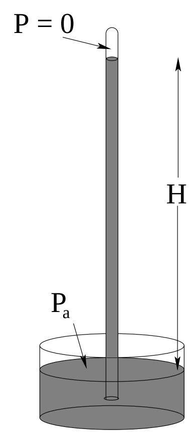
or

$$\boxed{v_1 = \frac{a}{A_1} v_p = 0.0025\sqrt{\frac{2\left((P_2 - P_{\text{atm}}) - \rho g(y_1 - y_5)\right)}{\rho}}}$$

and:

$$\boxed{v_2 = \frac{a}{A_2} v_p = 4 v_1 = 0.01\sqrt{\frac{2\left((P_2 - P_{\text{atm}}) - \rho g(y_1 - y_5)\right)}{\rho}}}$$

Problem 391. problems-1/fluids-pr-compare-barometers.tex



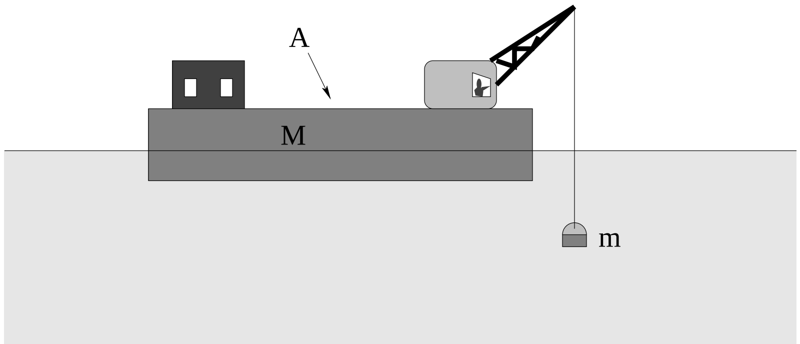
The idea of a barometer is a simple one. A tube filled with a suitable liquid is inverted into a reservoir. The tube empties (maintaining a seal so air bubbles cannot get into the tube) until the static pressure in the liquid is in balance with the *vacuum* that forms at the top of the tube and the ambient pressure of the surrounding air on the fluid surface of the reservoir at the bottom.

a) Suppose the fluid is water, with $\rho_w = 1000$ kg/m^3. Approximately how high will the water column be? Note that water is not an ideal fluid to make a barometer with because of the height of the column necessary and because of its annoying tendency to boil at room temperature into a vacuum.

b) Suppose the fluid is mercury, with a specific gravity of 13.6. How high will the mercury column be? Mercury, as you can see, *is* nearly ideal for fluids-pr-compare-barometers except for the minor problem with its extreme toxicity and high vapor pressure.

Fortunately, there are many other ways of making good fluids-pr-compare-barometers.

Problem 392. problems-1/fluids-pr-crane-2.tex

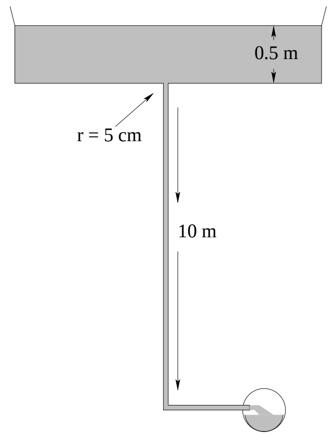


A barge with a crane mounted on it has a cross sectional area A, a total mass M, and straight sides. It is very slowly winching up a one of Blackbeard's treasure chests (of total mass m) from the ocean floor near Beaufort.

a) As the chest comes out of the water, does the boat sink or rise? Justify your answer with an equation or two and/ or a before and after figure.

b) Just before the crane turns to put the chest on the deck, Blackbeard's Ghost appears and cuts the cable of the crane so that the chest plunges back into the briny deep. Find an expression for the distance d the boat rises up in the water (after it stops bobbing) when this happens. Use the symbol ρ_s for the density of sea water.

Problem 393. problems-1/fluids-pr-dangerous-drain.tex



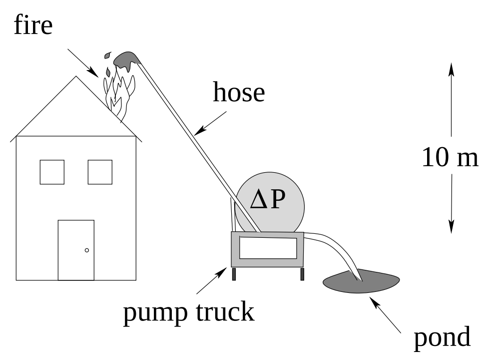
It is dangerous to build a drain for a pool or tub consisting of a single narrow pipe that drops down a long ways before encountering air at atmospheric pressure. This was demonstrated tragically in 1993 in an accident that occurred (no fooling!) within two miles from where you are sitting. A baby pool was built with just such a drain and one day a little girl sat down on the drain and was severely injured. In 2008 another young girl in Minneapolis was killed!

In this problem you will analyze why.

Suppose the mouth of a drain is a circle five centimeters in radius, and the pool has been draining long enough that its drain pipe is filled with water (and no bubbles) to a depth of ten meters below the top of the drain, where it exits in a sewer line open to atmospheric pressure. The pool is 50 cm deep. If a thin steel plate is dropped to suddenly cover the drain with a watertight seal, what is the force one would have to exert to remove it straight up?

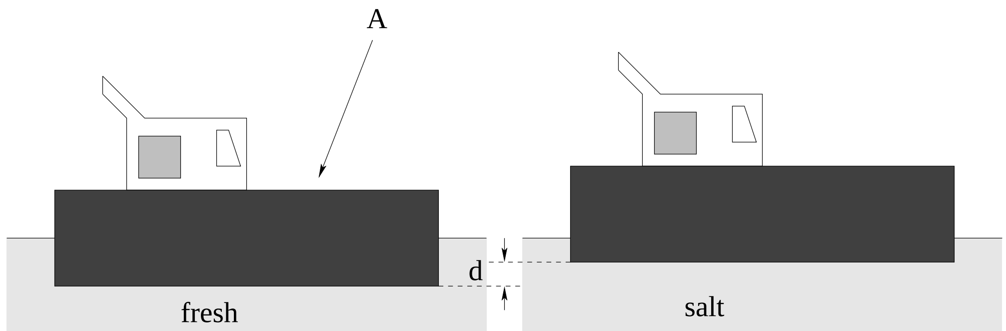
Note carefully this force relative to the likely strength of mere flesh and bone (or even thin steel plates!) Ignorance of physics can be actively dangerous.

Problem 394. problems-1/fluids-pr-firefighters-pump.tex



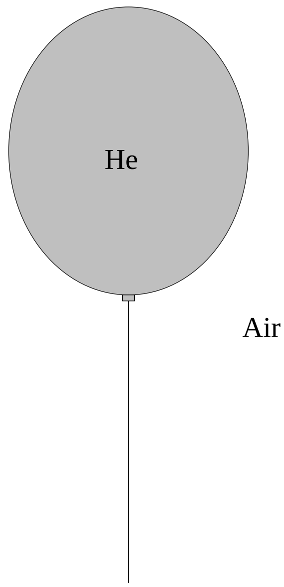
Firefighters arrive at a fire in the country and have to use water from the farm pond to try to battle the blaze. Their pump firetruck takes in water from the pond at one atmosphere (P_0) and increases the pressure at the **bottom of the hose** to an adjustable pressure $P_0 + \Delta P$ that can be set at any value of ΔP from 0 to 2 atmospheres of pressure. What is the **minimum** value ΔP_{min} one can set the pump to that will lift the water as high as the second floor (ten meters up above the ground, two meters above the fire)? ***Show all work and justify your answer with a physical principle or two!***

Problem 395. problems-1/fluids-pr-floating-freighter.tex



A rectangular ocean barge with horizontal area A (viewed from the top) floats in fresh water (ρ_w). It floats downriver and enters the ocean ($\rho_s = 1.1\rho_w$). As it does so, the ship bobs up an additional distance d from its earlier (freshwater) waterline. Find the total mass of the ship in terms of A, ρ_w, ρ_s and d. Hint – since you don't know either the height of the ship or its displacement in fresh water as given, concentrate on the *difference* in the forces (and the displacement) as it sails from fresh to salt.

Problem 396. problems-1/fluids-pr-helium-balloon.tex

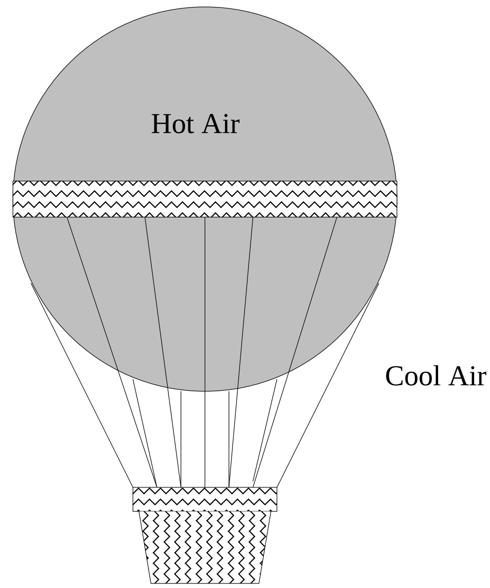


In the figure above, a helium balloon ($\rho_{\mathrm{He}} = 0.18$ kg/m^3) is suspended in air ($\rho_a = 1.28$ kg/m^3) by a string.

a) Assuming that the volume of the helium balloon is approximately 4000 cubic centimeters (4×10^{-3} m^3), find the total 'lift' of the balloon (the tension in the string). Neglect the mass of the balloon itself and the string.

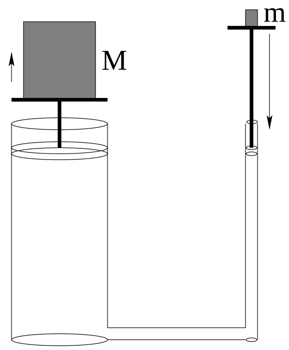
b) In the movies, humans are shown grabbing a few dozen helium balloons and being pulled up into the sky. Assuming that a reasonable human payload (including the mass of all of the balloon rubber and strings) is 100 kg, approximately how many balloons would *really* be required to lift a person?

Problem 397. problems-1/fluids-pr-hot-air-balloon.tex



A hot air balloon is drawn in the figure above. Estimate its total 'lift', assuming that the density of cool air is approximately constant at $\rho_a = 1.28$ kg/m^3, the density of hot air in the baloon is $\rho_h = 0.64$ kg/m^3, and that the balloon proper has a (filled) volume of 1000 m^3 (corresponding to a spherical balloon roughly 13 meters in diameter). If the balloon, basket, and rigging have a mass of 340 kg, what is the maximum payload it can carry?

Problem 398. problems-1/fluids-pr-hydraulic-lift.tex

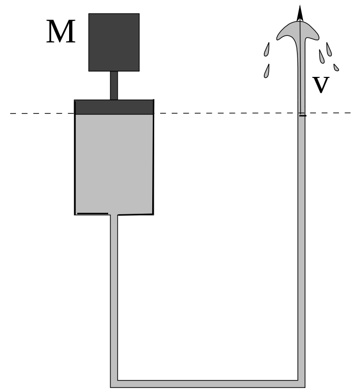


The figure above illustrates the principle of hydraulic lift. A pair of coupled cylinders are filled with an incompressible, very light fluid (assume that the mass of the fluid is zero compared to everything else).

a) If the mass M on the left is 1000 kilograms, the cross-sectional area of the left piston is 100 cm^2, and the cross sectional area of the right piston is 1 cm^2, what mass m should one place on the right for the two objects to be in balance?

b) Suppose one pushes the right piston down a distance of one meter. How much does the mass M rise?

Problem 399. problems-1/fluids-pr-piston-pump-1.tex

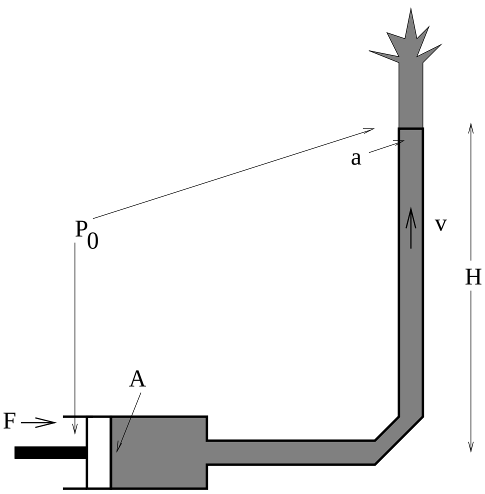


A piston and weight has a total mass M and is pressing on water confined in a cylinder of cross sectional area A. The water is then pushed into a pipe with a cross sectional area of a that is open to the air at the same height as the piston. Neglect viscosity.

a) What does M have to be to make the water spurt from the pipe with a speed v? You should use the approximation $a \ll A$ to develop your algebraic answer.

b) Find the numerical value for M that will produce a speed $v = 5$ m/sec for the following data: $A = 100$ cm^2, $a = 1$ cm^2. The density of water is $\rho_w = 10^3$ kg/meter^3.

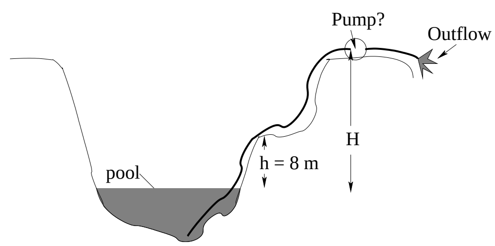
Problem 400. problems-1/fluids-pr-piston-pump-2.tex



A piston is pressed with a force $\vec{\mathbf{F}}$ on a hydraulic cylinder containing water ($\rho = 10^3$ kg/m^3). The cross sectional area of the cylinder is $A = 400$ cm^2. The water therein is forced into a pipe with a cross sectional area of $a = 2$ cm^2 that rises vertically a height $H = 40$ meters. Both the end of the pipe (at the top) and the back of the piston (at the bottom) are open to atmospheric pressure.

What does F have to be to make the water spurt from the pipe with a speed of 10 meters/sec at the top? Solve this problem beginning from (stated) physical principles, showing all work.

Problem 401. problems-1/fluids-pr-pump-water-up-cliff.tex



This problem will help you learn required concepts such as:

- Static Pressure
- Barometers

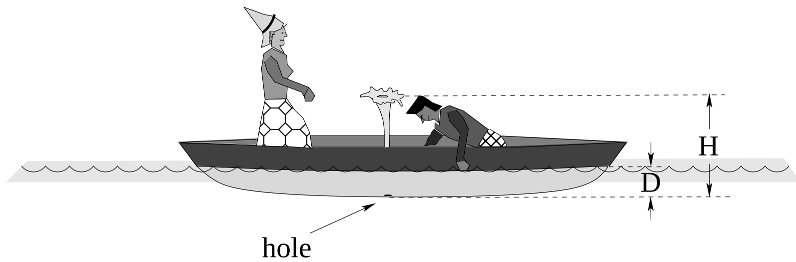
so please review them before you begin.

A pump is a machine that can maintain a pressure differential between its two sides. A particular pump that can maintain a pressure differential of as much as 10 atmospheres of pressure between the low pressure side and the high pressure side is being used on a construction site.

a) Your construction boss has just called you into her office to either explain why they aren't getting any water out of the pump on top of the $H = 25$ meter high cliff shown above. Examine the schematic above and show (algebraically) why it cannot possibly deliver water that high. Your explanation should include an invocation of the appropriate physical law(s) and an explicit calculation of the highest distance the a pump *could* lift water in this arrangement. Why is the notion that the pump "sucks water up" misleading? What really moves the water up?

b) If you answered a), you get to keep your job. If you answer b), you might even get a raise (or at least, get full credit on this problem)! Tell your boss where this single pump should be located to move water up to the top and show (draw a picture of) how it should be hooked up.

Problem 402. problems-1/fluids-pr-romeo-and-juliet.tex



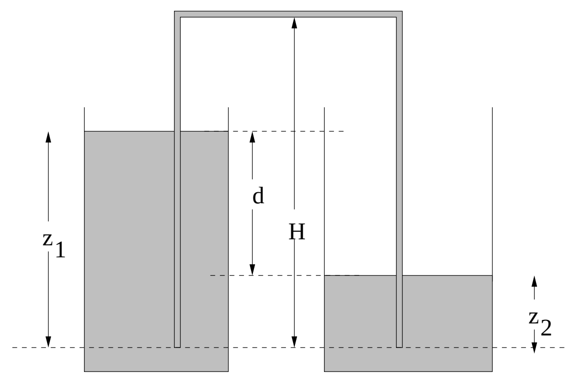
Romeo and Juliet are out in their boat again when Juliet's Salvatore Ferragamo heels poke a circular hole of radius r in the bottom of the boat. The boat has a ***draft*** of D (this is the distance the boat's bottom lies underwater as shown).

a) Romeo tries to cover the hole with his hand. What is the minimum force he must apply to keep it covered?

b) Juliet convinces Romeo that a little water fountain would be romantic, so he moves his hand. How fast does the water move through the hole?

c) To what height H does Juliet's fountain spout up from the bottom of the boat? (The height drawn is to illustrate the quantity H only and may not be at all correct.)

Problem 403. problems-1/fluids-pr-siphon-two-tank.tex



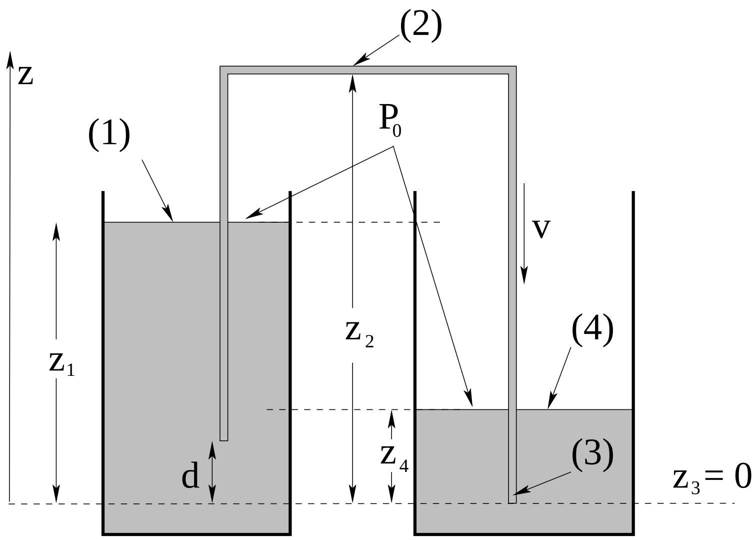
In the figure about two tanks are partly filled with fluid to the heights indicated. A siphon tube of a certain height H at its heighest point is started between them. The fluid is assumed to have a density ρ (which could well be "water", for example) and viscosity and fluid resistance are to be ignored. You may also assume that the surface area at the top of either tank is large compared to the cross-sectional area of the tube.

a) Find the velocity v of the fluid in the siphon tube at the instant shown.

b) Find the pressure P_H at the heighest point of the siphon tube.

c) Find the maximum height obstacle $H_{\max}$ that a siphon tube can go over (relative to the geometry shown) and still function.

Problem 404. problems-1/fluids-pr-siphon-two-tank-soln.tex



In the figure about two tanks are partly filled with fluid to the heights indicated. A siphon tube of a certain height H at its heighest point is started between them. The fluid is assumed to have a density ρ (which could well be "water", for example) and viscosity and fluid resistance are to be ignored. You may also assume that the surface area at the top of either tank is large compared to the cross-sectional area of the tube.

a) Find the velocity v of the fluid in the siphon tube at the instant shown.

b) Find the pressure P_H at the heighest point of the siphon tube.

c) Find the maximum height obstacle $H_{\rm max}$ that a siphon tube can go over (relative to the geometry shown) and still function.

Solution

It is useful to consider points 1 through 4 in the figure above. If we write Bernoulli's formula for points 1, 2 and 3, all three formulas must be equal for the fluid moving through the "pipe" represented by the two tanks and intermediary tube. Note that $P_1 = P_4 = P_0$, atmospheric pressure at the top of both tanks. Thus

$$\begin{aligned} P_0 + \rho g z_1 + \frac{1}{2}\rho v_1^2 \quad (1) &= P_0 + \rho g z_2 + \frac{1}{2}\rho v^2 \quad (2) \\ &= P_3 + \rho g(0) + \frac{1}{2}\rho v^2 \quad (3) \end{aligned}$$

where v is the desired velocity in the tube. We will assume that $v_1 \ll v$ and $v_2 \ll v$ and throw them both out relative to v.

In our previous tank problems like this, P_3 is the pressure in the fluid at the point where the system exits the fluid. The reason the fluid flows in the tube at all is that the pressure at this height is *different* on the dashed line in the two vessels. *Within* the two tanks (not in the tube) the fluid is nearly static, so the pressure $P_3 = P_0 + \rho g z_4$. This is the key to solving the problem, because if you naively write Bernoulli's formula for points 1 and 4 and equate them, you get a contradiction. For all values of z less than the top of the fluid, the pressure in the second tank is less than the pressure in the first at the same height.

We can now do some algebra between points 1 and 3:

$$\begin{aligned} P_0 + \rho g z_1 + \frac{1}{2}\rho v_1^2 \quad (1) &= P_0 + \rho g z_4 + \frac{1}{2}\rho v^2 \\ \frac{1}{2}\rho v^2 &= \rho g(z_1 - z_4) \\ v &= \sqrt{2g(z_1 - z_2)} \end{aligned}$$

We get an answer that looks "like" Torricelli's Law even though the tube exit per se is no longer the relevant height and even though the fluid at the tank *tops* is moving slowly compared to this in *both* tanks! The force that pushes the fluid from the first to the second tank is evidently the pressure in the *first* tank at the depth of the surface of the *second* tank, $\Delta z = z_1 - z_4$.

Now let's equate the Bernoulli formulas for points 1 and 2 and solve for P_2:

$$\begin{aligned} P_0 + \rho g z_1 + \frac{1}{2}\rho v_1^2 &= P_2 + \rho g z_2 + \frac{1}{2}\rho v^2 \\ P_2 &= P_0 + \rho g(z_1 - z_2) - \frac{1}{2}\rho v^2 \\ &= P_0 + \rho g(z_1 - z_2) - \rho g(z_1 - z_4) \\ &= P_0 - \rho g(z_2 - z_4) = P_0 - \rho g h_{\max} \end{aligned}$$

where we have substituted our answer for v in. This answer leads us to some "issues". If we start at the outflow pressure (inside the tube) and go uphill, the pressure must decrease. The pressure at point 2 must be lower than P_0 by $\rho g h_{\max}$, the decrease in static pressure with height, because the speed of the fluid in the tube does not change.

If, however, we keep lowering the second tank (increasing Δz between the surfaces of the two tanks) and thereby making v larger, we also make P_2 smaller until eventually it becomes zero! If we lower it any further, the pressure in the tube cannot be negative, as ***before*** it ever reaches 0 (a vacuum) nearly any real fluid will "come apart" at the molecular level in a process called cavitation. But what if we consider just the first line of $1 = 2$:

$$P_0 + \rho g z_1 = P_2 + \rho g z_2 + \frac{1}{2}\rho v^2$$

and suppose we set $P_2 = 0$, the point where cavitation occurs. This equation then becomes:

$$P_0 + \rho g z_1 = \rho g z_2 + \frac{1}{2}\rho v^2$$

and we can solve for v this way as well:

$$v = \sqrt{2\left(P_0 - \rho g(z_2 - z_1)\right)/\rho}$$

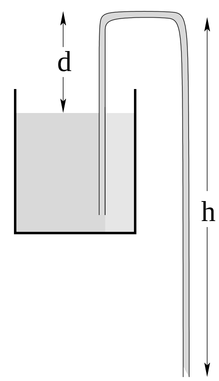
This equation *seems* to imply that as we increase z_2 holding z_1 constant, we decrease the maximum speed of flow in the tube until it is zero *and* the pressure is zero, but be careful. This is increasing z_2 ***holding*** $\boldsymbol{P_2 = 0}$, which is to say, maintaining $z_2 - z_4 = P_0/(\rho g)$!

We must therefore carefully think about the conditions for cavitation. Will the fluid cavitate at zero pressure *while in uniform motion in the tube* with $v \neq 0$? Or does the fluid *both* have to be *stationary* in the tube to cavitate? If we reach zero pressure at the top with $z_2 - z_1$ too small to make v zero, then if we increase $h_{\max}$ we lower the point on the right hand side of the tube where zero pressure exists and we have a serious problem. There is no longer any downward directed pressure gradient above that point because the pressure cannot go below zero. Gravity is pulling down fluid elements in the tube. If there were no pressure gradient to oppose gravity, those elements would *speed up*. But they cannot speed up and maintain a uniform flow (just as a uniformly falling stream of water splits up into droplets).

So when does the siphon "break" and e.g. water stop flowing? I think that the answer is best understood by considering that uniformly falling stream. If $z_2 - z_4 > 10$ meters, there is a stretch at the top where the velocity entering the vicinity of P_2 from a solution to Bernoulli's equation on the left hand side only ($1 = 2$ above) is ***smaller than*** v for the collective tube ($1 = 3$ above). There is a false continuity implied by $1 = 3$ through pressures that are implicitly less than zero, but this is impossible. The fluid in the tube cavitates *continually*, basically breaking up into drops that accelerate as they fall freely under gravity from right where the top of the tube bends down on the right until they match the flow velocity implied on the lower part of the tube continuously matched to the pressure at the bottom of the right hand side. Fluid flows, but it is no longer the case that it is flowing uniformly or that the tube itself remains continuously full.

As $z_2 - z_1$ is *separately* increased to 10 meters, v (now dominated by $z_2 - z_1$ at zero pressure at point 2 and P_0 at z_1) decreases to zero and the fluid on both sides of the tube stops flowing forming two "water barometers" on either side. Not so obvious!

Problem 405. problems-1/fluids-pr-siphon.tex

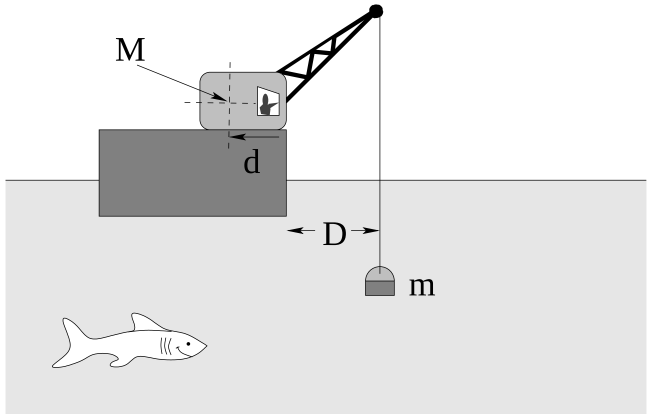


Water is being drained from a large container by means of a siphon as shown. The highest point in the siphon is distance d above the level of water in the container, and the total height of the long arm of the siphon is h. The distance h can be varied. The mass density of water is ρ_w, and air pressure is P_0. Express all answers in terms of d, h, ρ_w, P_0, and g.

a) What is the maximum possible value of h for which the siphon will work? (***Hint:*** The pressure cannot be negative anywhere in the siphon, in particular, in the long arm of the siphon.)

b) For that maximum value of h, what is the speed of the water coming out of the siphon?

Problem 406. problems-1/fluids-pr-static-crane.tex

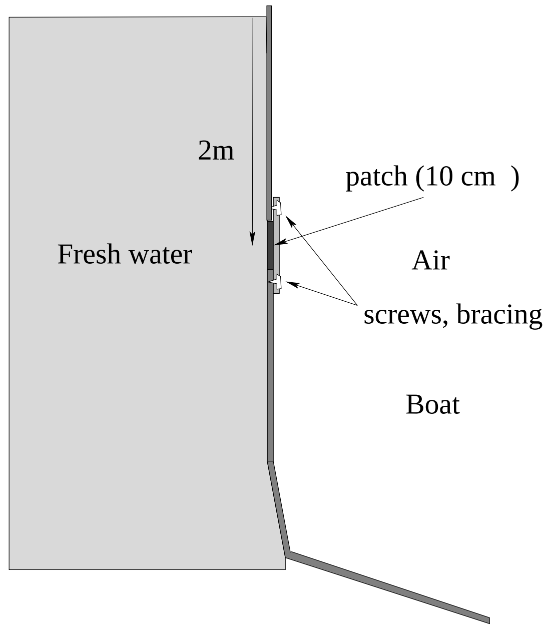


The crane above has a nearly massless boom. It is being used to salvage some of Blackbeard's treasure – a chest of mass m filled with very dense gold.

a) Find the maximum weight that the crane can lift, assuming that all of the weight of the crane itself acts downward at its center of mass to counterbalance it at the position shown, a horizontal distance d to the left of the bottom right corner of the crane. The crane's boom is fixed so that its moment arm (shown) is always D. Your answer should be expressed in M, g and the given lengths d and D.

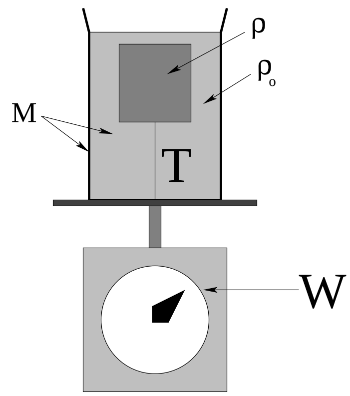
b) Suppose that Blackbeard's treasure is so massive that the crane is *almost* tipping over as it very slowly lifts it up **through the water**. What will happen when the crane tries to lift the mass out of the water, and **why**? "Why" should involve certain forces and a good before and after picture.

Problem 407. problems-1/fluids-pr-static-hole-in-a-boat.tex



Your yacht has a hole in it! Oh, no! The hole is 2 meters below the waterline, and has a cross-sectional area of 10 cm^2 (that's ten square centimeters, not ten centimeter's squared!). You patch it, and need to brace the patch with screws that can each hold at most a force of 5 Newtons. How many screws (at *least*) should you use to be sure of being able to withstand the force of the ocean pressing in against your patch?

Problem 408. problems-1/fluids-pr-tension-suspends-immersed-mass.tex



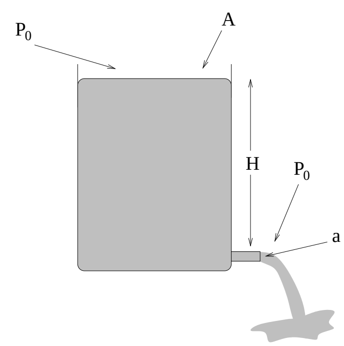
A floating block of density ρ and volume V is suspended, fully immersed, by a thin thread attached to the bottom in a jar of oil (density $\rho_o > \rho$) that is resting on a scale as shown. The total mass of the oil and jar (alone) is M.

a) What is the buoyant force exerted by the oil on the block?

b) What is the tension T in the thread?

c) What does the scale read?

Problem 409. problems-1/fluids-pr-time-to-empty-open-vat.tex



This problem will help you learn required concepts such as:

- Bernoulli's Equation
- Torricelli's Law

so please review them before you begin.

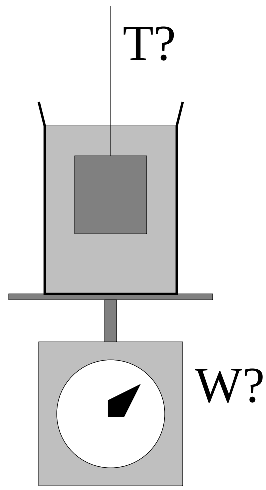
In the figure above, a large drum of water is open at the top and filled up to a height H above a tap at the bottom (which is also open to normal air pressure). The drum has a cross-sectional area A at the top and the tap has a cross sectional area of a at the bottom.

a) Find the speed with which the water emerges from the tap. Assume laminar flow without resistance. Compare your answer to the speed a mass has after falling a height H in a uniform gravitational field (after using $A \gg a$ to simplify your final answer, Torricelli's Law).

b) How long does it take for all of the water to flow out of the tap? (Hint: Start by *guessing* a reasonable answer using dimensional analysis and insight gained from a). That is, think about how you expect the time to vary with each quantity and form a simple expression with the relevant parameters that has the right units. Next, find an expression for the velocity of the top. Integrate to find the time it takes for the top to reach the bottom.) Compare your answer(s) to each other and the time it takes a mass to fall a height H in a uniform gravitational field. Does the correct answer make dimensional and physical sense?

c) Evaluate the answers to a) and b) for $A = 0.50$ m^2, $a = 0.5$ cm^2, $H = 100$ cm.

Problem 410. problems-1/fluids-pr-weight-of-immersed-mass.tex



A block of density ρ and volume V is suspended by a thin thread and is immersed completely in a jar of oil (density $\rho_o < \rho$) that is resting on a scale as shown. The total mass of the oil and jar (alone) is M.

a) What is the buoyant force exerted by the oil on the block?

b) What is the tension T in the thread?

c) What does the scale read?

Chapter 12

Oscillations

And now for one of the most important physical topics ever. We have seen that statics is pretty important. Atoms bond together to make molecules or solids that are in a sort of static equilibrium. Objects are glued, or stapled, or nailed together into bigger objects in static equilibrium. Anything that has ***persistent stable structure*** lives in, or near, a state of static equilibrium.

Did I just say near? I did. If you pull ***any system*** a little bit ***out*** of a stable equilibrium, it will usually be pushed back towards its equilibrium position because it is ***stable***. Furthermore, our friend the Taylor Series tells us that most often the restoring force (or torque) will be ***linear*** in the displacement for sufficiently small displacements from equilibrium. And what happens when you displace any mass from a stable equilibrium so that it experiences a linear restoring force?

It *oscillates*. ***Harmonically***.

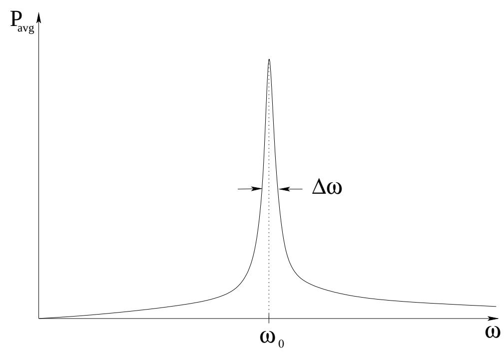
That doesn't mean it plays the harmonica – it means that the oscillation can be described by the harmonic functions", sine, cosine, and the (complex) exponential.

Oooo, I just used a bad word – ***complex*** exponential. Sorry, but in order to master a variety of concepts associated with oscillation and (very soon) waves, it ***really helps*** if you know something about complex numbers. And, of course, harmonic functions. And some trig identities that you ***almost certainly have forgotten***. Time to go brush up on all of this stuff, or (if necessary) learn it for the first time.

12.1 Oscillations

12.1.1 Multiple Choice

Problem 411. problems-1/oscillation-mc-change-resonance-3.tex



In the figure above, the curve shows the (average) power $P_{\text{avg}}(\omega)$ delivered to a damped, driven oscillator with equation of motion:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

Recall that the "width" of the curve $\Delta\omega$ is the ***full width at half maximum power***. Suppose the damping constant b is ***doubled*** while k of the spring, m, and the driving force magnitude F_0 are ***kept unchanged***. What happens to the curve?

a) The curve becomes narrower (smaller $\Delta\omega$) at the same frequency;

b) The curve becomes narrower at a higher frequency;

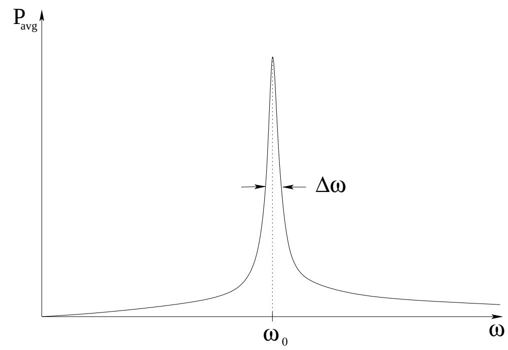
c) The curve becomes broader (larger $\Delta\omega$) at the same frequency

d) The curve becomes broader at a different frequency;

e) The curve does not change;

f) There is not enough information to determine the changes of the curve.

Problem 412. problems-1/oscillation-mc-increase-k-m-resonance-curve.tex



In the figure above, the curve shows the (average) power $P_{\text{avg}}(\omega)$ delivered to a damped, driven oscillator with equation of motion:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

Recall that the "width" of the curve $\Delta\omega$ is the ***full width at half maximum power***. If both k of the spring and m are ***doubled*** while the damping constant b and driving force magnitude F_0 are ***kept unchanged***, what happens to the curve?

a) The curve becomes narrower (smaller $\Delta\omega$) at the same frequency;

b) The curve becomes narrower at a higher frequency;

c) The curve becomes broader (larger $\Delta\omega$) at the same frequency

d) The curve becomes broader at a different frequency;

e) The curve does not change;

f) There is not enough information to determine the changes of the curve.

Problem 413. problems-1/oscillation-mc-stride-resonance.tex

You have to take a long hike on level ground, and are in a hurry to finish it. On the other hand, you don't want to waste energy and arrive more tired than you have to be.

Your *stride* is the length of your steps. Your *pace* is the frequency of your steps, basically the number of steps you take per minute. Your *average speed* is the product of your pace and your stride: the distance travelled per minute is the number of steps you take per minute times the distance you cover per step.

Your best strategy to cover the distance faster but with minimum additional energy consumed is to:

a) Increase your stride but keep your pace about the same.

b) Increase your pace, but keep your stride about the same.

c) Increase your pace and your stride.

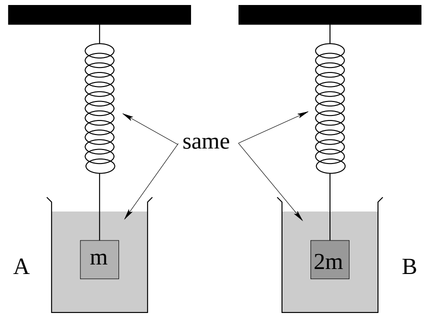
d) Increase your stride but decrease your pace.

e) Increase your pace but decrease your stride.

(in all cases so that your average speed increases).

Note well that this is a physics problem, so be sure to *justify your answer* with a physical argument. You might want to think about *why* one answer will probably accomplish your goal within the constraints and the others will not.

Problem 414. problems-1/oscillation-mc-two-damped-oscillators.tex



Two ***identical springs*** support two masses of the ***same size and shape*** in the ***same damping fluid***. However, $m_B = 2m_A$.

Both systems are pulled to an initial displacement from equilibrium of X_0 and released, and the exponential decay times τ_A and τ_B required for the initial amplitude of oscillation of each mass to decay to $X_0 e^{-1}$ is measured. We expect that:

a) $\tau_A = 2\tau_B$

b) $2\tau_A = \tau_B$

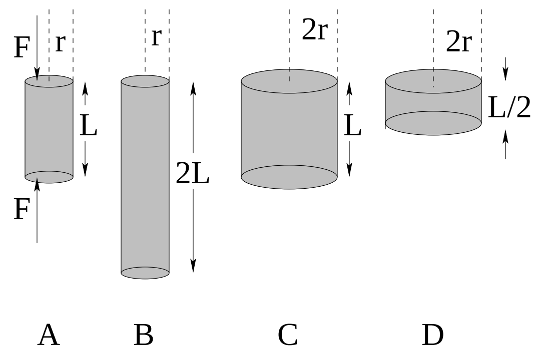
c) $\tau_A = \tau_B$

d) $4\tau_A = \tau_B$

e) We cannot predict the ***relative*** decay times without more information.

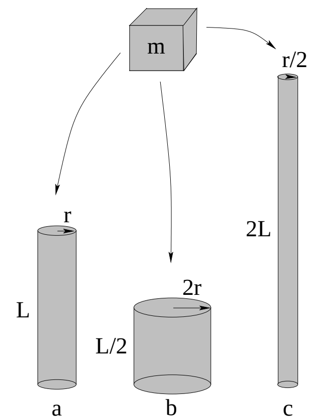
12.1.2 Ranking/Scaling

Problem 415. problems-1/oscillation-ra-compressed-rods-youngs-modulus-scaling.tex



Rank the magnitude of the compression ΔL of the rods (made of the same material) above when a force with magnitude F is exerted between the ends as shown in case A. Equality is a possibility. Your answer should look something like $C = D > A > B$.

Problem 416. problems-1/oscillation-ra-compression-three-rods-1.tex



In the figure above three rods made out of copper are shown with the dimensions given. In (a), a mass m is placed on top of the rod (which rests on a rigid table) and the rod is observed to be compressed and shrinks by a length ΔL. By what length ΔL_i do you expect rods (b) and (c) to be compressed by if the *same* mass m is placed on top of them? (Express your answer as a pure number times ΔL_a.)

$\Delta L_b = \boxed{}$ $\Delta L_c = \boxed{}$

Problem 417. problems-1/oscillation-ra-mass-spring-double-displacement.tex

Two identical masses are attached to two identical springs. The first mass is pulled to a distance x_0 from equilibrium. The second one is pulled to a distance $2x_0$ from equilibrium. At time $t = 0$ they are released. The first mass reaches its equilibrium point at time t_1, the second one at time t_2.

What is the ratio t_2/t_1?

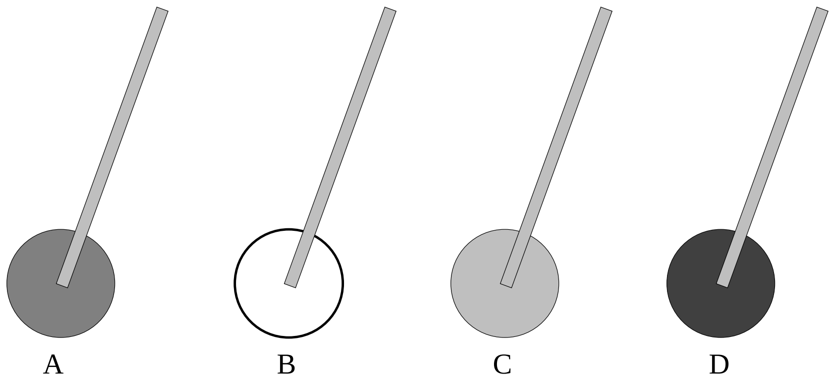
$$\frac{t_2}{t_1} = \boxed{}$$

Problem 418. problems-1/oscillation-ra-mass-swing-double-displacement.tex

Two kids are sitting on swings of equal length. One of them has about twice the mass of the other (but they are about the same height). The lighter one is pulled back to an initial (small) angle θ_0. The heavier one is pulled back to a (still small!) angle $2\theta_0$. At $t = 0$ they are both released. It takes the lighter one a time t_l to reach the lowest point of his trajectory, and the heavier one a time t_h.

What is the ratio t_h/t_l?

Problem 419. problems-1/oscillation-ra-physical-pendula-periods.tex



In the figures above, four physical pendulums are drawn. All consist of a light (massless) rod of length L to the center of mass of different shaped masses connected to the end. All of the shapes have the same mass M and the same primary length scale R. Rank the **periods** of the physical pendulums from lowest (highest frequency!) to the highest (lowest frequency!). Equality is a possibility.

The moments of inertia of the round objects (about their centers of mass) are:

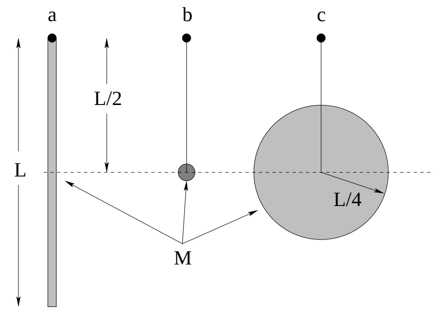
A) $I = \frac{1}{2}MR^2$ (disk)

B) $I = MR^2$ (hoop)

C) $I = \frac{2}{3}MR^2$ (hollow ball)

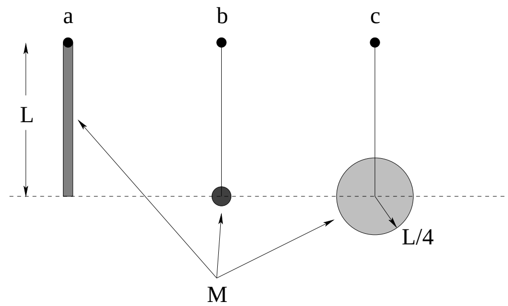
D) $I = \frac{2}{5}MR^2$ (solid ball)

Problem 420. problems-1/oscillation-ra-physical-pendulums-1.tex



In the figure above, three pendulums are suspended from frictionless pivots. The first is a rod of mass M and length L. The second is a "point" mass M with negligible radius. The third is a disk of mass M and radius $L/2$. In all three cases, the center of mass of the pendulum is a distance $L/2$ from the pivot and the mass is constrained to rotate around the pivot (physical pendulum). Rank the angular frequencies (where equality is allowed) so that an answer might be (but probably isn't) $\omega_a > \omega_b = \omega_c$.

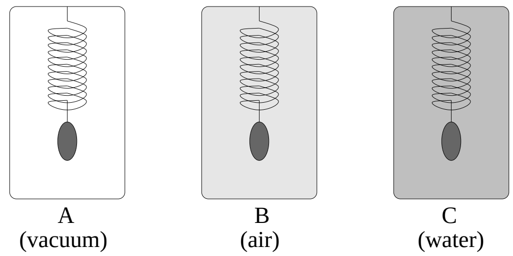
Problem 421. problems-1/oscillation-ra-physical-pendulums-2.tex



In the figure above, three pendulums are suspended from frictionless pivots. The first is a thick rod of mass M and length L. The second is a "point" mass M with negligible radius on a thin (massless) rod of length L. The third is a disk of mass M and radius $L/4$ on the end of a thin (massless) rod of so that its center of mass is a distance L away from the pivot. In all three cases, the mass is constrained to rotate around the pivot as a ***physical pendulum***.

Rank the ***angular frequencies in increasing order*** (where equality is allowed) so that an answer might be (but probably isn't) $\omega_a > \omega_b = \omega_c$.

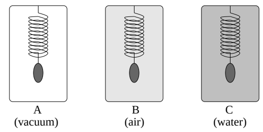
Problem 422. problems-1/oscillation-ra-rank-the-damped-frequency.tex



In the figure above identical masses are connected to identical springs and located in three different labelled containers. All three masses are pulled to the same distance from equilibrium and are released from rest. The container A contains a vacuum, container B is filled with ordinary room-temperature air at 1 atmosphere of pressure, and container C contains water.

Rank the ***frequencies*** of the oscillation of the three masses by their container letter, where (precise) equality is a possibility. That is, a possible answer might be $f_A = f_C < f_B$ (but probably isn't). (It is wise to explain your answer with a few words or an equation.)

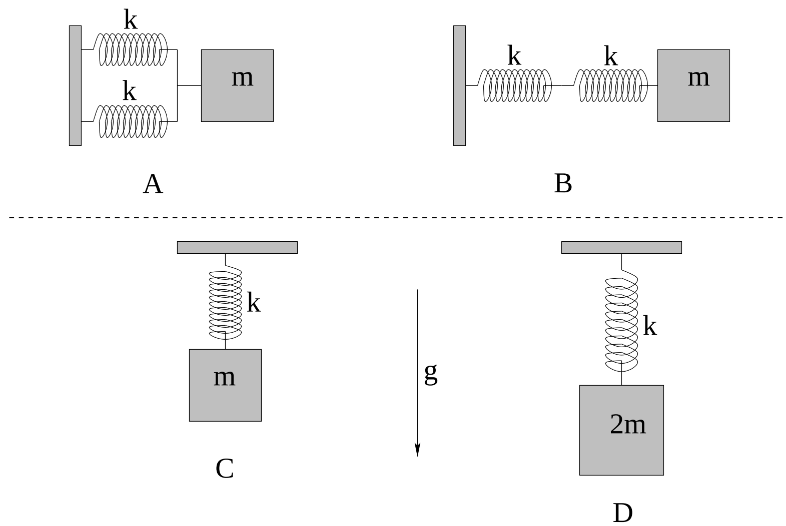
Problem 423. problems-1/oscillation-ra-rank-the-damped-periods.tex



In the figure above identical masses are connected to identical springs and located in three different labelled containers. All three masses are pulled to the same distance from equilibrium and are released from rest. The container A contains a vacuum, container B is filled with ordinary room-temperature air at 1 atmosphere of pressure, and container C contains water.

Rank the ***period*** of the oscillation of the three masses by their container number, where (precise) equality is a possibility. That is, a possible answer might be $T_a = T_c < T_b$ (but probably isn't). Explain your answer with a few words or an equation.

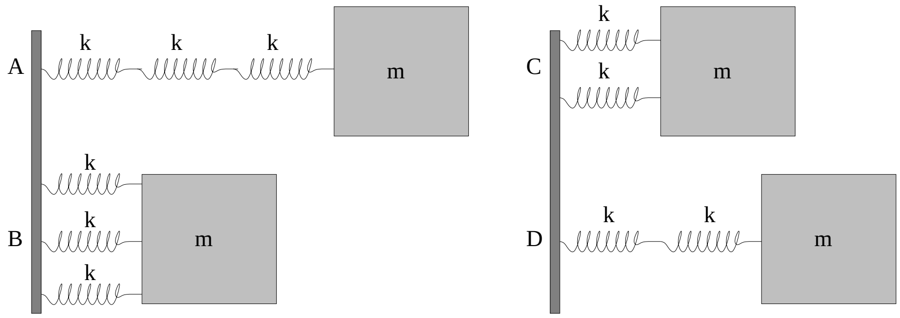
Problem 424. problems-1/oscillation-ra-rank-the-periods.tex



In the figure above, rank the ***periods*** of each ***pair*** of oscillators shown (where equality is allowed). That is, fill in the boxes in the two expressions below with a $<, >, =$ sign as appropriate.

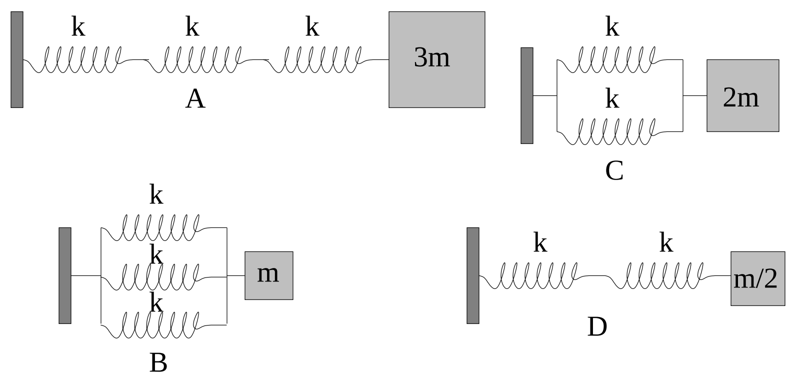
$T_A \boxed{} T_B$ $T_C \boxed{} T_D$

Problem 425. problems-1/oscillation-ra-series-parallel-frequency-easy.tex



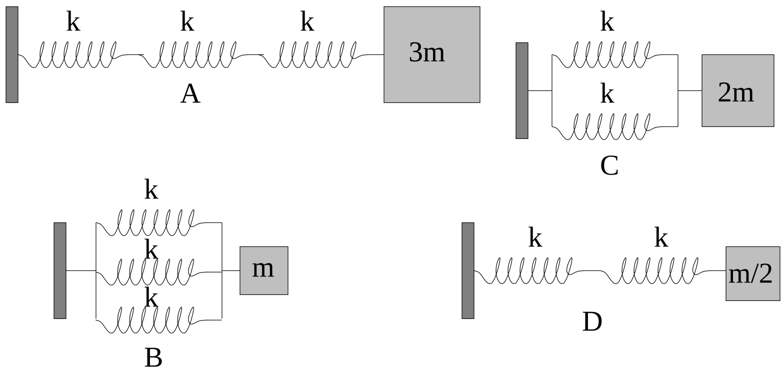
Rank the ***oscillation frequencies*** of the identical masses m connected to the springs in the figure above from ***lowest to highest*** with equality a possibility. The springs have spring constant k, and you should neglect damping. A possible answer is (as always) $D < A = B < C$ or the like.

Problem 426. problems-1/oscillation-ra-series-parallel-frequency.tex



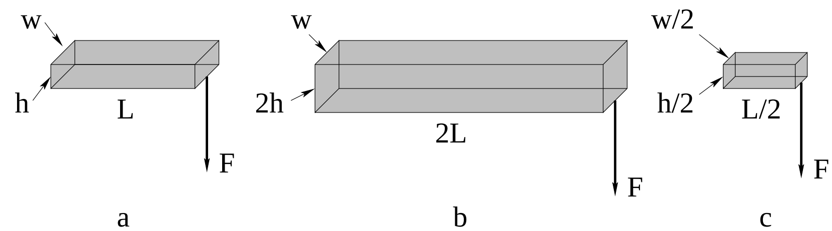
Rank the frequencies of the masses on the spring arrangements in the figure above, from **lowest to highest** with equality a possibility. Neglect damping. A possible answer is (as always) $D < A = B < C$ or the like.

Problem 427. problems-1/oscillation-ra-series-parallel-period.tex



Rank the ***period of oscillation*** of the masses on the spring arrangements in the figure above, from ***lowest to highest*** with equality a possibility. Neglect damping. A possible answer could be (as always) $D < A = B < C$ but probably isn't.

Problem 428. problems-1/oscillation-ra-shear-three-rods-1.tex



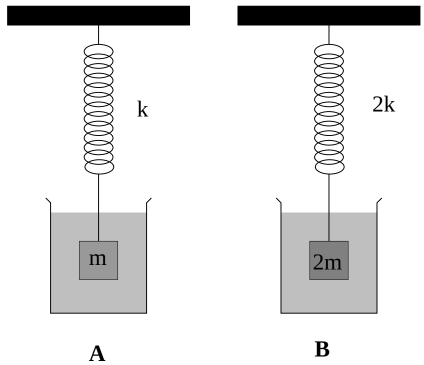
In the figure above, three light wooden boards and their relative dimensions are shown. The boards are each fixed in a vise (not shown) on the left hand side so that the left end of each board cannot move. A downward force $\vec{F}$ is applied at the right hand end of each board. The first board is bent by this force so that its right hand end is displaced downward by a distance Δh. By how much are the right hand ends of the other two boards displaced downward? (Express your answer in terms of Δh.)

b:

c:

12.1.3 Short Answer

Problem 429. problems-1/oscillation-sa-damped-oscillation.tex



Two springs with different spring constants (k and $2k$, respectively) support two blocks of the ***same size and shape***, but different masses, $m_B = 2m_A$. The blocks are fully submerged in the ***same damping fluid***, therefore, they have the same coefficient of damping.

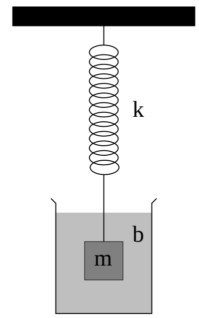
Both systems are pulled to an initial displacement from equilibrium and released to undergo damped oscillations with the blocks remaining fully submerged in the fluid at all times. The natural frequencies of two systems are ω_A and ω_B, respectively. The measured exponential decay times of the oscillation amplitude are τ_A and τ_B, respectively.

a) Write down the ratio for natural frequencies: $\dfrac{\omega_B}{\omega_A} = \boxed{}$

b) Write down the ratio for damping times: $\dfrac{\tau_B}{\tau_A} = \boxed{}$

c) Which oscillator will damp out its initial energy faster/sooner (A or B): $\boxed{}$.

Problem 430. problems-1/oscillation-sa-damping-variation.tex



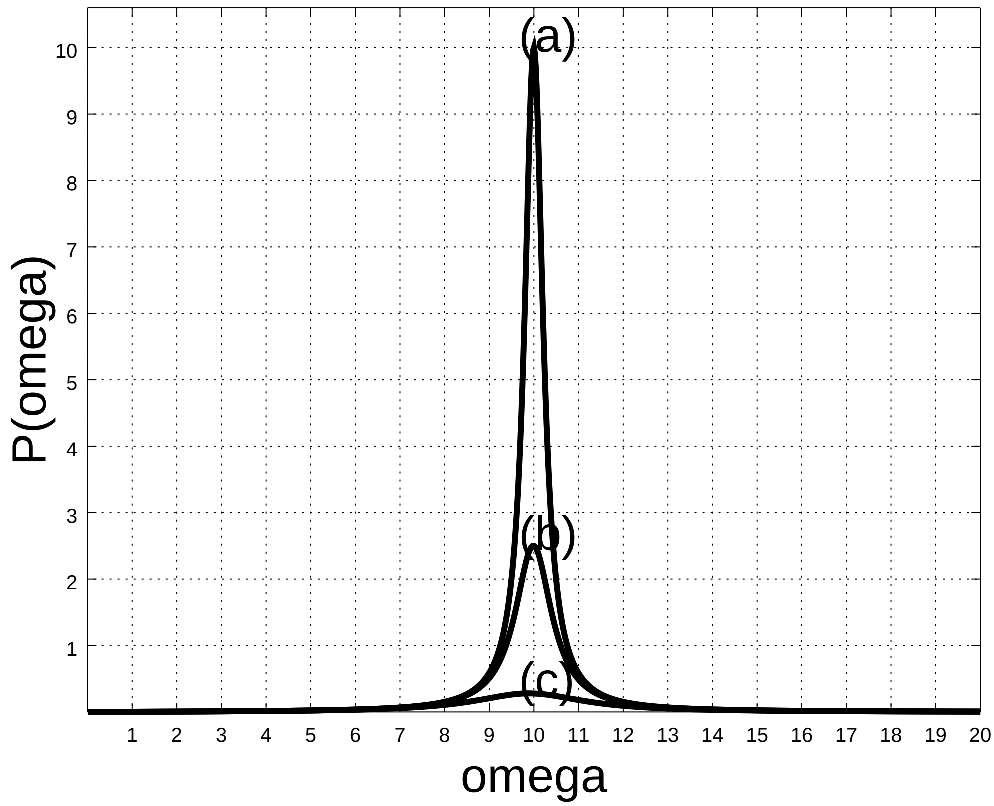
The damped oscillator above is set in motion at time $t = 0$. Fill in the following table with x's in the provided boxes. τ is the exponential damping time of the amplitude, and ω_0 is the natural frequency.:

If b increases: τ increases decreases remains unchanged. ω_0 increases decreases remains unchanged

If m increases: τ increases decreases remains unchanged. ω_0 increases decreases remains unchanged

If k increases: τ increases decreases remains unchanged. ω_0 increases decreases remains unchanged

Problem 431. problems-1/oscillation-sa-estimate-Q-resonance-curve.tex



In the figure above, three resonance curves showing the amplitude of steady-state driven oscillation $A(\omega)$ as functions of ω. In all three cases the resonance frequency ω_0 is the same. Put down an *estimate* of the Q-value of each oscillator by looking at the graph. It may help for you to put down the definition of Q most relevant to the process of estimation on the page.

a)

b)

c)

Problem 432. problems-1/oscillation-sa-match-the-waveform.tex

You are presented with three identical simple harmonic oscillators, **A,B,C**, which oscillate with a known harmonic frequency ω. They differ ***only in their initial conditions***. At time $t = 0$, the attached masses have an initial position and velocity (x, v) given by:

A $(x_A = x_0 v_A = 0)$

B $(x_B = 0, v_B = v_0)$

C $(x_C = x_0, v_C = x_0 * \omega)$

where x_0 and v_0 are ***positive numbers not equal to zero*** in the appropriate units.

Match each set of initial conditions to the corresponding solution from the list of possible solution ***forms*** (put A, B or C in three of the four boxes) below:

$x(t) = A\cos(\omega t)$

$x(t) = A\cos(\omega t - \pi/4)$

$x(t) = A\sin(\omega t)$

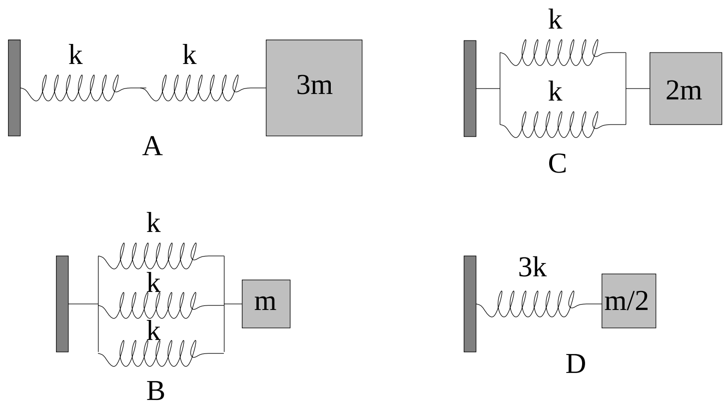
$x(t) = A\cos(\omega t + \pi/4)$

Consider one of oscillator solutions, $x(t) = Asin(\omega t)$. Find the velocity and acceleration at $t = T/4$, where T is the period of oscillations: that is, please evaluate $v_x(t = T/4)$ and $a_x(t = T/4)$.

Problem 433. problems-1/oscillation-sa-roman-soldiers-bridge-resonance.tex

Roman soldiers (like soldiers the world over even today) marched in step at a constant frequency – except when crossing wooden bridges, when they broke their march and walked over with random pacing. Why? What might have happened (and originally did sometimes happen) if they marched across with a collective periodic step?

Problem 434. problems-1/oscillation-sa-series-parallel-frequency.tex



Find the ratio of the angular frequencies of each spring-mass combination above to $\omega_0 = \sqrt{k/m}$.

$$\frac{\omega_A}{\omega_0} = \boxed{} \qquad \frac{\omega_B}{\omega_0} = \boxed{} \qquad \frac{\omega_C}{\omega_0} = \boxed{} \qquad \frac{\omega_D}{\omega_0} = \boxed{}$$

Problem 435. problems-1/oscillation-sa-sho-true-facts.tex

The one-dimensional motion of a mass m is described by $x(t) = A\sin(\omega t)$. Identify the true and false statements among the following by placing a T in the provided box for true statements and an F in the provided box for false statements:

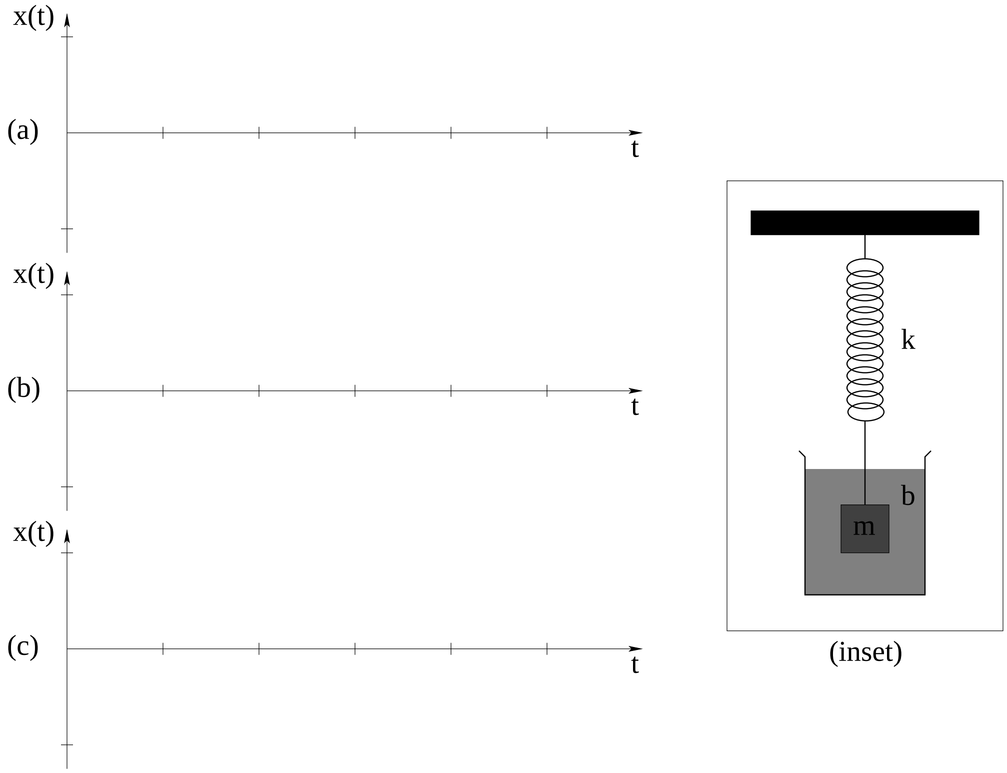
a) ☐ If A and ω are constant (i.e. – independent of time t) the motion is simple harmonic motion.

b) ☐ The mass m starts at $t = 0$ with zero velocity.

c) ☐ If the motion of mass m is simple harmonic oscillation, the potential energy of the mass can be written $U(x) = \frac{1}{2}m\omega^2 x^2$.

d) ☐ If the motion of the mass m is simple harmonic oscillation, the total force acting on the mass can be written $F_x = -m\omega^2 x$.

Problem 436. problems-1/oscillation-sa-sketch-damped-oscillation.tex



A mass m is attached to a spring with spring constant k and immersed in a damping fluid with linear damping coefficient b as shown in the inset figure above. Equilibrium is at $x = 0$ meters. At time $t = 0$ seconds the mass is pulled to $x(0) = 1$ meters and released from rest. The ***period*** of the oscillator in the *absence* of damping is $T = 2$ seconds. On the provided axes with integer tick-marks above, sketch the following:

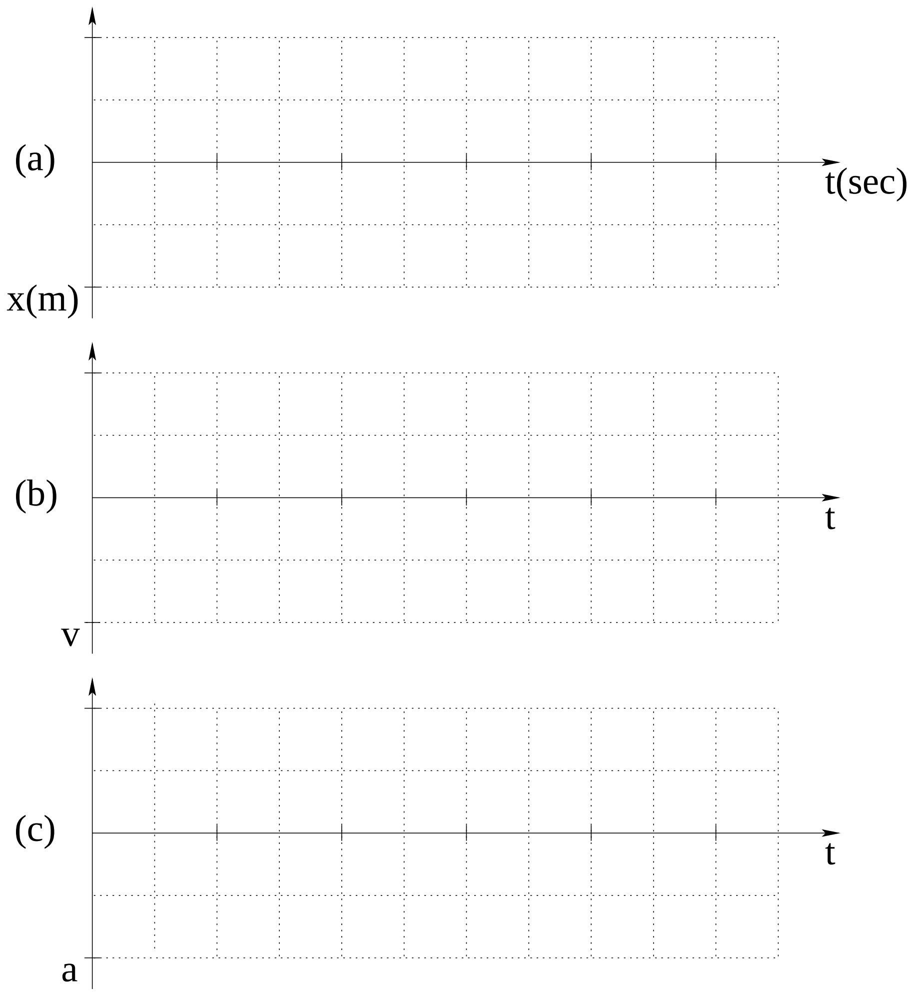
a) $x(t)$ in the absence of damping.

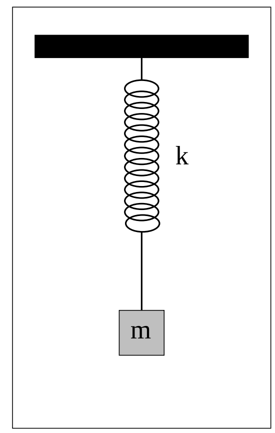
b) $x(t)$ if $b/2m = 1/3$ (underdamped, assume that $\omega' \approx \omega_0$).

c) $x(t)$ in the case where $b/2m = \pi$ (critically damped).

The second two curves only need to be *qualitatively* correct (you don't have to plot them exactly), but they should also not be crazily out of scale. You may use $e = 2.72 \approx 3$ to make drawing the curves easier without needing a calculator.

Problem 437. problems-1/oscillation-sa-sketch-oscillation-plus-damping.tex



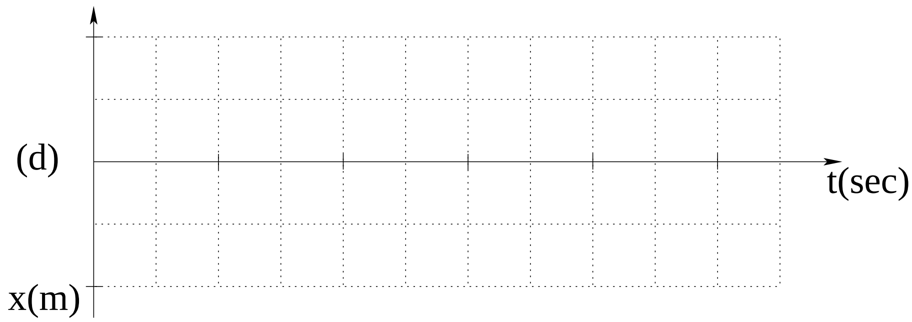


(inset)

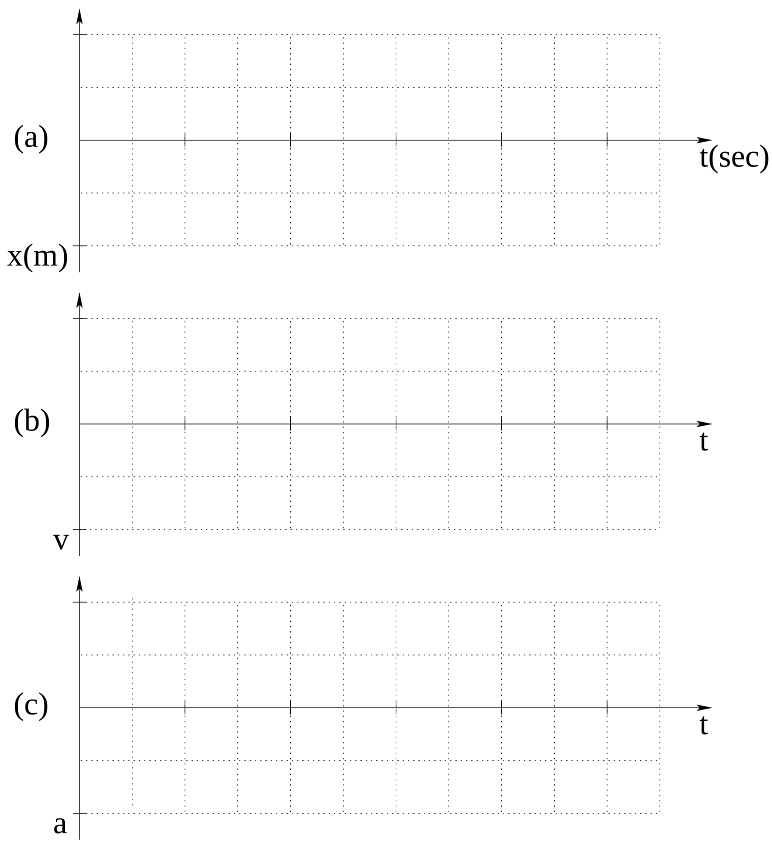
A mass m is attached to a spring with spring constant k as shown in the inset figure above.

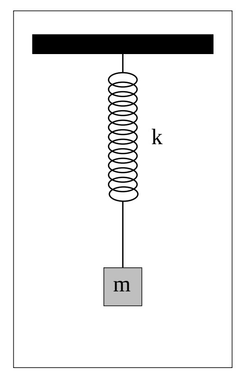
On the provided axes above, sketch $x(t)$, $v(t)$ and $a(t)$, given that at time $t = 0$ the mass is pulled to $x(0) = X_0 = 1$ meter (relative to equilibrium) and released from rest, assuming no damping. The ***period*** is $T = 1$ second, and you should use the tic-marks on the t axis as seconds. Your graphs should have the correct sign, phase, period, and you should label the peak positive value in terms of the givens on the ordinate axes.

Suppose that the block is then placed in a damping fluid. On the axes labelled d) below, sketch the position as a function of time for an oscillator with period $T = 1$ seconds and damping time $\tau = 4$ seconds, again assuming that $x(0) = X_0 = 1$ meter.



Problem 438. problems-1/oscillation-sa-sketch-oscillation.tex





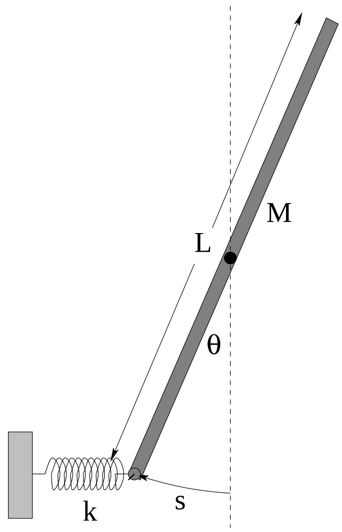
(inset)

A mass m is attached to a spring with spring constant k as shown in the inset figure above. There ***is no damping***.

On the provided axes above, sketch $x(t)$, $v(t)$ and $a(t)$, given that at time $t = 0$ the mass is pulled to $x(0) = X_0 = 1$ (relative to equilibrium) and released from rest. The ***period*** is $T = 2$ seconds, and you should use the tic-marks on the t axis as seconds. Your graphs should have the correct sign, phase, period, and you should label the peak positive value in terms of the givens on the ordinate axes.

12.1.4 Long Problems

Problem 439. problems-1/oscillation-pr-bar-and-spring-1.tex



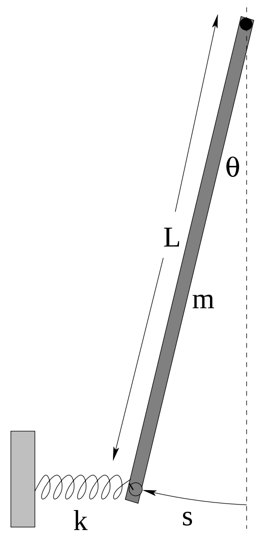
In the figure above a rigid rod of mass M and length L is pivoted in the center with a frictionless bearing. Its lower end is attached to a spring with spring constant k as shown that is unstretched (at equilibrium) when the rod is vertical and $\theta = 0$.

For *small displacements* $s \ll L$ (where one can use the small angle approximation), the spring will exert a restoring force $F_s = -ks \approx -k(L/2)\theta$ along the arc of motion of the end of the rod. It is pulled to an initial small displacement angle θ_0 and released at time $t = 0$.

a) What is the period of this oscillator for small oscillations?

b) What is the ***angular velocity*** Ω of the rod when it reaches its equilibrium position at $\theta = 0$? (Note well: Do not confuse ω_0, the angular frequency of oscillation, and $\Omega = \frac{d\theta}{dt}$, the angular velocity of the rod! Don't forget direction!)

Problem 440. problems-1/oscillation-pr-bar-and-spring-2.tex



In the figure above a rigid rod of mass m and length L is pivoted at the end with a frictionless bearing. Its lower end is attached to a spring with spring constant k as shown that is unstretched (at equilibrium) when the rod is vertical and $\theta = 0$.

For *small displacements* $s \ll L$ (where one can use the small angle approximation), the spring will exert a restoring force $F_s = -ks$ along the arc of motion of the end of the rod. It is pulled to an initial small displacement angle θ_0 and released at time $t = 0$, at which point it will begin to oscillate with angular frequency ω_0.

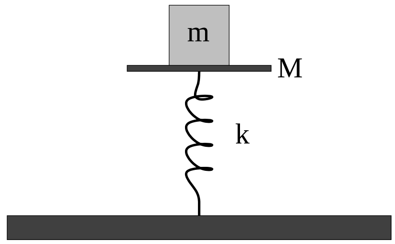
a) **Neglecting damping**, find the period T_0 of this oscillator for small oscillations and **sketch a qualitatively correct graph** of $\theta(t)$ for the rod. (Note well: *both* the spring *and* gravity contribute to the motion of the rod!)

b) What is the angular velocity of the rod $\omega = \frac{d\theta}{dt}$ when it reaches its equilibrium position at $\theta = 0$? Do not confuse the angular velocity of the rod with its angular frequency.

c) Suppose one compares the predicted motion $\theta(t)$ to the motion one would actually observe in the real world, where the system surely would be at least weakly damped. **Sketch a graph** that is **qualitatively** correct illustrating what $\theta(t)$ might **really** look like when weak damping is taken into account.

(Hint: The moment of inertia of a rod pivoted about one end is $\frac{1}{3}ML^2$.)

Problem 441. problems-1/oscillation-pr-block-on-vertically-oscillating-plate.tex



A block of mass m is sitting on a plate of mass M. It is supported by a vertical ideal massless spring with spring constant k. Gravity points down.

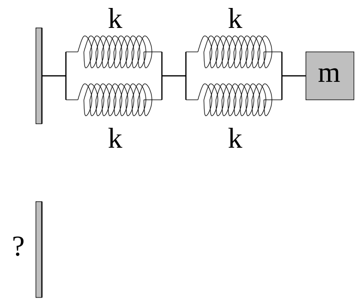
a) When the system is at rest, how much is the spring compressed from its completely uncompressed length?

b) The spring is pushed down an ***extra*** distance A and released. Assuming that the mass m ***remains on the plate***, what is its frequency of vertical oscillation?

c) What is the maximum value of A such that the small mass m will not leave the plate at any point in the motion?

Express all answers in terms of m, M, k, g.

Problem 442. problems-1/oscillation-pr-box-of-springs.tex



You are given a mass m, a box full of identical springs each with spring constant k, and a bunch of stiff wire you can bend and use to fasten the springs together to the wall and the mass in any combination of series and parallel you like.

I've drawn one such arrangement for you, one that will cause the mass m to oscillate harmonically on a smooth surface at angular frequency ω. Your job is to design an arrangement of springs that will make the mass oscillate at an angular frequency of $\sqrt{\frac{3k}{2m}}$, using only the (uncut) springs in the box.

a) Find the angular frequency of the four-spring oscillator I've drawn.

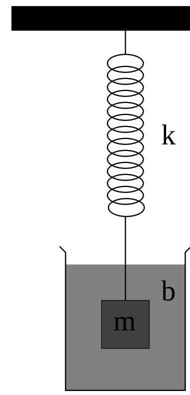
b) Draw a new arrangement on the bar underneath (or elsewhere on your paper) that will have an angular frequency of $\sqrt{\frac{3k}{2m}}$. Note well that there is more than one way to get the right answer, but some ways need (a lot) more springs than others. Try to get an answer with no more than six springs

c) Prove/show that your answer is correct.

Problem 443. problems-1/oscillation-pr-car-on-springs-resonance.tex

A car with a mass of $M = 1000$ kg rests on shock absorber springs with a collective spring constant of $k = 10^5$ N/m. It is driving down a road which has raised expansion joints every 5 meters that bounce the car. At what speed would you expect the ride to be roughest?

Problem 444. problems-1/oscillation-pr-damped-oscillation.tex



A mass m is attached to a spring with spring constant k and immersed in a medium with damping coefficient b. (Gravity, if present at all, is irrelevant as shown in class). The net force on the mass when displaced by x from equilibrium and moving with velocity v_x is thus:

$$F_x = ma_x = -kx - bv_x$$

(in one dimension).

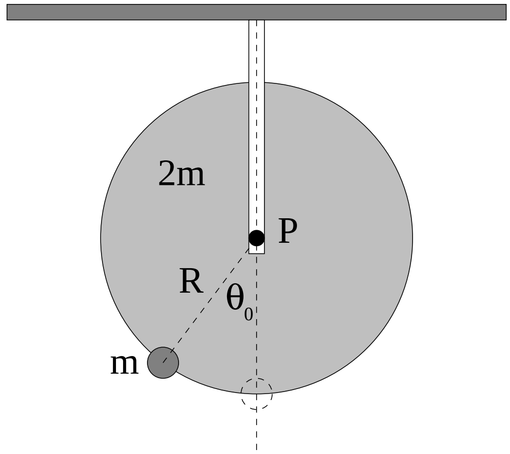
a) Convert this equation (Newton's second law for the mass/spring/damping fluid arrangement) into the equation of motion for the system, a "second order linear homogeneous differential equation" as done in class.

b) Optionally *solve* this equation, finding in particular the exponential damping rate of the solution (the real part of the exponential time constant) and the shifted frequency ω', assuming that the motion is underdamped. You can put down any form you like for the answer; the easiest is probably a sum of exponential forms. However, you may also simply **put down the solution** derived in class if you plan to *just* memorize this solution instead of learn to derive and understand it.

c) Using your answer for ω' from part b), write down the criteria for damped, underdamped, and critically damped oscillation.

d) Draw *three* qualitatively correct graphs of $x(t)$ if the oscillator is pulled to a position x_0 and released at rest at time $t = 0$, one for each damping. Note that you should be able to do this part even if you cannot derive the curves that you draw or ω'. Clearly label each curve.

Problem 445. problems-1/oscillation-pr-disk-with-rim-weight.tex



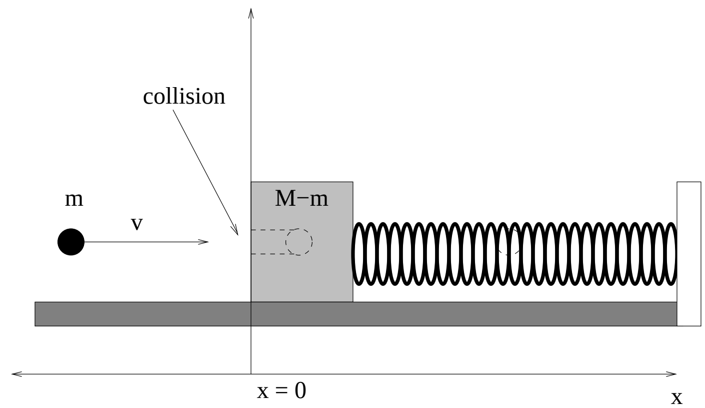
A uniform disk of radius R and mass $2m$ can freely rotate about a fixed frictionless horizontal axis passing through its fixed center P as shown. It has a point mass m fixed on its rim, so that in equilibrium, the disk is oriented such that $\theta = 0$. At time $t = 0$, the disk is gently rotated by the ***small, postive*** angle θ_0 as shown and released ***from rest***.

a) Just after it is released, what is the net torque ***vector*** $\vec{\tau}$ about P acting on the disk (as a function of θ and the givens)?

b) After the disk is released, it oscillates. What is the angular ***frequency*** ω of the oscillation?

c) Find $\theta(t)$, i.e., the angular position of the point mass as a function of time.

Problem 446. problems-1/oscillation-pr-inelastic-collision-mass-on-spring.tex



A bullet of mass m, travelling at speed v, hits a block of mass $M - m$ with a pre-drilled hole resting connected at the equilibrium position to a connected spring with constant k and ***sticks in the hole***. The block is sitting on a frictionless table (i.e. – ignore damping). Assume that the collision occurs at $t = 0$. All answers below should be given in terms of m, M, k, v.

a) What is the maximum displacement X_0 of the block?

b) What is the angular frequency ω of oscillation of the combined bullet-block system?

c) Write down $x(t)$, the position of the block as a function of time.

Problem 447. problems-1/oscillation-pr-inelastic-collision-mass-on-spring-soln.tex

a) What is the maximum displacement X_0 of the block?

First, momentum conservation in the inelastic collision:

$$p = p_i = mv = Mv_f = p_f$$

Second, energy conservation ***after*** the collision:

$$E_i = K_f = \frac{p^2}{2M} = \frac{1}{2}kX_0^2 = U_f = E_f$$

b) What is the angular frequency ω of oscillation of the combined bullet-block system?

It's just the usual angular frequency for the combined bullet+block mass M:

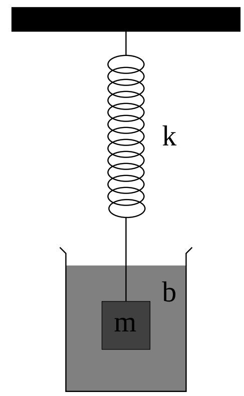
$$\omega = \sqrt{\frac{k}{M}}$$

c) Write down $x(t)$, the position of the block as a function of time

This is a simple harmonic oscillator. It starts ***at the origin*** at time $t = 0$, so:

$$x(t) = X_0 \sin(\omega t)$$

Problem 448. problems-1/oscillation-pr-mass-on-spring-damped.tex



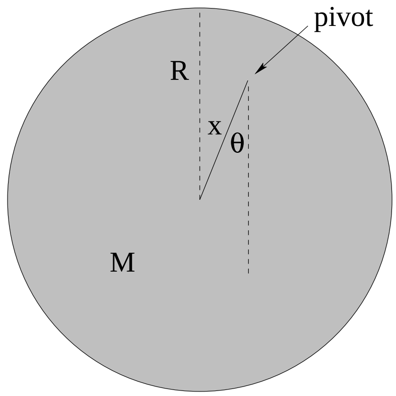
A mass m is attached to a spring with spring constant k and immersed in a medium with damping coefficient b. The net force on the mass when displaced by x from its equilibrium position is thus:

$$F_x = ma_x = -kx - bv_x$$

Convert this equation (Newton's second law for the mass/spring/damping fluid arrangement) into a second order linear homogeneous differential equation and solve it, finding the damping rate and the shifted frequency ω'. You may leave the final answer in exponential form or convert it to cosine as you wish.

Also Draw a qualitatively correct graph of $x(t)$ if the oscillator is pulled to a position x_0 and released at rest at time $t = 0$. Note that you should be able to do this part even if you cannot derive the curves that you draw or ω'.

Problem 449. problems-1/oscillation-pr-minimize-period-of-disk.tex



A uniform disk of mass M and radius R has a hole drilled in it a distance $0 \leq x < R$ from its center. It is then hung on a (frictionless) pivot, pulled to the side through a *small* angle θ_0, and released from rest to oscillate harmonically.

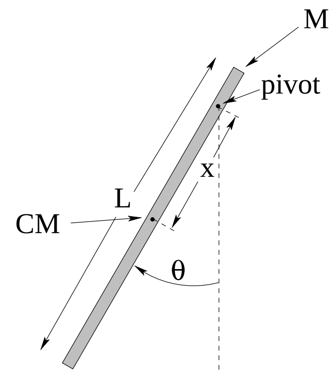
a) What is the moment of inertia of the disk about this pivot?

b) Write $\tau = I\alpha$ for this disk, make the small angle approximation, and turn it into the differential *equation of motion.*

c) Write an expression for T, the period of oscillation of the disk, as a function of d.

d) 5 point extra credit bonus question! What value of d ***minimizes*** this period? That is, if we wanted to make a disk oscillate with the shortest possible period, how far from the end would we drill a pivot hole?

Problem 450. problems-1/oscillation-pr-minimize-period-of-rod.tex



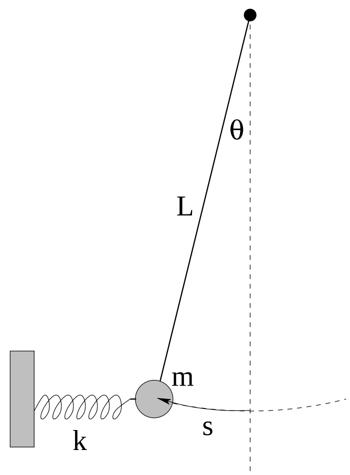
A rod of mass M and length L is pivoted a distance x from the center as shown above. Gravity acts on the rod, pulling it down (as usual) at its center of mass.

a) What is the moment of inertia of the rod about this pivot?

b) Write $\tau = I\alpha$ for this rod, make the small angle approximation, and turn it into the differential *equation of motion*. Use this to write an expression for T, the period of oscillation of the rod, as a function of x.

c) What value of x ***minimizes*** this period? That is, if we wanted to make a rod oscillate with the shortest possible period, how far from the end would we drill a pivot hole?

Problem 451. problems-1/oscillation-pr-pendulum-with-spring.tex

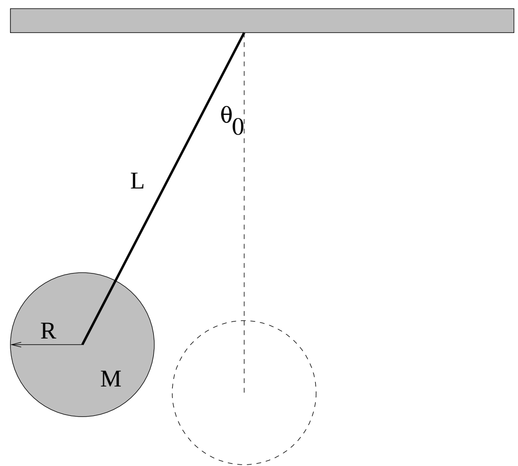


In the figure above a mass m on the end of a massless string of length L forms a pendulum. A light (massless) spring of spring constant k is attached to the mass so that for *small oscillations* $s \ll L$ (where one can use the small angle approximation), $F_s = -ks$ where s is the distance along the arc of motion from the equilibrium position in the center. When released, both gravity and the spring contribute to its motion, with the force exerted by the spring remaining **approximately tangent to the trajectory throughout**.

a) Find the period of this oscillator for small oscillations.

b) If it is started at an angle θ_0 and released, how fast is the mass m moving as it crosses equilibrium at $\theta = 0$?

Problem 452. problems-1/oscillation-pr-physical-pendulum-ball-on-stick.tex



A physical pendulum is constructed from a thin rod of negligible mass inserted into a uniform ball of mass M and radius R. The rod has length L from the pivot point to the center of the ball. At time $t = 0$ the ball is released from rest when the rod is at an initial **small** angle θ_0 with respect to its vertical equilibrium position.

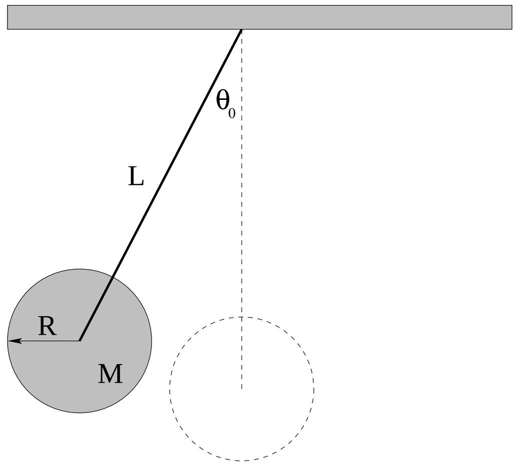
Answer all the questions below in terms of M, R, L, g, θ_0. You may make the small angle approximation where appropriate.

a) Determine the equation of motion for the system, solving for $\alpha = \frac{d^2\theta}{dt^2}$.

b) Determine the angular frequency of oscillation ω and write down $\theta(t)$ for the ball.

c) Find the maximum speed v of the ball. Is this larger or smaller than it would have been if the ball had been a point mass M at the end of the rod? Why?

Problem 453. problems-1/oscillation-pr-physical-pendulum-disk-on-stick-1.tex



A physical pendulum is constructed from a thin rod of negligible mass rigidly inserted into a uniform disk of mass M and radius R. The rod has length L from the pivot point at the top of the rod to the center of the disk. At time $t = 0$ the disk is released from rest when the rod is at an initial ***small*** angle θ_0 with respect to its vertical equilibrium position.

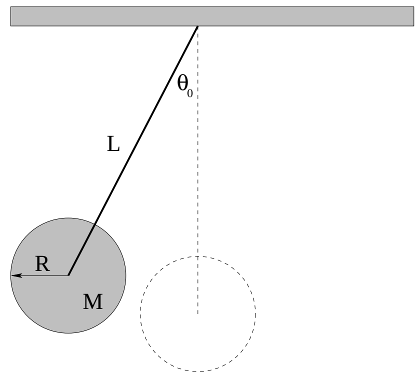
Answer all the questions below in terms of M, R, L, g, θ_0. You may make the **small angle approximation** where appropriate.

a) Find the ***vector torque*** $\vec{\tau}$ about the pivot point *at the instant the ball is released*, assuming $\theta_0 > 0$ (positive) as drawn.

b) Determine the ***period*** T of the resulting oscillation.

c) Find the ***maximum speed*** v of the center of mass of the disk as it oscillates.

Problem 454. problems-1/oscillation-pr-physical-pendulum-disk-on-stick.tex



A physical pendulum is constructed from a thin rod of negligible mass inserted into a uniform disk of mass M and radius R. The rod has length L from the pivot point to the center of the disk. At time $t = 0$ the disk is released from rest when the rod is at an initial ***small*** angle θ_0 with respect to its vertical equilibrium position. You may make the small angle approximation where appropriate.

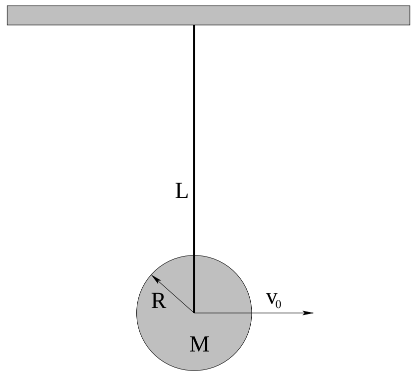
a) Determine the ***equation of motion*** for the system, solving for $\alpha = \frac{d^2\theta}{dt^2}$.

b) Determine the angular frequency of oscillation and write down the harmonic motion solution $\theta(t)$ for the disk.

c) Find the maximum speed v of the disk.

d) Is this larger or smaller than it would have been if the disk had been a point mass M at the end of the rod? Why?

Problem 455. problems-1/oscillation-pr-physical-pendulum-disk-on-stick-v0-only.tex



A physical pendulum is constructed from a thin rod of negligible mass inserted into a uniform disk of mass M and radius R. The rod has length L from the pivot point to the center of the disk. At time $t = 0$ the disk is sitting in its ***equilibrium position*** $\boldsymbol{\theta = 0}$ and is given as sharp blow so that it has an ***initial speed of*** $\boldsymbol{v_0}$ ***to the right***. The resulting oscillation is "small": you may make the ***small angle approximation*** where appropriate.

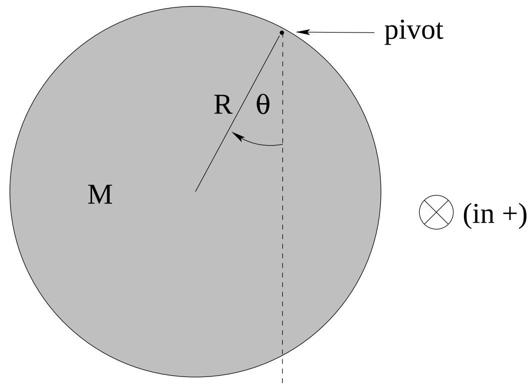
a) Draw the situation at a time that the pendulum has swung through an arbitrary angle θ. Determine the ***equation of motion*** for the system, solving for $\alpha = \frac{d^2\theta}{dt^2}$.

b) Determine the angular frequency of oscillation and write down the ***harmonic motion solution*** $\theta(t)$ for the disk. (Hint: What is the maximum angular velocity of the pendulum?)

c) Find the maximum angle θ_{max} that the disk reaches.

d) Is angle θ_{max} larger or smaller than it would have been if the ball had been a point mass M at the end of the rod started with the same initial velocity? Why?

Problem 456. problems-1/oscillation-pr-physical-pendulum-disk.tex



A disk of mass M and radius R is pivoted at the rim and hung from a wall as shown above. Gravity acts on the disk, pulling its center of mass down (as usual).

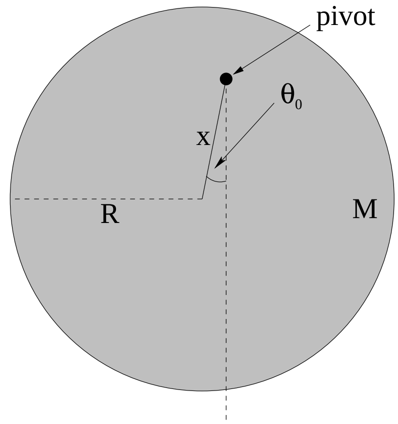
a) What is the moment of inertia of the disk ***about this pivot***?

b) Find the *differential equation of motion* for this system.

c) Write an expression for T, the period of oscillation of the disk.

d) Write down $\theta(t)$ for the disk, assuming that it starts at time $t = 0$ with angular position $\theta(t = 0) = 0$ and angular velocity $\dfrac{d\theta}{dt} = \omega(t = 0) = \omega_0$.

Problem 457. problems-1/oscillation-pr-physical-pendulum-disk-with-hole.tex



A uniform disk of mass M and radius R has a hole drilled in it a distance $0 \leq x < R$ from its center. It is then hung on a (frictionless) pivot, pulled to the side through a *small* angle θ_0, and released from rest to oscillate harmonically.

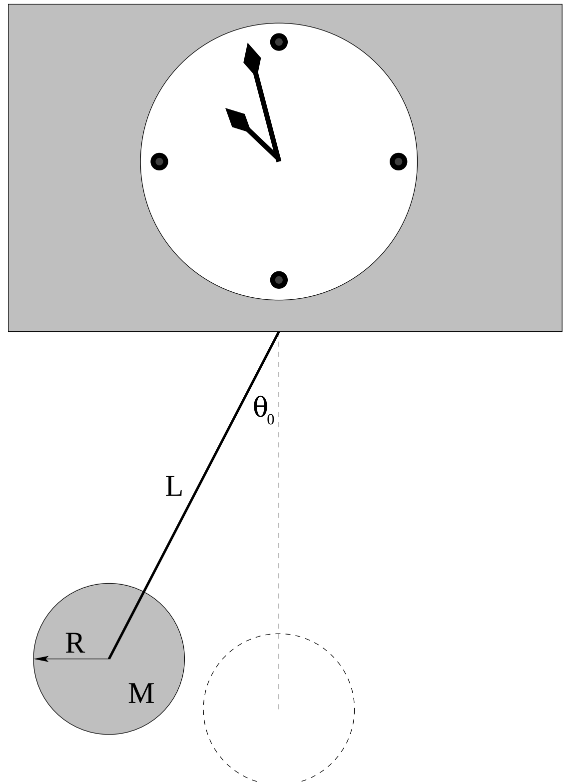
a) What is the moment of inertia of the disk around this pivot?

b) Write down the differential equation of motion for this physical pendulum. Circle ω^2.

c) Find the period of the physical pendulum as a function of (possibly) x, M, R, and g.

d) Write down the solution to the equation of motion, $\theta(t)$.

Problem 458. problems-1/oscillation-pr-physical-pendulum-grandfather-clock.tex



A Grandfather clock's pendulum is constructed from a thin rod of negligible mass inserted into a uniform **disk** of mass $M = 1.0$ kg and radius $R = 5.0$ cm. The rod has a length L from the pivot point to the center of the disk that can be adjusted from 0.20 m to 0.30 m in length so that the clock keeps the correct time. When the clock runs, its pendulum oscillates through a maximum angle of $\theta_0 = 0.05$ radians, which is a "small angle". Use $g = 10$ m/sec^2 and neglect drag.

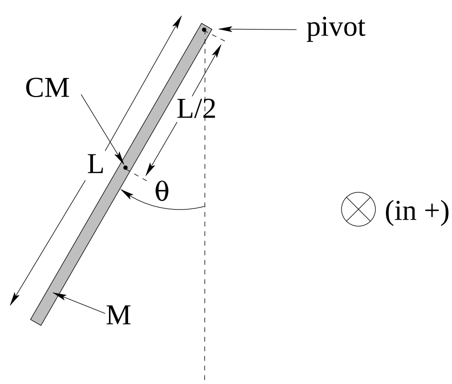
a) **Algebraically** determine the (differential) equation of motion for the system, making the small angle approximation to put it in the form of a simple harmonic oscillator equation.

b) Write down the algebraic function that describes $\theta(t)$, the angle that the pendulum makes as a function of time, assuming it starts from rest at $\theta(0) = \theta_0$ at $t = 0$.

c) The clock keeps correct time when the period of its pendulum is $T = 1$ second. What should L be (to **2** significant digits) so that this is true. (Use the algebraic form for ω^2 from your answer to part a to solve for L.)

d) Suppose one replaces the disk at the end with an identical mass concentrated in a very small (point-like) sphere. Will the clock run fast or slow?

Problem 459. problems-1/oscillation-pr-physical-pendulum-rod.tex



A rod of mass M and length L is pivoted at one end, a distance $L/2$ from the center as shown above. Gravity acts on the rod, pulling it down (as usual) at its center of mass.

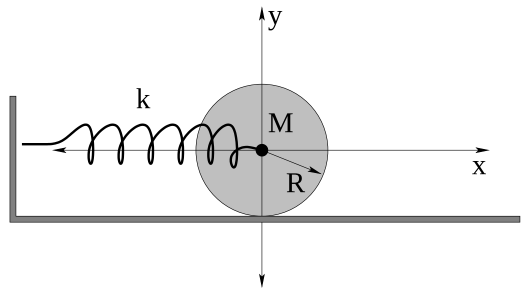
a) What is the moment of inertia of the rod about this pivot?

b) Find the *differential equation of motion* for this system.

c) Write an expression for T, the period of oscillation of the rod.

d) Write down $\theta(t)$ for the rod, assuming that it starts at time $t = 0$ with angular position $\theta(t = 0) = 0$ and angular velocity $\dfrac{d\theta}{dt} = \omega(t = 0) = \omega_0$.

Problem 460. problems-1/oscillation-pr-rolling-wheel-1.tex



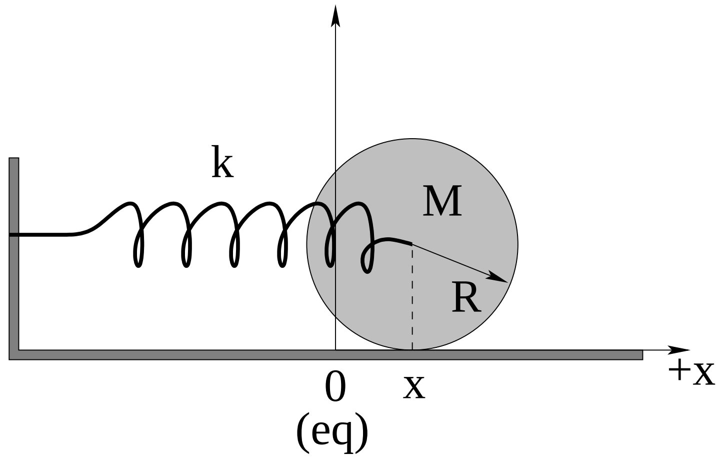
A spring with spring constant k is attached to a wall and to the axle of a wheel of radius R, mass M, and moment of inertia $I = \beta MR^2$ that is sitting on a rough floor. The wheel is stretched a distance A from its equilibrium position and is released at rest at time $t = 0$. The rough floor provides enough static friction that the wheel rolls without slipping.

a) When the displacement of the wheel from its equilibrium position is x and the speed of center of mass of the wheel is v, what is its total mechanical energy?

b) What is the maximum velocity v_{max} of the wheel?

c) What is the angular frequency of **oscillation**, ω, for the axle of the wheel as it rolls back and forth? (Note that this is *not* the angular velocity of the rolling wheel!)

Problem 461. problems-1/oscillation-pr-rolling-wheel-and-spring.tex



A spring with spring constant k is attached to a wall and to the axle of a wheel of radius R, mass M, and moment of inertia $I = \beta M R^2$ that is sitting on a rough floor. The wheel is stretched a distance A from its equilibrium position and is released at rest at time $t = 0$. The rough floor provides enough static friction that, for this value of A, the wheel ***rolls without slipping***.

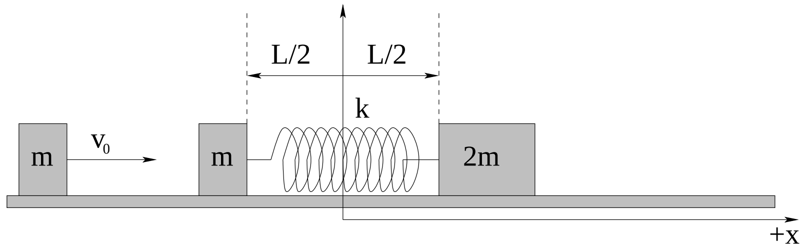
a) Assuming that the figure above illustrates the position x of the wheel right after being released (before it reaches the equilibrium position of the spring the first time) ***draw and label*** arrows on the diagram representing the force of static friction $\vec{f_s}$, the force exerted by the spring, and the direction of its center of mass motion and its rotation.

b) Show that the motion of the wheel is simple harmonic, and find its period.

c) What is the total energy of the wheel? From this or from the solution to the equation of motion, what is the maximum speed $v_{\max}$ of the wheel and where does it occur?

d) **Challenge!** Find the largest amplitude $A_{\max}$ that the wheel can have before it starts to slip.

Problem 462. problems-1/oscillation-pr-three-block-inelastic-collision.tex



Two blocks of mass m and $2m$ are resting on a frictionless table, connected by an ideal (massless) spring with spring constant k at its equilibrium length L. A third block of mass m is moving to the right with speed v_0 as shown. It collides with and sticks to the block of mass m connected to the spring (forming a new "block" of mass $2m$ on the left hand end of the spring).

We wish to find the position of both the left and the right hand blocks as functions of time. This is a challenging problem and will require several steps of work. **Hints:** Think about ***what is conserved*** both during the collision and during the subsequent motion of the blocks. Try to visualize this motion. Finally, the motion of the blocks is simplest in the ***center of mass frame***.

The following questions will guide you through the work:

a) Let the origin of the laboratory frame be the location of the center of mass of the system at the instant of collision. Write an expression for $x_{\rm cm}(t)$, the position of the center of mass as a function of time.

b) What is the total kinetic energy of the system ***immediately after*** the collision?

c) What is the kinetic energy of the system at the instant (some time later) that the blocks are travelling with the same speed? (This is the kinetic energy of the center of mass motion alone.)

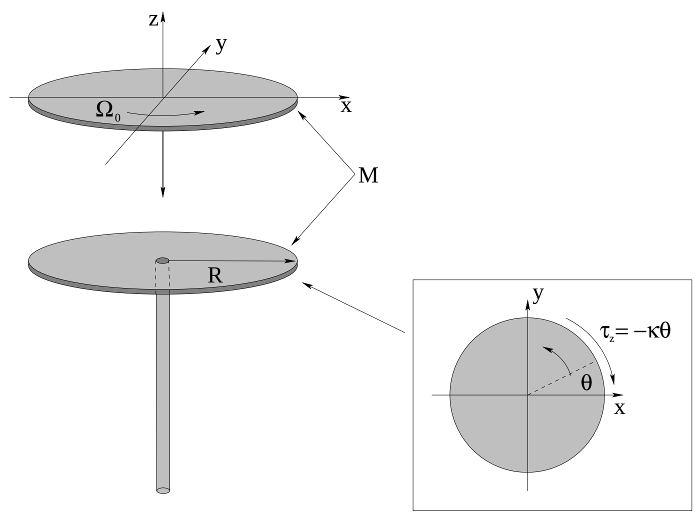
d) At this instant, the total compression of the spring is maximum with some magnitude $x_{\rm max}$. Find $x_{\rm max}$.

e) Write expressions for $x'_l(t)$ and $x'_r(t)$, the position of the left hand and right hand blocks *relative* to the center of mass of the system.

f) Add these functions to $x_{\rm cm}(t)$ to find $x_l(t)$ and $x_r(t)$, the position of the two blocks as a function of time.

g) Differentiate these solutions to find $v_l(t)$ and $v_r(t)$, and verify that your answer obeys the initial condition $v_l(0) = v_0/2$, $v_r(0) = 0$. Your overall solution should describe an "inchworm" crawl of the spring as first one mass momentarily moves at speed $v_0/2$ with the other momentarily at rest, then vice versa.

Problem 463. problems-1/oscillation-pr-torsional-oscillator-collision.tex



The ***torsional oscillator*** above consists of a disk of mass M and radius R connected to a stiff supporting rod. The rod acts like a ***torsional spring***, exerting a restoring torque:

$$\tau_z = -\kappa\theta$$

if it is twisted through an angle θ counterclockwise around the z axis (see inset above). κ is the positive torsional spring constant. This torque will make any object with a moment of inertia that is symmetrically attached to the rod ***rotationally oscillate*** around the z axis of the rod as shown.

A second identical disk also of mass M and radius R, rotating around their mutual axis at an angular speed Ω_0, is dropped gently onto the ***stationary*** first disk from above and sticks to it (so that they rotate together after the collision). At the instant of this angular collision, the disks have zero angular displacement (i.e. are at the equilibrium angle, $\theta_0 = 0$)[1].

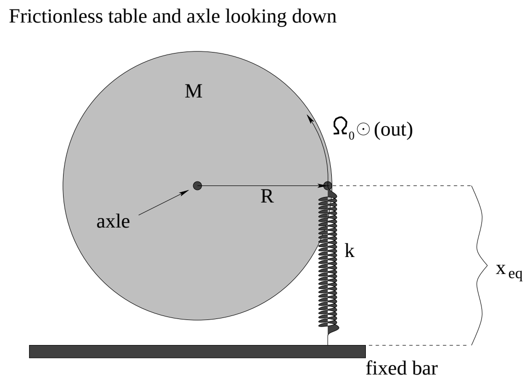
a) Find the ***final*** angular speed Ω_f of the two disks moving together immediately *after* the collision (and before the disks have time to rotate).

b) Find the energy that was ***lost*** in this (inelastic) rotational collision.

c) From the torque equation given above, find the differential ***equation of motion*** for $\theta(t)$ for the two disks moving together *after* the collision. Identify ω^2 (the angular frequency of the ***oscillator*** after the collision) in this equation, and write down the solution $\theta(t)$ in terms of Ω_f, κ, M and R. You do not have to substitute your answer to a) for Ω_f.

[1] Note that I'm using a capital omega $\Omega = d\theta/dt$ to help you keep track of the angular speed Ω of the *disks* and angular frequency ω of the *oscillator* separately below. If you cannot remember the moment of inertia of a disk in terms of M and R, use the symbol I_d for the moment of inertia of a single disk where appropriate in your answers (and lose 2 points).

Problem 464. problems-1/oscillation-pr-torsional-oscillator-spring-hard.tex



In the figure above, a disk of radius R and mass M is mounted on a nearly frictionless axle. A massless spring with spring constant k is attached to a point on its circumference so that it is in equilibrium as shown. The disk is then lightly struck at time $t = 0$ so that it is given a small instantaneous counterclockwise angular velocity of ω_0 while it is still at the equilibrium position, and it subsequently oscillates approximately harmonically through a small maximum angle θ_0. Note: $I_{\rm disk} = \frac{1}{2}MR^2$ about its center of mass.

a) Find the angular *frequency* ω_f of its oscillation, assuming that the axle is frictionless and exerts no torque on the disk. (**Note well** that this is not the same thing as the initial angular velocity of the disk!)

b) Find the angle θ_0 through which it will rotate before (first) coming momentarily to rest in this frictionless case.

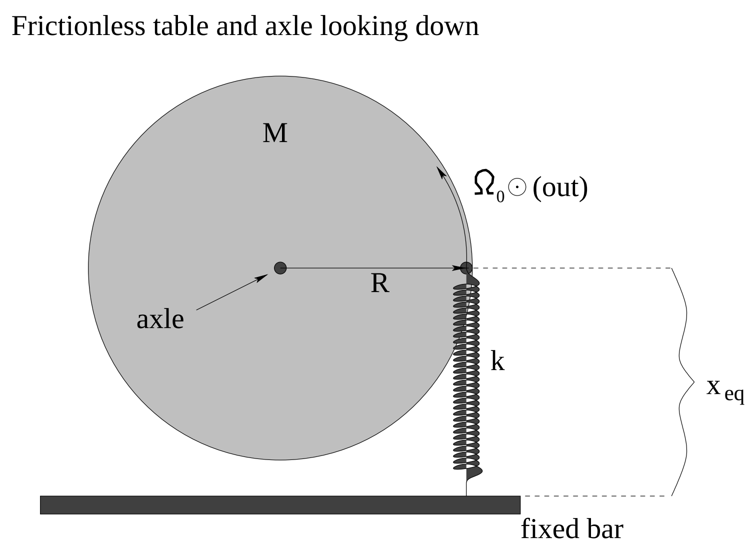
c) Suppose that the axle exerts a weak "drag" torque on the disk when the disk rotates. Do you expect the frequency of oscillation to be larger, smaller, or the same as ω_f once drag is taken into account? (Note that you do not have to derive an answer, but you should justify it on intuitive grounds.)

d) Draw a *qualitatively correct* graph of $\theta(t)$, the angle the disk has rotated through (relative to equilibrium) as a function of time when drag/friction is included as in c).

(Continued workspace on next page)

(Continuation of oscillator problem)

Problem 465. problems-1/oscillation-pr-torsional-oscillator-spring-init-omega0.tex



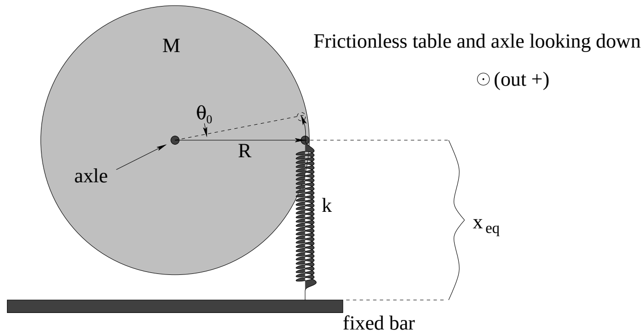
In the figure above, a disk of radius R and mass M is mounted on a frictionless axle. A massless spring with spring constant k is attached to a point on its circumference so that it is in equilibrium as shown. The disk is then lightly struck at time $t = 0$ so that it is given a ***small*** instantaneous counterclockwise angular velocity of Ω_0 while it is still at the equilibrium position, and it subsequently oscillates approximately harmonically through a ***small*** maximum angle θ_0.

a) Find the angular *frequency* ω of its oscillation. You may want to obtain the differential equation of motion first.

b) Find the angle θ_0 through which it will rotate before (first) coming momentarily to rest.

c) Write down (or find) $\theta(t)$, the angle the disk rotates through (relative to equilibrium) as a function of time.

Problem 466. problems-1/oscillation-pr-torsional-oscillator-spring-init-theta0.tex



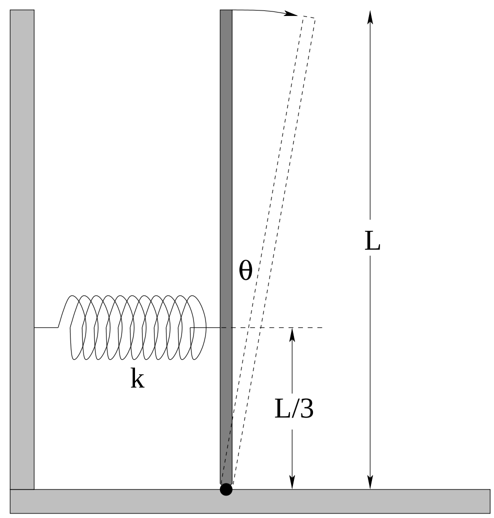
In the figure above, a **disk** of radius R and mass M is mounted on a vertical frictionless axle. A massless spring with spring constant k is attached to a point on its circumference so that it is in equilibrium as shown. The disk is then rotated through a ***small*** angle θ_0 and is released, from rest, at time $t = 0$. It subsequently oscillates approximately harmonically. (Use out of the page for positive θ.)

a) Find the angular *frequency* ω of its oscillation. You may want to obtain the differential equation of motion first.

b) Write down (or find) $\theta(t)$, the angle the disk rotates through (relative to equilibrium) as a function of time.

c) Find the maximum rotational angular *velocity* Ω_0 of the disk as it rotates.

Problem 467. problems-1/oscillation-pr-vertical-bar-and-spring-2.tex



A uniform vertical bar of mass M and length L is pivoted at the bottom. A spring with spring constant k is attached a height $L/3$ over the pivot. This spring is strong enough that the bar will oscillate harmonically about the vertical if it is tipped over to a ***small angle*** θ and released.

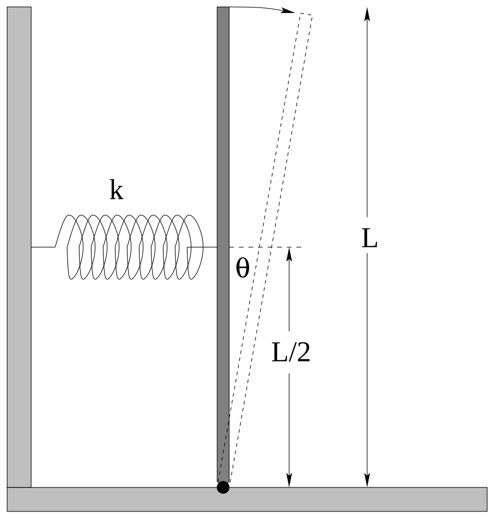
Find:

a) The total torque (magnitude and direction, where ***θ is positive into the page*** as shown) due to ***both*** gravity and the spring as a function of θ.

b) What is the angular frequency ω of the bar as it oscillates? Recall that the moment of inertia of a uniform bar is $\frac{1}{3}ML^2$.

c) What is the smallest value that k can have such that the bar is in stable equilibrium in the vertical position? [If the spring constant is ***larger*** than this smallest value of k, the spring can sustain oscillations of the bar and does not fall over if perturbed from equilibrium.]

Problem 468. problems-1/oscillation-pr-vertical-bar-and-spring.tex



A uniform vertical bar of mass M and length L is pivoted at the bottom. A spring with spring constant k is attached a height $L/2$ over the pivot. This spring is strong enough that the bar will oscillate harmonically about the vertical if it is tipped over to a ***small*** angle θ and released.

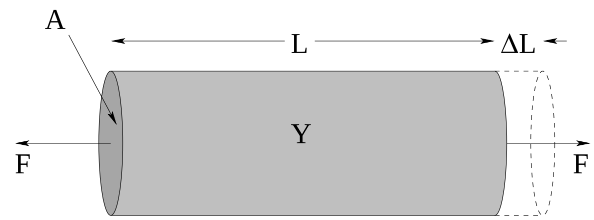
Find:

a) The total torque (magnitude and direction, where **into the page is positive** $\boldsymbol{\theta}$ as shown) due to both gravity and the spring as a function of θ.

b) What is the angular frequency ω of the bar as it oscillates? Recall that the moment of inertia of a uniform bar is $\frac{1}{3}ML^2$.

c) What is the smallest value that k can have such that the bar is in stable equilibrium in the vertical position? [If the spring constant is ***larger*** than this smallest value of k, the spring can sustain oscillations of the bar and does not fall over if perturbed from equilibrium.]

Problem 469. problems-1/oscillation-pr-youngs-modulus.tex



A cylindrical bar of material with cross-sectional area A, unstressed length L, and a Young's Modulus Y is subjected to a force F that stretches the bar as shown. The bar behaves like an elastic "spring", pulling back with a force $F = -k\Delta L$.

a) Show that the "spring constant" of the bar is $k = AY/L$.

b) Show that the energy stored in the bar when it is stretched by length ΔL is $U = \frac{1}{2}F\Delta L$. This will be easiest if you assume that the bar is a "spring" with the spring constant k determined in part a).

Chapter 13

Waves

What do you get when you cross an owl with a bungee cord? Ooo, wrong riddle[1].

We'll try again. What do you get when you *stretch out* a bungee cord and pluck it like an owl?

A ***wave***. A wave is basically what you get when you have a whole bunch of harmonic oscillators all coupled together so each one can push on its neighbors. Or when one part of a string can pull on its neighbors. Or when one piece of matter can push on its neighbor. Or when one studies any of the fields that are the basic laws of nature. Or when one studies quantum ***wave*** mechanics.

But wait! Wave mechanics is chemistry, chemistry is biology, and biology is us. Does that mean that we are, basically, a ***very complex*** wave phenomenon?

Damn skippy it does. It also means that if you drill down to the fundamentals of *nearly any science* you're gonna find waves down there, tormenting you with their perplexity and complexity, unless you master them now. And understanding waves on a humble guitar string, or the wave pulses you can put on a slinky or garden hose is the very first baby step in that direction.

[1]Interested parties might want to google up Kung Pow Owl Bungee Cord" or the like and watch a youtube clip from the movie to learn the highly politically incorrect but hilarious answer.

13.1 Waves

13.1.1 Multiple Choice

Problem 470. problems-1/waves-mc-fixed-and-free-fundamental-review.tex

Consider a vibrating string of length $L = 2$ m. It is found that there are successive standing wave resonances at 50 Hz and 70 Hz. Then the standing wave with the lowest possible frequency (i.e. the first mode or fundamental mode) has frequency:

a) 10 Hz.

b) 20 Hz.

c) 30 Hz.

d) 40 Hz.

e) 50 Hz.

and the string has (check one):

☐ Either both ends fixed or both ends free

☐ One end (either end) fixed and the other free

Problem 471. problems-1/waves-mc-fixed-and-free-fundamental-review-soln.tex

Let's do this one with verbal/conceptual reasoning and not get lost in algebra. The difference between two successive frequencies is clearly 20 Hz. The frequencies themselves are not integer multiples of 20 Hz. Therefore this must be the ***odd harmonic*** series corresponding to a string fixed at one end (either end!) and free at the other. We count backwards in our heads by 20's: 70, 50, 30, 10 – and conclude that the fundamental frequency must be 10 Hz. Note that the length of the string is irrelevant, although it might be important in some other problem. You will often have more information than you need in any given problem! Make sure that you know how to pick out what *is* important!

Hence:

a) **10 Hz**

and

☒ One end (either end) fixed and the other free

Problem 472. problems-1/waves-mc-unknown-fixed-and-free-bcs-fundamental.tex

You are given the following information resulting from measurements of the resonant modes of a string of length L with unknown boundary conditions. You are told that two ***successive*** resonant frequencies are $f_i = 125$ Hz and $f_{i+1} = 175$ Hz where ***mode*** index i counts the frequencies from the bottom. Select the true statement from the following list:

a) The fundamental frequency is 25 Hz, the string is definitely fixed at both ends, and 125 Hz is the $m = 5$ fifth harmonic (fifth ***multiple*** of the fundamental frequency).

b) The fundamental frequency is 25 Hz, the string is definitely free at both ends, and 125 Hz is the $m = 5$ fifth harmonic.

c) The fundamental frequency is 25 Hz, the string is definitely fixed at one end and free at the other, and 125 Hz is the $m = 3$ third harmonic.

d) The fundamental frequency is 25 Hz, the string is definitely fixed at one end and free at the other, and 125 Hz is the $m = 5$ fifth harmonic.

e) None of the above are correct. We cannot tell whether the string is fixed at both ends or free at both ends and/or what harmonic 125 Hz is from this data.

Problem 473. problems-1/waves-mc-unknown-fixed-bcs-fundamental-1.tex

You are given the following information resulting from measurements of the resonant frequencies of a string of length L with unknown boundary conditions. You are told that two ***successive*** frequencies are $f_i = 200$ Hz and $f_{i+1} = 250$ Hz for some index i that just counts the frequencies observed from the lowest one (principle harmonic) and is not necessarily a harmonic index. Select the true statement from the following list:

a) The fundamental frequency is 50 Hz, the string is definitely free at both ends, and $i = 4$.

b) The fundamental frequency is 50 Hz, the string is definitely fixed at one end and free at the other, and $i + 1 = 5$.

c) The fundamental frequency is 50 Hz, the string might be fixed at both ends ***or*** free at both ends and $i = 4$.

d) The fundamental frequency is 25 Hz, the string might be fixed at both ends ***or*** free at both ends, and $i = 8$.

e) The fundamental frequency is 100 Hz, the string might be fixed at both ends ***or*** free at both ends, and $i + 1 = 2.5$.

Problem 474. problems-1/waves-mc-unknown-fixed-bcs-fundamental.tex

You are given the following information resulting from measurements of the resonant modes of a string of length L with unknown boundary conditions. You are told that two ***successive*** resonant modes have frequencies of $f_m = 350$ Hz and $f_{m+1} = 400$ Hz for some mode index m. Select the true statement from the following list:

a) The fundamental frequency is 50 Hz, the string is definitely fixed at both ends, and $m = 7$.

b) The fundamental frequency is 50 Hz, the string is definitely free at both ends, and $m = 7$.

c) The fundamental frequency is 50 Hz, the string is definitely fixed at one end and free at the other, and $m = 4$.

d) The fundamental frequency is 50 Hz, the string might be fixed ***or*** free at both ends, and $m = 7$.

e) The fundamental frequency is 100 Hz, the string might be fixed ***or*** free at both ends, and $m = 3$.

13.1.2 Short Answer

13.1.3 Short Answer

Problem 475. problems-1/waves-sa-breaking-guitar-string.tex

A certain guitar string is tuned to vibrate at the (principle harmonic) frequency f when its tension is adjusted to T. The string will break at a tension $3T$.

a) Can you double the frequency of the string by increasing the tension (only) without breaking the string?

b) What is the maximum frequency that you *can* make the string have, in terms of f, without (quite) breaking the string?

Problem 476. problems-1/waves-sa-heavy-to-light.tex

One end of a heavy rope is tied to a lighter rope as shown in the figure. An upright wave pulse is incident from then left and travels to the right reaching the junction between the ropes at time $t = 0$, so that, for time $t > 0$, there are two pulses - a transmitted pulse in the light rope and a reflected pulse in the heavy rope.

Compare the transmitted and reflected pulses ***to the incident pulse*** by filling in the table below (each answer is "relative to the same property of the incident pulse"):

	Transmitted	**Reflected**
speed (greater, lesser, equal)		
orientation (upright, inverted)		
energy (greater, lesser, equal)		

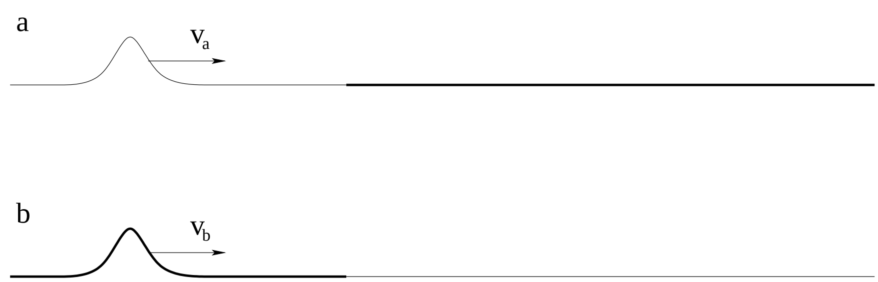
Problem 477. problems-1/waves-sa-reflected-wave-energy.tex

A string of some mass density μ is smoothly joined to a string of greater mass density and the combined string is stretched to a uniform tension T_{en} (the same in both wires). The speed of a wave pulse on the thinner wire is ***twice the speed of a pulse*** on the thicker wire. A wave pulse reflected from the thin-to-thick junction has ***half the amplitude*** of the original pulse. Assuming no loss of energy in the wire:

a) What fraction of the incident energy is reflected at the junction?

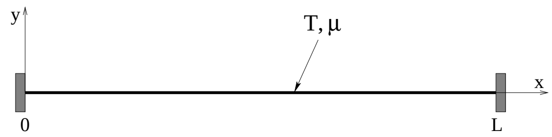
b) Is the reflected pulse upside down or right side up?

Problem 478. problems-1/waves-sa-reflection-transmission-at-junction.tex



Two combinations of two strings with different mass densities are drawn above that are connected in the middle. In both cases the string with the greatest mass density is drawn darker and thicker than the lighter one, and the strings have the same tension T in both a and b. A wave pulse is generated on the string pairs that is travelling from left to right as shown. The wave pulse will arrive at the junction between the strings at time t_a (for a) and t_b (for b). Sketch reasonable *estimates* for the transmitted and reflected wave pulses onto the a and b figures at time $2t_a$ and $2t_b$ respectively. Your sketch should correctly represent things like the relative speed of the reflected and transmitted wave and any changes you might reasonably expect for the amplitude and appearance of the pulses.

Problem 479. problems-1/waves-sa-string-fixed-both-ends.tex



A string of mass density μ is stretched to a tension T and fixed at $x = 0$ and $x = L$. The transverse string displacement is measured in the y direction. All answers should be given ***in terms of these quantities*** or new quantities (such as v) you ***define in terms of these quantities***.

Write down $k_n, \omega_n, f_n, \lambda_n$ for the first three modes supported by the string. Sketch them in on the axes below, labelling nodes and antinodes. You do not have to derive them, although of course you may want to justify your answers to some extent for partial credit in case your answer is carelessly wrong.

Problem 480. problems-1/waves-sa-two-string-densities-frequency.tex

Two identical strings of length L, fixed at both ends, have an identical tension T, but have different mass densities. One string has a mass density of μ, the other a mass density of 16μ

When plucked, the first string produces a (principle harmonic) tone at frequency f_1. What is the frequency of the tone produced by the second string?

a) $f_2 = 4f_1$

b) $f_2 = 2f_1$

c) $f_2 = f_1$

d) $f_2 = \frac{1}{2}f_1$

e) $f_2 = \frac{1}{4}f_1$

Problem 481. problems-1/waves-sa-wave-energies.tex

A wave on a string with mass density μ travels to the right ($+x$) according to the formula:

$$y(x,t) = A\sin(kx - \omega t)$$

Suppose this wave has an average energy per unit length E_0. Identify all of the changes one can make to this wave that will produce a wave with a average energy per unit length of $4E_0$. In all cases the changes indicated are the ***only*** changes in the string or wave formula.

a) Change $A \to 2A$ and change $\mu \to 2\mu$.

b) Change $A \to 2A$.

c) Change $\mu \to 2\mu$.

d) Change $\mu \to 2\mu$ and $k \to 2k$.

e) Change $\mu \to 2\mu$ and $\omega \to 2\omega$.

f) Change $\omega \to 2\omega$.

g) Change $k \to 2k$.

Problem 482. problems-1/waves-sa-wave-facts.tex

Answer the five short questions below:

a) Suppose you are given string A with mass density μ that is stretched until it has tension T_A. You are given a second string B with the ***same*** mass density stretched to ***twice the tension***, $T_B = 2T_A$.

What is the speed of a wave v_B on string B relative to v_A, the speed on string A?

$$v_B = \boxed{} \times v_A$$

b) Suppose you are given string A with mass density μ_A that is stretched until it has tension T. You are given a second string B with ***four times the mass density*** of A $\mu_B = 4\mu_A$ but at the ***same*** tension.

What is the speed of a wave v_B on string B relative to v_A, the speed on string A?

$$v_B = \boxed{} \times v_A$$

c) Suppose you are given string A with mass density μ that is stretched until it has tension T_A. You are given a second (identical) string B with mass density μ that is stretched to ***twice the tension***, $T_B = 2T_A$. Both strings are carrying a ***travelling harmonic wave at the same frequency***.

What is the wave number k_B on string B in terms of the wave number k_A on string A?

$$k_B = \boxed{} \times k_A$$

d) Suppose you are given string A with mass density μ_A that is stretched until it has tension T_A. You are given a second string B with four times the mass density of A $\mu_B = 4\mu_A$ but at the same tension. Both strings are carrying waves with the same wavelength λ.

What is the (regular) frequency f_B on string B in terms of the frequency f_A on string A?

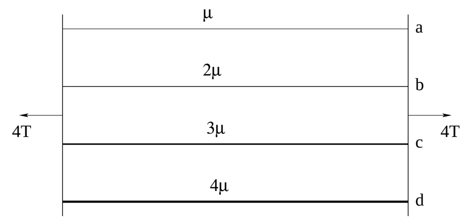
$$f_B = \boxed{} \times f_A$$

e) Suppose you are given string A with mass density μ_A that is stretched until it has tension T_A. You are given a second string B with four times the mass density of A $\mu_B = 4\mu_A$ and a tension four times the tension of A $T_B = 4T_A$. Both strings carry a wave with the same frequency f.

What is the wavelength λ_B in terms of the wavelength λ_A?

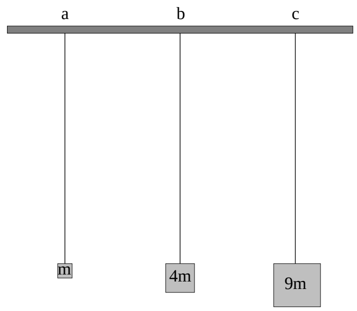
$$\lambda_B = \boxed{} \times \lambda_A$$

Problem 483. problems-1/waves-sa-wave-speed-vary-mu.tex



In the figure above, the neck of a stringed instrument is schmatized. Four strings of different *thickness* and the same length are stretched in such a way that the tension in each is about the same (T) for a total of $4T$ between the end bridges – if this were not so, the neck of the guitar or ukelele or violin would tend to bow towards the side with the greater tension. If the velocity of a wave on the first (lightest) string is v_1, what is the speed of a wave of the other three in terms of v_1?

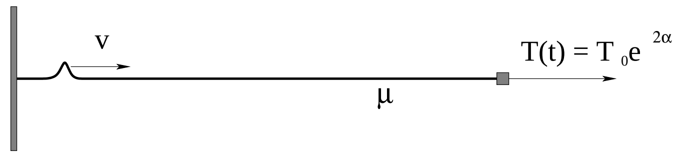
Problem 484. problems-1/waves-sa-wave-speed-vary-T.tex



Three strings of length L (not shown) with the same mass per unit length μ are suspended vertically and blocks of mass m, $4m$ and $9m$ are hung from them. The total mass of each string $\mu L \ll m$ (the strings are *much* lighter than the masses hanging from them). If the speed of a wave pulse on the first string (a) is v_0, what is the speed of the same wave pulse on the second (b) and third (c) strings?

13.1.4 Long Problems

Problem 485. problems-1/waves-pr-accelerating-wave-pulse.tex



A wave pulse is started on a string with mass density μ with an applied tension that *increases* like $T_0 e^{2\alpha t}$.

a) Find the initial velocity of the wave pulse at time $t = 0$.

b) Find the acceleration of the wave pulse as a function of time.

Problem 486. problems-1/waves-pr-construct-transverse-travelling-wave-1.tex

A very long string aligned with the x-axis is being shaken at the ends in such a way that there is a ***travelling harmonic wave*** on it. y is the vertical direction perpendicular to the string in the direction of the string's displacement. Given the following data (**note units**):

Amplitude $A = 1$ cm
Wavelength $\lambda = 0.5$ m
Period $T = 0.001$ sec

a) Write down the formula for a transverse wave travelling ***in the $-x$ direction*** (that is, to the ***left***) corresponding to these numerical parameters. You may use π in your answer as a symbol as needed.

b) What is the speed of the wave on the string in terms of the givens?

c) Suppose one wished to ***double the power*** transmitted by the string. Enter T or F in the provided boxes if the stated change to one of the givens would accomplish this ***assuming no other change to the givens is made***.

- Increase the amplitude from 1 cm to 2 cm. ☐
- Decrease the period of the wave to 0.000707. ☐
- Increase the wavelength of the wave to 1 meter. ☐
- Increase the amplitude from 1 cm to 1.414 cm. ☐

Problem 487. problems-1/waves-pr-fixed-both-ends.tex

A string with mass density μ and under tension T vibrates in the y-direction. The string is ***fixed at both ends*** at $x = 0$ and $x = L$. Answer all questions in terms of these givens.

a) What are the ***two lowest frequencies*** f_1 and f_2 that a standing wave can have for this string?

b) Write down an equation for $y(x, t)$, the y-displacement of the string as a function of position x along the string and time t for the ***standing wave*** corresponding to the ***second lowest frequency*** $\boldsymbol{f_2}$ (the second mode) that you just computed. Assume that the standing wave has a maximum vertical displacement of $y = A$.

c) On the graph below, plot the y-displacement for the second mode versus horizontal position x at an instant when the the string achieves its maximum displacement. Indicate the positions on the x-axis of any nodes or antinodes.

Problem 488. problems-1/waves-pr-fixed-one-end.tex

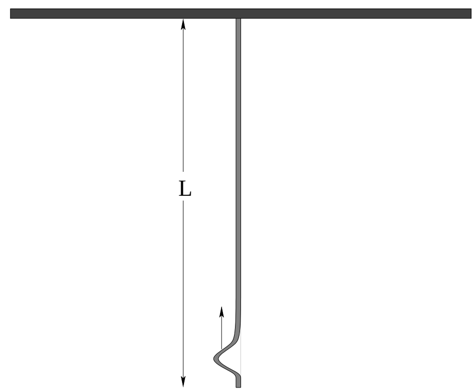
A string with mass density μ and under tension T vibrates in the y-direction. The string is ***fixed*** at $x = 0$ and ***free*** at $x = L$. Answer all questions in terms of these givens.

a) What are the ***two lowest frequencies*** f_1 and f_2 that a standing wave can have for this string?

b) Write down an equation for $y(x, t)$, the y-displacement of the string as a function of position x along the string and time t for the ***standing wave*** corresponding to the ***second lowest frequency*** $\boldsymbol{f_2}$ (the second mode) that you just computed. Assume that the standing wave has a maximum vertical displacement of $y = A$.

c) On the graph below, plot the y-displacement for the second mode versus horizontal position x at an instant when the the string achieves its maximum displacement. Indicate the positions on the x-axis of any nodes or antinodes.

Problem 489. problems-1/waves-pr-speed-on-hanging-string.tex



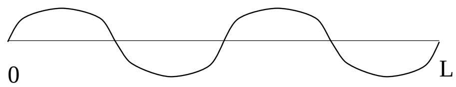
A string of total length L with a mass density μ is shown hanging from the ceiling above.

a) Find the tension in the string as a function of y, the distance *up* from its bottom end. Note that the string is not massless, so each small bit of string must be in static equilibrium.

b) Find the velocity $v(y)$ of a small wave pulse cast into the string at the bottom that is travelling upward.

c) Find the amount of time it will take this pulse to reach the top of the string, reflect, and return to the bottom. Neglect the size (width in y) of the pulse relative to the length of the string.

Problem 490. problems-1/waves-pr-standing-wave-mode-energy.tex



A string of total mass M and total length L is fixed at both ends, stretched so that the speed of waves on the string is v. It is plucked so that it harmonically vibrates in its $n = 4$ mode:

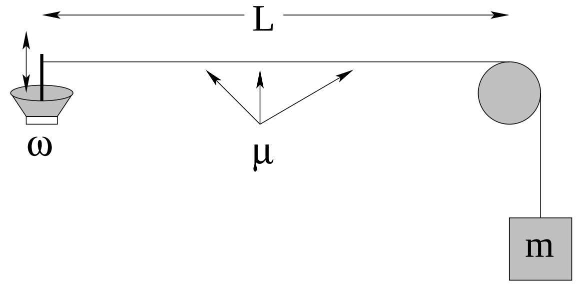
$$y(x,t) = A_4 sin(k_4 x) cos(\omega_4 t).$$

Find (derive) the instantaneous total kinetic energy in the string in terms of M, L, $n = 4$, v and A_4 (although it will simplify matters to use k_4 and ω_4 **once you define them**).

Remember (FYI):

$$\int_0^{n\pi} \sin^2(u) du = \int_0^{n\pi} \cos^2(u) du = \frac{n\pi}{2}$$

Problem 491. problems-1/waves-pr-string-and-hanging-mass.tex



In the figure above, a string of length L and mass density μ is run over a pulley and maintained at some tension by a stationary hanging mass m. The string is driven with tiny oscillations at a tunable frequency ω by a speaker attached to one end as shown. You may neglect the weight of the string compared to the weight of the mass m.

a) For a given mass m, write an expression for the velocity of waves on the string.

b) Find the frequency of the ***third harmonic*** of the string (expressed in terms of the givens).

c) What is the angular frequency of the ***sound wave*** that the string will produce when it is driven in resonance with its third harmonic frequency?

d) What is the wavelength of the sound wave produced by the string vibrating at this frequency? You may express your answer algebraically in terms of v_{a} (the speed of sound in air).

Problem 492. problems-1/waves-pr-travelling-wave-analysis.tex

A travelling wave on a string of mass $\mu = 0.01$ kg/meter is given by the expression:

$$y(x,t) = 2.0\sin(0.02\pi x + 2\pi t) \qquad \text{(meters)}$$

Answer the following questions about this wave. All of the arithmetic *should* be doable without a calculator, but if you have any doubt feel free to leave arithmetical expressions of the algebra unevaluated.

a) What is the amplitude of this wave?

b) What is its wavelength?

c) What is its period?

d) What is the ***velocity*** of this wave (include direction!)?

e) Write an *algebraic* expression for the kinetic energy per unit length in the string as a function of time.

Chapter 14

Sound

What's that again? Did you say something? I couldn't hear you, because the ***sound*** of your voice wasn't intense enough for me to hear. This isn't your fault – I'm aging and hence growing deaf(er). And besides, you're probably not even in the same room with me.

In any event, understanding sound seems once again like it would be very useful to physicians, engineers, physicists, musicians, communications specialists, and maybe even ordinary people who want to learn a bit about waves in what is still a relatively simple and ubiquitous application before tackling some bit of science that has some *difficult* wave mechanics in it.

In the problems below we will have our first encounter with 3 dimensional waves (the symmetric pretty easy kind), $1/r^2$ laws, the idea of decibels" and the use of logarithmic scales to describe things that vary over many orders of magnitude efficiently. We'll also get our first very tiny taste of interference arising from wave superposition, although the main course is put off until you cover electromagnetic waves and the interference and diffraction of light waves.

Ping.

14.1 Sound Waves

14.1.1 Multiple Choice

Problem 493. problems-1/sound-mc-car-horn-decibels.tex

You are stuck in freeway traffic and need to get home. So does the driver next to you – she starts blowing the horn of her car, which you hear as a sound with a sound level of 90 dB. Not to be outdone, the driver behind you, in front of you, and to the other side of you all lean on their horn as well, so that now you are hearing all **four** horns (which reach your ears with equal intensities) at once. The sound level you *now* hear is:

a) 93 db

b) 96 dB

c) 180 dB

d) 360 dB

e) Unchanged.

Problem 494. problems-1/sound-mc-exam-noise-in-dB.tex

200 students are taking an examination in a room, and the sounds of pens scratching on paper, sighs, groans, and muttered imprecations has created a more or less continuous sound level of this noise of 60 dB. Assuming each student contributes equally to this noise and nothing else changes or adds to it, what will the sound level in the room be when only 50 students are left?

a) 50 dB

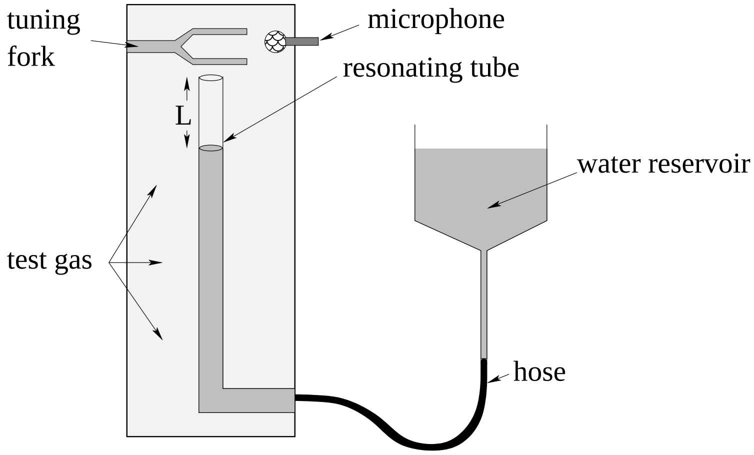
b) 15 dB

c) 66 dB

d) 54 dB

e) 57 dB

Problem 495. problems-1/sound-mc-measuring-speed-of-sound.tex



A simple method for measuring the speed of sound in a reservoir filled with gas is to hold a tuning fork at a fixed, known frequency above a tube connected with a flexible hose to a reservoir such that the height of the water in the tube can easily be varied. The sound one detects with a microphone is then the loudest when the tuning fork is in ***resonance*** with standing wave modes in the tube.

If you hold a 2000 Hz tuning fork above the tube when it is completely full and then lower the reservoir slowly to drop the water level in the tube, you hear the fork resonate most loudly when the water is $L = 2.5$, 7.5, and 12.5 cm beneath the end of the tube. The speed of sound in the gas is therefore:

a) 50 m/sec

b) 100 m/sec

c) 200 m/sec

d) 500 m/sec

e) 750 m/sec

Problem 496. problems-1/sound-mc-shooting-a-gun-dB.tex

A 30-06 rifle makes a bang that peaks at 170 decibels 1 meter away from the muzzle. If you are standing 100 meters away (approximately) what sound level do you hear in decibels?

a) 120 dB

b) 130 dB

c) 140 dB

d) 150 dB

e) 160 dB

Problem 497. problems-1/sound-mc-siren-in-dB.tex

A siren radiates sound energy uniformly in all directions. When you stand a distance 100 m away from the siren you hear a sound level of 90 dB. If you move to a distance of 10 m from the siren, the sound level is:

a) 90 dB, no change.

b) 100 dB.

c) 110 dB.

d) 120 dB.

e) 130 dB.

Problem 498. problems-1/sound-mc-sound-level-to-pressure-1.tex

You measure the intensity level of a single frequency sound wave produced by a loudspeaker with a calibrated microphone to be 80 dB. At that intensity, the *peak* pressure in the sound wave at the microphone is $P_0 + P_a$, where P_a is the baseline atmospheric pressure and P_0 is the pressure over that associated with the wave. The loudspeaker's amplitude is turned up until the measured intensity level is 120 dB. What is the peak pressure of the sound wave now?

a) $4P_0 + P_a$

b) $10P_0 + P_a$

c) $40P_0 + P_a$

d) $100P_0 + P_a$

e) $100(P_0 + P_a)$

Problem 499. problems-1/sound-mc-sound-level-to-pressure-2.tex

You measure the sound level of a single frequency sound wave produced by a loudspeaker with a calibrated microphone to be ***80 dB***. At that intensity, the *peak* pressure in the sound wave at the microphone is $P_0 + P_a$, where P_a is the baseline atmospheric pressure and P_0 is the pressure over that associated with the wave. The loudspeaker's amplitude is turned up until the measured sound level is ***100 dB***. What is the peak pressure of the sound wave now?

a) $4P_0 + P_a$

b) $10P_0 + P_a$

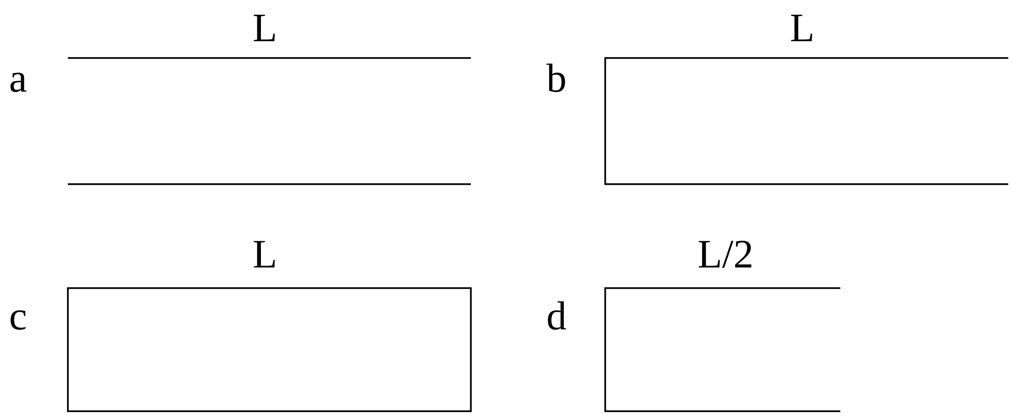
c) $40P_0 + P_a$

d) $100(P_0 + P_a)$

e) $100P_0 + P_a$

14.1.2 Ranking/Scaling

Problem 500. problems-1/sound-ra-sound-resonances-pressure.tex

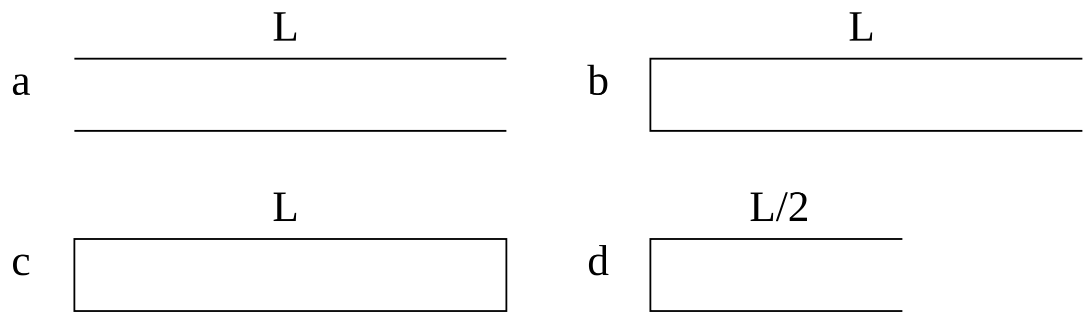


a) Rank the fundamental harmonic resonant ***frequencies*** ($n = 1$) of the four open/closed pipes drawn above, where equality is a possible answer. An answer might be (but probably isn't) $f_a < f_b = f_c < f_d$.

b) Draw into each pipe a representation of a the ***pressure wave*** associated with each resonance.

c) Label the nodes (in your representation of the waves) with an **N** and antinodes with an **A**.

Problem 501. problems-1/sound-ra-sound-resonances.tex



a) Rank the fundamental harmonic resonant ***frequencies*** ($n = 1$) of the four open/closed pipes drawn above, where equality is a possible answer. An answer might be (but probably isn't) $f_a < f_b = f_c < f_d$.

b) Draw into each pipe a representation of a the ***displacement wave*** associated with each resonance.

c) Label the nodes (in your representation of the waves) with an **N** and antinodes with an **A**.

14.1.3 Short Answer

Problem 502. problems-1/sound-sa-alarm-clocks.tex

Wal Mart had a special on alarm clocks, and you bought ten of them just to make sure that you will wake up in time for your physics final exam. Each alarm clock produces an incoherent sound level in your ears of 90 dB when you place the clock on the nightstand one meter from your head. Ignore reflection of sound energy from walls, etc and treat the clocks like point sound sources.

a) If you put 4 clocks on the nightstand one meter from your head, you will hear a sound level of (approximately to the nearest integer): $\boxed{}$ dB

b) If you put 8 clocks on the dresser 2 meters from your head, you will hear a sound level of: $\boxed{}$ dB

c) If you put all 10 clocks in the far corner of the room 4 meters from your head, you will hear a sound level of: $\boxed{}$ dB

Problem 503. problems-1/sound-sa-beats.tex

Two identical strings of length L have mass μ and are fixed at both ends. One string has tension T. The other has tension $1.21T$. When plucked, the first string produces a tone at frequency f_1. What is the *beat* frequency produced if the second string is plucked at the same time, producing a tone f_2? Are the beats likely to be audible if f_1 is 500 Hz?

Problem 504. problems-1/sound-sa-decibels-sun-human-body.tex

Sunlight reaches the surface of the earth with *roughly* 1000 Watts/meter^2 of intensity. What is the "intensity level" of a *sound* wave that carries as much energy per square meter, in **decibels**? In table 15-1 in Tipler and Mosca, what kind of sound sources produce this sort of intensity? Bear in mind that the Sun is *150 million kilometers away* where sound sources capable of reaching the same intensity are typically only a few meters away. Hmmm, seems as though the Sun produces a *lot* of (electromagnetic) energy compared to terrestrial sources of (sound) energy.

While you are at it, the human body produces energy at the rate of roughly 100 Watts. *Estimate* the fraction of this energy that goes into my lecture when I am speaking in a loud voice in front of the class (loud enough to be heard as loudly as normal conversation ten meters away).

Problem 505. problems-1/sound-sa-principle-harmonic-series.tex

Two pipes used in ***different*** musical instruments have the ***same length L***, but the fundamental frequency (frequency of the principal harmonic, $m = 1$) of one is ***twice that of the other***. Explain how this could be, illustrating your answer with a drawing of two pipes and the principle modes such that this is true. Make sure you indicate which pipe has the higher frequency and which pipe has the lower one, and whether your diagram is illustrating pressure or displacement standing waves!

Problem 506. problems-1/sound-sa-scaling-thunder-dB.tex

Lightning strikes one kilometer away, and the resulting thunderclap has an intensity of 5×10^{-3} Watts/meter^2. What is the intensity level in decibels? If one is instead 10 kilometers away, approximately how many decibels lower would the intenstity level be?

Problem 507. problems-1/sound-sa-scaling-time-thunder-dB.tex

You see the flash of lightning and ***three*** seconds later you hear a thunderclap with a peak sound level of 120 dB. A few minutes later you see a second flash of lightning and ***twelve*** seconds later you hear the thunderclap.

a) Approximately what peak sound level do you hear in the second (presumably "identically produced") thunderclap?

   Second thunderclap is: $\boxed{}$ dB

b) Roughly – to the nearest kilometer – how far away are the two lightning flashes?

   First (three seconds): $\boxed{}$ km

   Second (twelve seconds): $\boxed{}$ km

Problem 508. problems-1/sound-sa-sound-speed-decibels.tex

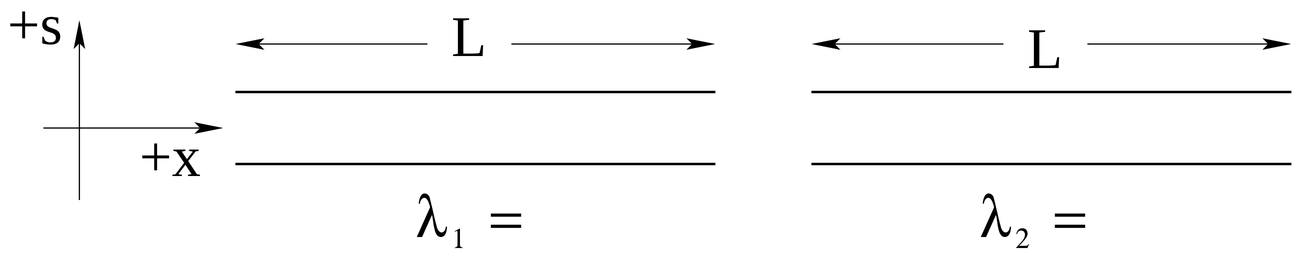
(12 points) Some short questions about sound:

a) Show that doubling the intensity of a sound wave corresponds to an increase in its intensity level or loundness by about 3 dB.

b) I sometimes work as a timer at my son's swim meets. We are told to start our watches when we see a light flash on the starter's console, not when we hear the starting horn. If I am timing a lane on the far side of the pool some 17 meters away from the starter and start when the sound of the horn reaches me, how much will the times I measure (on average) change? Will the swimmer have an advantage or a disadvantage relative to a swimmer timed by someone that starts on the flash of light?

c) Suppose I turn the knob on my surround-sound amplifier and decrease the loudness where I'm listening by 6 dB. By roughly what fraction has the *amplitude* of oscillation of the speakers changed?

Problem 509. problems-1/sound-sa-tube-open-both-ends-1.tex



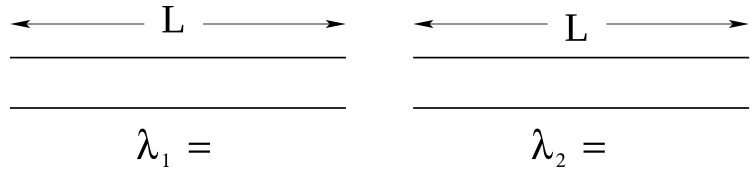
A tube open at both ends used as a "panpipe" musical instrument. It has length $L = 34$ centimeters.

a) Sketch the ***first two displacement modes*** (or harmonics) in the provided tubes.

b) Label the nodes and antinodes, and underneath each tube indicate the wavelength of the mode/harmonic.

c) What is the ***frequency*** of the ***second harmonic*** of the tube (an actual number, please, in Hertz or cycles per second).

Problem 510. problems-1/sound-sa-tube-open-both-ends-2.tex



A tube open at both ends used as a "panpipe" musical instrument. It has length $L = 34$ centimeters.

a) Sketch the ***first two displacement modes*** (or harmonics) in the provided tubes.

b) Label the nodes and antinodes, and underneath each tube indicate the wavelength of the mode/harmonic.

c) What is the ***frequency*** of the ***principle harmonic*** of the tube (an actual number, please, in Hertz or cycles per second).

Problem 511. problems-1/sound-sa-two-wave-speeds-frequency.tex

Two identical pipes, both closed at both ends, are filled with two ***different*** gases. In the first gas, the speed of sound is $v_1 = \sqrt{B_1/\rho_1}$, in the second the speed of sound is $v_2 = \sqrt{B_2/\rho_2} = 2v_1$. Both pipes are driven by speakers in resonance with their ***fundamental harmonic frequency***, f_1 and f_2 respectively.

If f_1 is the fundamental frequency in the first pipe, what is the fundamental frequency f_2 in the second pipe?

a) $f_2 = 4f_1$

b) $f_2 = 2f_1$

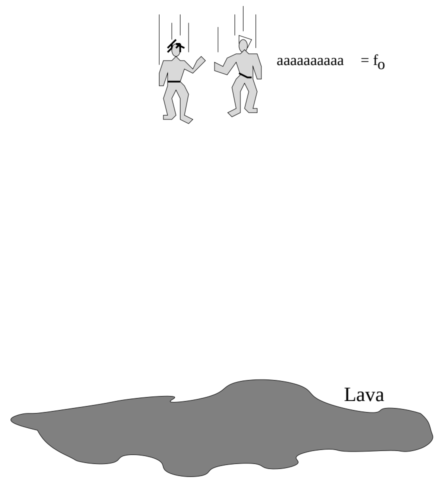
c) $f_2 = f_1$

d) $f_2 = \frac{1}{2}f_1$

e) $f_2 = \frac{1}{4}f_1$

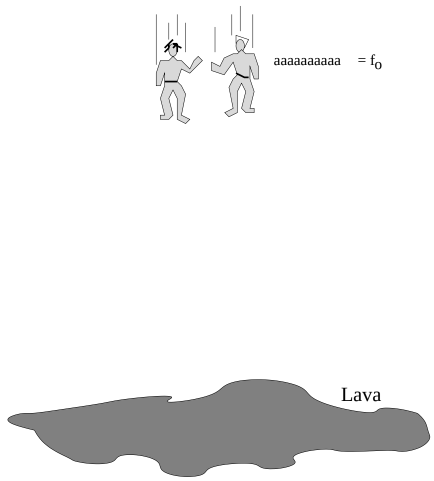
14.1.4 Long Problems

Problem 512. problems-1/sound-pr-bill-and-ted-double-doppler-1.tex



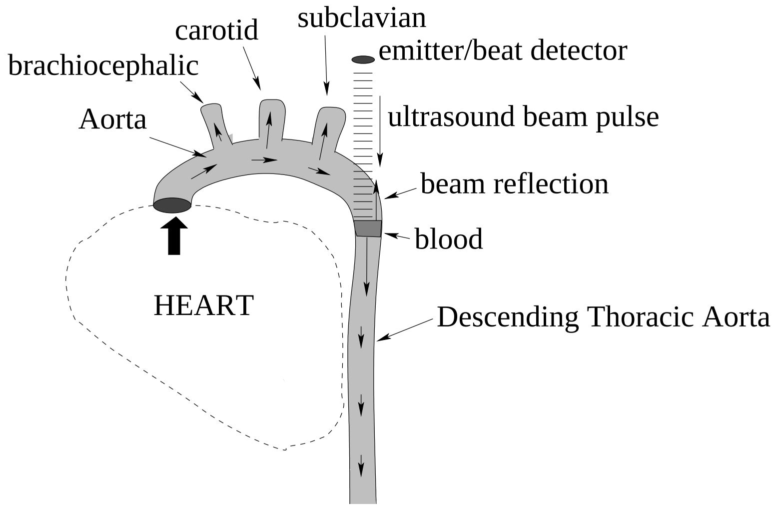
Bill and Ted are falling at a constant speed (terminal velocity) into hell, and are screaming at a frequency f_0. They hear their own voices reflecting back to them from the puddle of molten rock that lies below at a frequency of $2f_0$. How fast are they falling relative to the speed of sound?

Problem 513. problems-1/sound-pr-bill-and-ted-double-doppler-2.tex



Bill and Ted are falling at a constant speed (terminal velocity) into hell, and are screaming at a frequency f_0. They hear their own voices reflecting back to them from the puddle of molten rock that lies below at a frequency of $1.21 f_0$. How fast are they falling relative to the speed of sound?

Problem 514. problems-1/sound-pr-doppler-electrocardiograph.tex



During a cardiac cycle, blood is ejected by the heart into the aorta with a typical peak speed around 0.5 m/sec for a healthy adult. However, in a patient with an obstruction, the peak speed can be much higher. The peak blood speed can be detected noninvasively using a pulsed ultrasound beam.

Let us model this process as a simple highly directional ultrasound beam of frequency f_0 that is being directed through a patient's descending thoracic aorta parallel to the artery as shown. We will assume that the ultrasound beam is reflected off of just one small (shaded) section of the flowing blood fluid that is travelling at a speed v in the direction shown the same way it would be reflected off of a moving object. Use $v_{\rm us}$ for the speed of ultrasonic sound in blood or living tissue.

patient

a) Write an *expression* for the the frequency f we expect the detector to detect in terms of f_0, $v_{\rm us}$, and v. Is f higher than or lower than the beam frequency f_0?

$f =$

b) The detector *measures* f, but we wish to know v. Solve for $v/v_{\rm us}$ in terms of f_0, and f.

$$\frac{v}{v_{\rm us}} =$$

The next two questions involve actual numbers. Suppose $f_0 = 2 \times 10^6$ Hz and $v_{\rm us} = 1.5 \times 10^3$ m/sec.

c) What is the wavelength of the incident beam? $\lambda =$ $\boxed{}$

d) **Extra Credit (2 points):** If a beat detector detects a beat frequency of $\Delta f = 8 \times 10^3$ Hz between the incident and reflected ultrasound beams, find the blood speed and then determine whether the patient is likely to have an obstructed descending thoracic aorta based upon information provided above. (The speed of the blood is expected to be much smaller than that of the ultrasound so that beats can be detected comparing the outgoing to the incoming doppler shifted wave.)

Problem 515. problems-1/sound-pr-doppler-moving-receiver-derive.tex

A microphone mounted on a cart is moved directly toward a harmonic source at a speed of $v_r = 34$ m/sec. The harmonic source is emitting sound waves at a frequency of $f_0 = 1000$ Hz.

a) **Derive** an expression for the frequency of the waves picked up by the moving microphone.

b) What is that frequency?

Problem 516. problems-1/sound-pr-doppler-moving-source-derive.tex

A speaker mounted on a cart is moved directly toward a stationary microphone at a speed $v_s = 34$ m/sec. It is emitting harmonic sound waves at a source frequency of $f_0 = 1000$ Hz. $v_a = 340$ m/sec is the speed of sound in air.

a) ***Derive*** an algebraic expression for the frequency f' of the waves picked up by the stationary microphone, beginning with a suitable picture of the wave fronts. Limited partial credit will be awarded for ***just*** correctly remembering it if you cannot derive it.

b) What is the frequency f' in Hz? You should be able to do the arithmetic without a calculator.

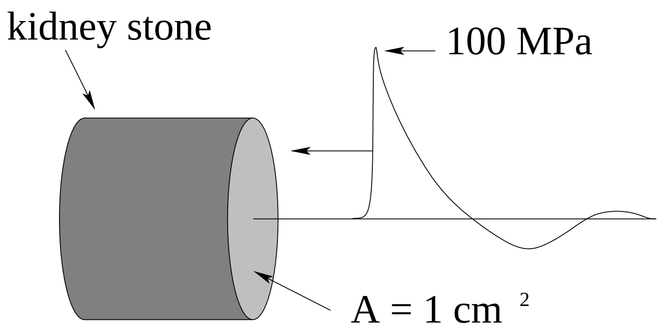
Problem 517. problems-1/sound-pr-doppler-moving-source.tex

A speaker mounted on a cart is moved directly toward a stationary microphone at a speed $v_s = 34$ m/sec. It is emitting harmonic sound waves at a source frequency of $f_0 = 1000$ Hz. $v_a = 340$ m/sec is the speed of sound in air.

a) What is the frequency f' of the waves picked up by the microphone in Hz? You should be able to do the arithmetic without a calculator.

b) Suppose a second source with the same frequency f_0 was located ***at rest*** an identical distance to the right of the microphone receiver. What would be the frequency of the ***beats*** recorded by the microphone?

Problem 518. problems-1/sound-pr-lithotripsy-decibels.tex



Modern ***lithotripsy*** machines create a focused acoustical shock wave (SW) pulse with an overpressures that range from $P_0 = 4 \times 10^7$ to over 10^8 Pascals[1]. A harmonic wave in water with this amplitude would have an intensity $I \sim P_0^2 \times 10^4$ when P_0 is expressed in ***atmospheres*** of pressure and I is the usual ***watts per square meter***. Although this expression will not be exact for a non-harmonic shock wave pulse, it should give the right order of magnitude for the average intensity in the initial peak.

a) Estimate I for an acoustical pulse with a peak amplitude of 10^8 Pascals. Algebra first! Careful with the units!

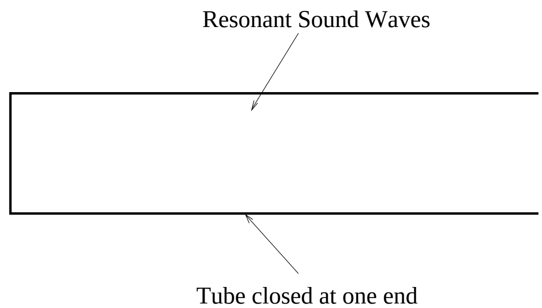
b) Express this intensity in decibels. Use the usual reference intensity for sound waves (the threshold of hearing).

c) Estimate the "instantaneous" peak force (rise time on the order of nanoseconds) exerted by the shock wave overpressure on the front face of a cylindrical kidney stone with an area of 1 square centimeter.

d) Assuming that this primary pulse lasts for $\Delta t = 10$ nanoseconds (or 10^{-8} seconds), what is the total impulse imparted to the front face of the kidney stone by this force?

[1]This dynamic pressure is comparable to the *static* pressure in the deep ocean trenches ten kilometers beneath the surface, where even "incompressible" water compresses by around 4 or 5%.

Problem 519. problems-1/sound-pr-standing-waves-organ-pipe.tex



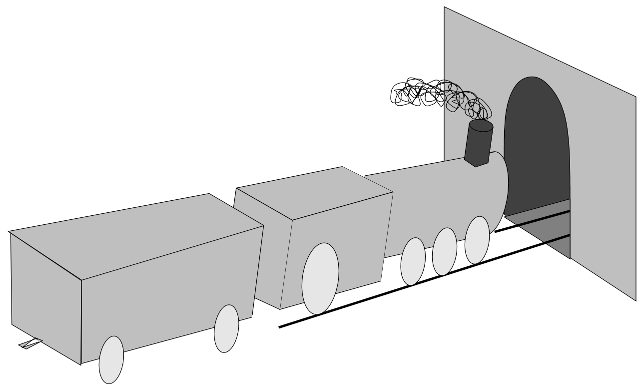
An organ pipe is made from a brass tube closed at one end as shown. The pipe is 3.4 meters long. When driven it produces a sound that is a mixture of the first, third and sixth harmonic (mode).

a) What are the frequencies of these modes?

b) Sketch the wave amplitudes for the third harmonic mode (only) in on the figure, indicating the nodes and antinodes. Be sure to indicate whether the nodes or antinodes are for **pressure/density** waves or **displacement** waves!

c) The temperature in the church where the organ plays varies by around 30° C between summer and winter. By how much (**approximately**) does this vary the frequency of the fundamental harmonic? (Indicate **why** your answer is what it is, don't just put down a guess).

Problem 520. problems-1/sound-pr-train-double-doppler-shift-2.tex



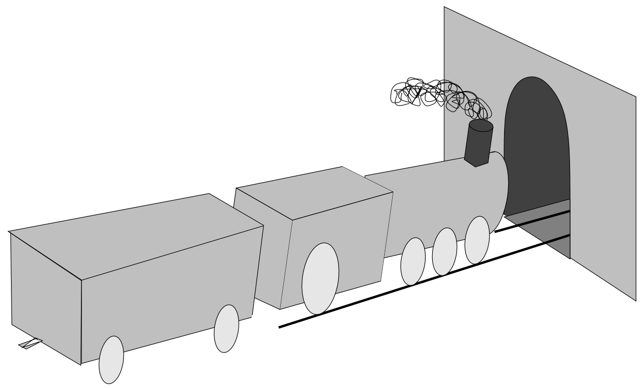
A train approaches a tunnel in a sheer cliff at speed v_{train}. The train blows a whistle of frequency 1000 Hz. A listener on the train hears a beat frequency of 10 Hz between the original whistle and the reflected sound.

a) What is the frequency of the reflected wave as heard by the passengers on the train?

b) Find the speed of the train relative to the speed of sound in air:

$$\frac{v_{\text{train}}}{v_{\text{air}}} =$$

Problem 521. problems-1/sound-pr-train-double-doppler-shift.tex



A train approaches a tunnel in a sheer cliff. The train is moving at 34 m/s, and it blows a horn of frequency 900 Hz. The speed of sounds is 340 m/s.

a) What frequency would a listener at the base of the cliff hear?

b) What frequency do the train passengers hear from the echo (the reflection from the cliff face)?

Chapter 15

Gravity

We began this book and the study of mechanics and dynamics with gravity. It is only fitting that we end it with gravity as well, but this time ***gravity done right!***

What, the force of gravity is not really mg? Sadly, no, it's not. This is only its form near the surface of the earth, at least for the $g = 10$ m/sec^2 we've grown to know and love.

Newton's Law of Gravitation is, at long last, our very first ***force law of nature***, not a composite of other force laws (like normal forces, springs, and so on). It is also our first inverse square law, and is an excellent preview of Coulomb's Law (which will start out our studies of electromagnetism shortly).

But the story of gravity is the story of the Universe itself. It is also the story of the Enlightenment, the discovery/invention of the scientific method, the systematic process by which we build up reliable, reproducible knowledge and reject mythology, lies, errors, fantasies, and just plain (probably) incorrect hypotheses of all sorts. Its importance thus goes far beyond its direct content or direct utility.

Physicians probably don't need to know much about the laws of gravitation other than that it holds us down to the Earth and holds the solar system and galaxy together. But gravity is ***so cool*** that personally, I think *everybody* should know a bunch of what is covered in this chapter if only so that they can understand a little bit of what they see when they go outside on a cold, clear night and look up at the stars.

That's a very *human* thing to do, and trust me – it is better when you can understand a tiny bit of the enormously beautiful *structure* of what you see.

15.1 Gravity

15.1.1 Multiple Choice

Problem 522. problems-1/gravitation-mc-drag-changes-orbit.tex

A satellite in a low-Earth (circular) orbit will slowly lose energy to frictional drag forces ***while remaining in an approximately circular orbit***. What happens to its orbit radius and speed?

a) Its orbit radius increases and its speed increases;

b) Its orbit radius increases and its speed decreases;

c) Its orbit radius decreases and its speed increases;

d) Its orbit radius decreases and its speed decreases;

e) There is no enough information to determine the change to its orbit radius and speed.

Briefly explain or justify your answer.

Problem 523. problems-1/gravitation-sa-escape-condition.tex

Answer the following short questions about escaping from a planet's gravitational field at its surface to "infinity". The answer to each is best given as an equation or short derivation or by a single sentence that correct captures the concept involved and explains or answers the question.

a) What is the ***condition*** for an object sitting on a planetary surface to escape to infinity?

b) Use the condition from part a) to derive (in a couple of lines of algebra) the escape ***speed*** from a planet of mass M and radius R. This is the smallest speed the object be moving with to escape to infinity.

c) Does it matter what direction the object leaves the surface (that is, does it have to leave travelling straight up or can it leave at an angle) as long as its path doesn't intersect the surface itself?

d) Assume that the planet is Earth, with mass M_e and radius R_e. Show that the escape speed from Earth can be written $v_e = \sqrt{2gR_e}$ where g is the usual gravitational field (acceleration) near the surface of the Earth.

Problem 524. problems-1/gravitation-sa-escape-condition-soln.tex

a) The escape condition is fundamentally $E_{\rm tot} = U + K = 0$, so that the object can reach $r \to \infty$ and arrive there at rest. This ($E_{\rm tot} \approx 0$) is also a good assumption to make for *any* object that falls to an attractor from far far away, for example for a falling asteroid or comet, for the purposes of estimation.

Hence:

$$E_{\rm tot} = \frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0$$

b)

$$v_e = \sqrt{\frac{2GM}{R}}$$

is both the (minimum) escape speed and a good estimate for the speed of e.g. a falling asteroid as it enters the Earth's atmosphere.

c) No. This is an energy condition, and does not depend on direction, as long as one doesn't run into something (like the planet itself) along the way!

d) Use $g = \dfrac{GM_e}{R_e^2}$ as follows:

$$v_e = \sqrt{\frac{2GM_e}{R_e} \times \frac{R_e}{R_e}} = \sqrt{2\left(\frac{GM_e}{R_e^2}\right) R_e} = \sqrt{2gR_e}$$

Problem 525. problems-1/gravitation-mc-kepler-and-scaling.tex

True or False:

a) Kepler's law of equal areas implies that gravity varies inversely with the square of the distance. **T** **F**

b) The planet closest to the sun on average (smallest semimajor axis) has the shortest period of revolution about the sun. **T** **F**

c) The acceleration of an apple near the surface of the earth, compared to the acceleration of the moon as it orbits the earth, is in the ratio of R_m/R_e, where R_m is the radius of the moon's orbit and R_e is the radius of the earth. **T** **F**

Problem 526. problems-1/gravitation-mc-period-of-missing-planet.tex

The Kepler project is surveying the night sky for stars with planets (and so far 1800 "exoplanets" have been discovered, with more being found every day). Suppose the Kepler telescope discovers that a gas giant similar to Jupiter (the easiest kind of planet to detect) is orbiting a particular star at a distance of 4 astronomical units (the radius of the Earth's orbit around the Sun). The period of the planet's orbit is determined to be 16 Earth years. What would the period of a possible Earth-like planet that was orbiting that star at 1 astronomical unit be?

a) 1/2 Earth year

b) 1 Earth year

c) $\sqrt{2}/2$ Earth years

d) 2 Earth years

e) 3 Earth years

Problem 527. problems-1/gravitation-mc-scaling-moon-orbit.tex

Planet Bongo has a moon, Mongo, that orbits it in a circular orbit much like the Moon orbits the Earth. You are told that

$$M_{\text{Bongo}} = 3M_{\text{Earth}} \qquad R_{\text{Bongo}} = 2R_{\text{Earth}} \qquad r_{\text{Mongo}} = 2r_{\text{Earth}}$$

where M is each planet's mass, R is its planetary radius, and r is the orbital radius of the respective moon about the center of its planet.

a) Compared to the speed of the Moon, the moon Mongo's speed is:

   A) larger;

   B) the same;

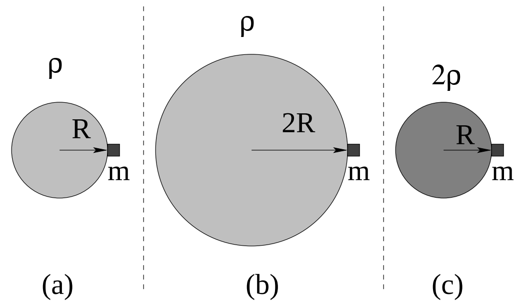
   C) smaller;

   D) unknown, as there is not enough information to decide;

b) Find the ratio of the period of the circular motion between the two:

$$\frac{T_{\text{Mongo}}}{T_{\text{Moon}}} = \boxed{}$$

Problem 528. problems-1/gravitation-mc-surface-gravity-scaling.tex



In the figure above, a small mass m is sitting on the surface of three planets. The density and radius of the planets are as shown:

a) ρ, R

b) $\rho, 2R$

c) $2\rho, R$

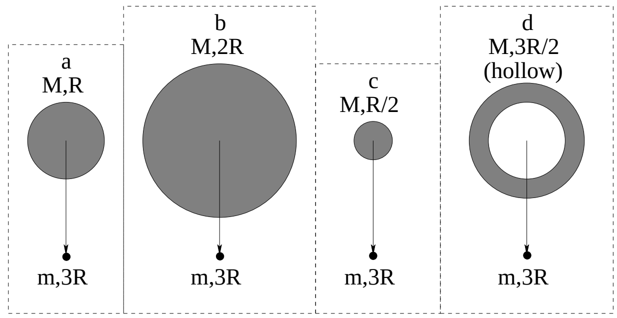
If the force on m due to gravity for the first planet is F_a, find and express F_b and F_c **in terms of $\boldsymbol{F_a}$**.

$F_b =$

$F_c =$

15.1.2 Ranking/Scaling

Problem 529. problems-1/gravitation-ra-four-planets.tex



(6 points) Four planets of mass M are drawn to scale above, each exerting a gravitational force of magnitude F_i (for $i = a, b, c, d$) on the small mass m at the position $3R$ from the center of each planet as shown. Rank the F_i from least to greatest including possible equalities. Indicate *why* you are answering the way that you answer in words or an equation or two.

15.1.3 Short Answer

Problem 530. problems-1/gravitation-sa-circular-orbit-vs-escape.tex

It is very costly (in energy) to lift a payload from the surface of the earth into a circular orbit, but once you are there, it only costs you that same amount of energy again to get from that circular orbit to anywhere you like – if you are willing to wait a long time to get there. Science Fiction author Robert A. Heinlein succinctly stated this as: "By the time you are in orbit, you're halfway to anywhere."

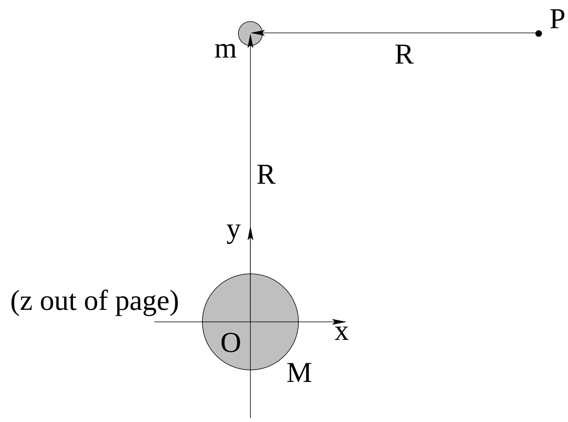
Prove this by comparing the total energy of a mass:

a) On the ground. Neglect its kinetic energy due to the rotation of the Earth.

b) In a (very low) circular orbit with at radius $R \approx R_E$ – assume that it is still more or less the same distance from the center of the Earth as it was when it was on the ground.

c) The orbit with minimal escape energy (that will arrive, at rest, "at infinity" after an infinite amount of time).

Problem 531. problems-1/gravitation-sa-force-and-torque.tex



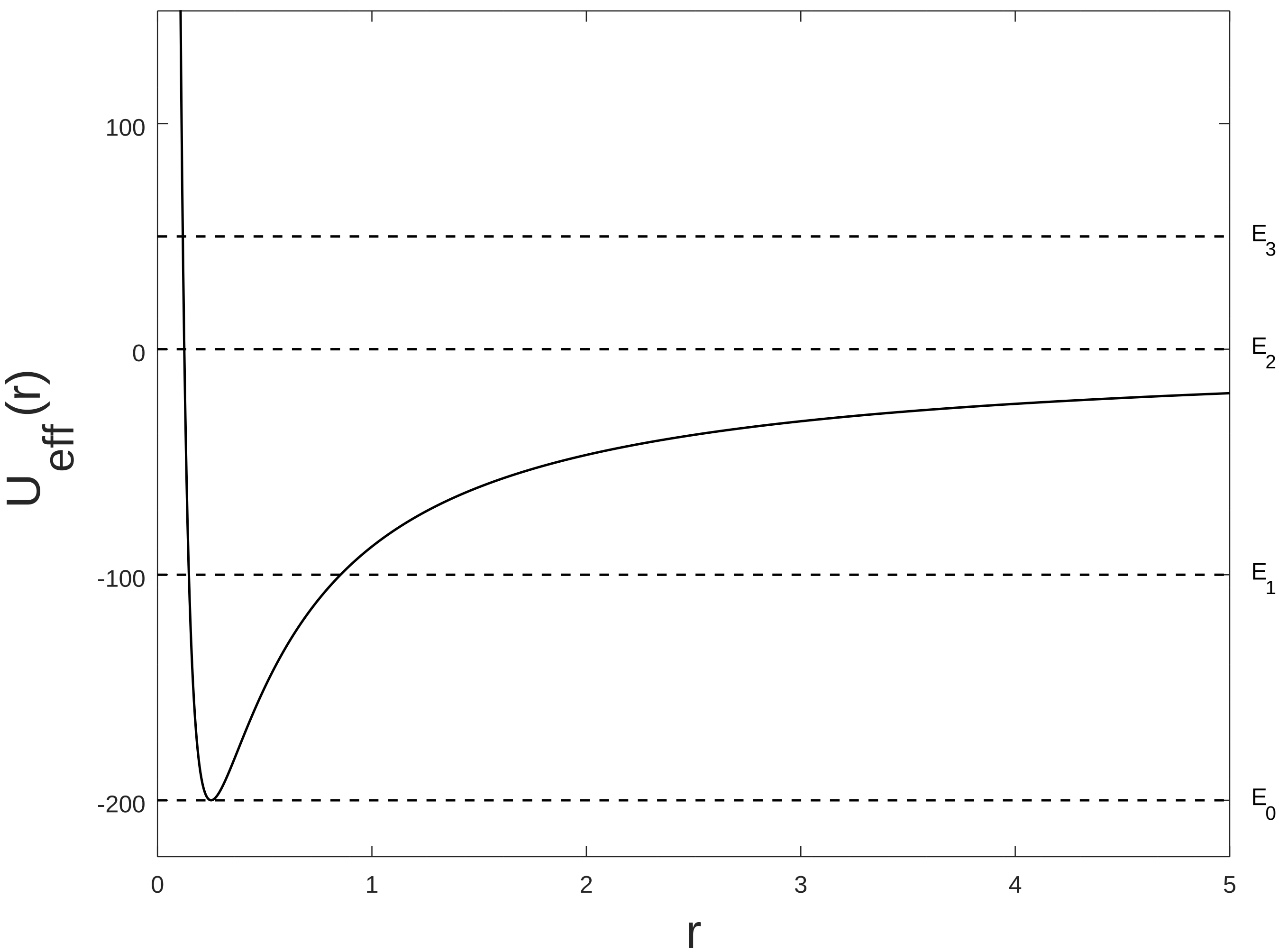
In the figure above, a mass M is located at the origin, and a mass m is located at $(0, R)$ as drawn. The z-axis in the figure comes ***out of the page***. All ***vector*** answers below may be indicated in any of the permissible ways.

a) Find the gravitional force acting on mass little m.

b) Find the torque around the origin O.

c) Find the torque on mass m relative to the pivot P. Draw and label an arrow symbol onto the figure above to explicitly indicate its direction

Problem 532. problems-1/gravitation-sa-identify-four-orbits.tex



The *effective radial potential* of a planetary object of mass m in an orbit around a star of mass M is:

$$U_{\text{eff}}(r) = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

The total energies E_0, E_1, E_2, E_3 of four orbits are drawn as dashed lines on the figure above for $G = 1$, $M = 100$, $m = 1$ and $L = 5$ (in some system of units). Name the kind of orbit (circular, elliptical, parabolic, hyperbolic) each energy represents and mark its turning point(s) in on the graph.

E_0 [ ]

E_1 [ ]

E_2

E_3

Problem 533. problems-1/gravitation-sa-kepler-3-circular-orbits.tex

In your homework, you studied several different cases of a mass m in a **circular orbit** around (or inside) another mass M, with different radial force laws. Suppose you are given a radial force law of the form:

$$\vec{\boldsymbol{F}} = -\frac{A}{r^n}\hat{\boldsymbol{r}}$$

Prove that (for circular orbits in particular):

$$r^{n+1} = CT^2$$

where T is the period of the orbit and r is the radius of the circle, and find the constant C. ($A = GMm$, $n = 2$ then leads to Kepler's third law, and $A = GMm/R^3$, $n = -1$ leads to the relation you derived for the mass in the tunnel through the death star).

Problem 534. problems-1/gravitation-sa-period-of-saturn.tex

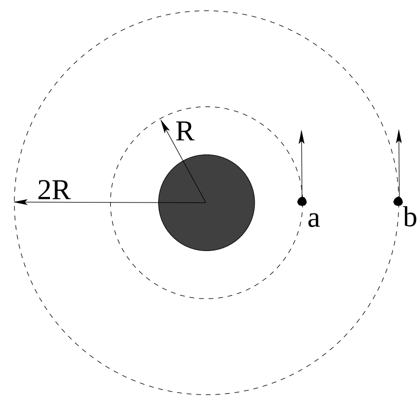
The earth's orbit is "one astronomical unit" (AU) in radius (this turns out to be about 150 million kilometers). The period of its orbit is one year. The mean radius of Saturn's orbit is (roughly) 10 AU. What is its "year" (period of revolution around the sun) in years? (You may express your answer as a power of a number without a calculator.)

Problem 535. problems-1/gravitation-sa-speed-of-jupiter.tex

The Earth's approximately circular orbit about the Sun is "one astronomical unit" (AU) in radius (this turns out to be about 150 million kilometers). The mean radius of Jupiter's approximately circular orbit is (roughly) 5 AU. What is the average speed of Jupiter $v_{jupiter}$ in terms of the average speed of the Earth v_{earth} as it moves around the Sun?

$\boldsymbol{v_{jupiter}} =$ $\boxed{}$

Problem 536. problems-1/gravitation-sa-two-orbits-scaling.tex



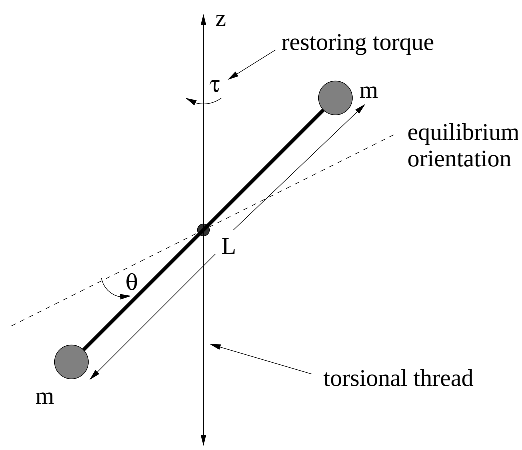
Two satellites are in circular orbits around the earth, one at radius R and the other at $2R$.

a) **Circle** the satellite that is moving **faster**.

b) How *much* faster is it moving? (Express the faster satellite's speed in terms of the speed of the slower satellite.)

15.1.4 Long Problems

Problem 537. problems-1/gravitation-pr-cavendish-torsional-oscillator.tex



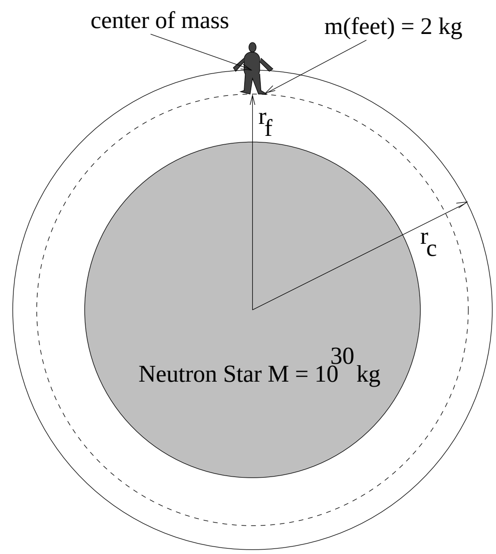
In the Cavendish experiment, the gravitational force is measured between two big masses M (not shown) acting on two small masses m on a rod of length L (assumed to be of negligible mass in this problem, although it isn't really) attached to a thin thread such that it makes a torsional pendulum (as drawn above). The twisting thread exerts a restoring torque of magnitude $\tau = -\kappa\theta$ on the rod connecting the small masses, where theta is measured from the equilibrium angle of the rod as shown.

In the experiment the two large masses are placed symmetrically so that they exert a torque on the small mass arrangement aligned with the torsional thread. The two small masses twist the thread toward the big masses until the gravitational torque is balanced by the torque of the thread. *If* κ is known, a measurement of the angle of deviation θ_0 suffices to determine the gravitional torque, hence the gravitational force, hence the gravitational constant G.

There's only one catch – one needs κ, and most spools of thread don't come labeled with their torsional response properties.

Show and tell how you can do a simple experiment to measure κ with nothing but an accurate stop watch, a measurement of the mass(es) m, and a measurement of the length L of the connecting rod. (Describe the experiment and derive the relation between the quantity you choose to measure and the desired result, κ).

Problem 538. problems-1/gravitation-pr-dangerous-tides.tex

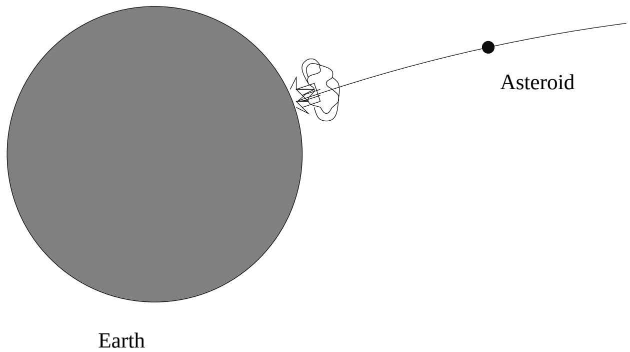


Tides can be dangerous. You are a scientist in orbit around a neutron star with a mass $M = 10^{30}$ kg and a radius of 8 km. Your center of mass moves in a perfect circle 10 km around the center of the star. You have just enough angular momentum that your feet always point "down" toward the center of the star and your head points away. Your feet are therefore also in a circular trajectory around the center of the star, but they cannot also be in orbit (free fall).

Assuming that your feet have a mass of approximately 2 kg and are located approximately 1 meter closer to the star than your center of mass, how much force do your legs have to provide to keep your feet from falling off? Do they fall off?

Hints: Proceed by finding the centripetal acceleration/force of your center of mass in terms of the gravitational field/force of the star at that location. Repeat this for your feet separately, assuming that they have the same angular frequency of circular motion as your center of mass but are in a (much!) stronger gravitational field. The difference in the force required to keep the feet in a circular orbit (the total centripetal force) and the actual gravitational force must be provided by your legs. Also, the binomial expansion might well be useful here...

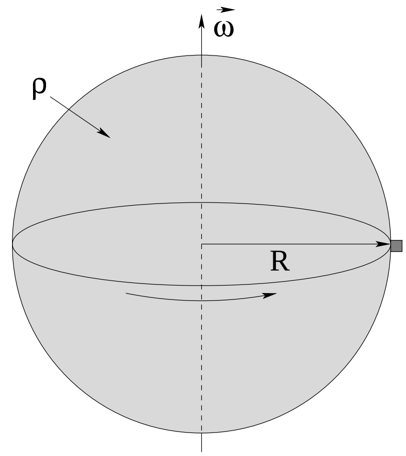
Problem 539. problems-1/gravitation-pr-dinosaur-killer-asteroid.tex



Estimate the total energy released when a spherical "Dinosaur Killer" asteroid with a density $\rho = 10$ kg/m^3 and radius $R = 1000$ meters falls onto the surface of the earth from "outer space" (far away). Obviously your answer should be justified by a good physical argument.

Note that this is a *lot* of energy – more than enough to wipe out all life within perhaps 1000 km of the point of impact (or more) and to change the climate of the planet.

Problem 540. problems-1/gravitation-pr-equatorial-weight.tex



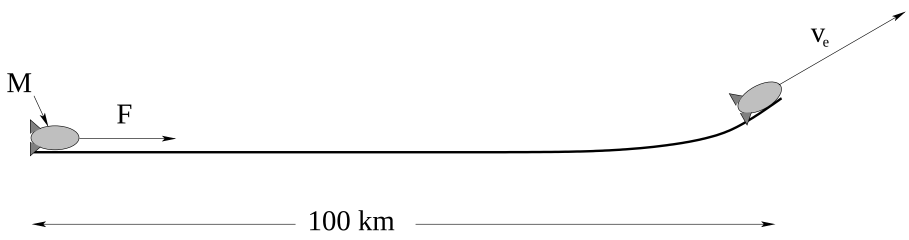
In the figure above, a planet with uniform mass density ρ and radius r rotates at a constant angular velocity $\vec{\omega}$ around its N-S axis. A small block of mass m is located on the planet's equator and at the instant shown is at rest relative to the surface (meaning that it too is rotating around the axis with constant angular velocity $\vec{\omega}$). Express all answers in terms of G, ρ, m and ω as needed or appropriate.

a) Draw the forces acting on the block into the picture, assuming that the planet's rotation is slow enough for the block to remain on the surface.

b) Derive an expression for the apparent weight of the block as a function of ω (the magnitude of the angular velocity).

c) If the planet's rotational speed is very slowly increased, at some point the normal force will go to zero. Find T_0, the period of the planet's rotation when this occurs.

Problem 541. problems-1/gravitation-pr-escape-velocity-linear-accelerator.tex



One way to reduce the cost of lifting mass into orbit is to use a linear accelerator to drive a payload up to escape velocity (or thereabouts) and then let it go. This way one doesn't have to lift the fuel used to lift the fuel used to lift the ... (almost all the fuel used in a rocket is used to lift fuel, not payload).

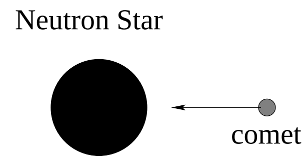
Assume that fusion energy has been developed and electricity is cheap, and that high temperature superconductors have made such a mass driver feasible. Your job is to do a first estimate of the design parameters.

A proposed plan for the mass driver is shown above. The track is 100 kilometers long and slopes gently upwards. The payload capsule has a mass of 2×10^3 kg (two metric tons). The head of the track is high in the Andes, $R = 6375$ kilometers from the center of the earth.

a) Neglecting air resistance, find the escape velocity for the capsule. Although bound orbits will not require quite as much energy, air resistance will dissipate some energy. Either way, this is a reasonable estimate of the velocity the driver must be able to produce.

b) Assuming that the capsule is started from rest and that a constant tangential force accelerates it, find the tangential force necessary to achieve escape velocity at the end of the track. Note: Ignore the normal force that the track must exert to divert it so that it departs at an upward angle.) From this find the acceleration of the capsule, in multiples of g. Is this acceleration likely to be tolerable to humans?

Problem 542. problems-1/gravitation-pr-escape-velocity-neutron-star.tex



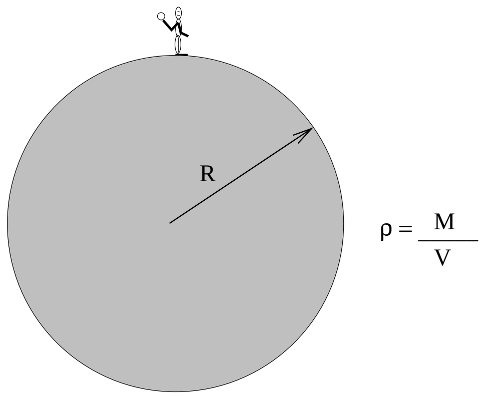
Cool stuff about gravity. A neutron star has a mass $M = 10^{30}$ kg and a radius R of 8 km. Answer the following problems **algebraically** using the variables M, m, G, R **first**, then (if you have a calculator handy or can do the arithmetic by hand) do the arithmetic and put down numbers. You can get full credit from the algebra alone, but the number answers are pretty interesting.

a) What is the escape velocity from the surface of the neutron star? (If you do the arithmetic, express the result as a fraction of c, the speed of light: $c = 3 \times 10^8$ m/sec).

b) A comet with a mass $m = 10^{14}$ kg falls from infinity into the neutron star. What is the energy liberated as it (inelastically) hits?

c) Compare this energy to the total (rest) mass energy of the comet, mc^2.

Problem 543. problems-1/gravitation-pr-escape-velocity-of-baseball.tex



Suppose that planetary rock has an average density ρ_p. Assuming that you can throw a fastball in baseball at v_f find an expression (in terms of G, ρ, v_e and known constants) for he radius R of the largest spherical planet where you can stand on the surface and throw a baseball away ***to "infinity"*** (so that it never comes back)?

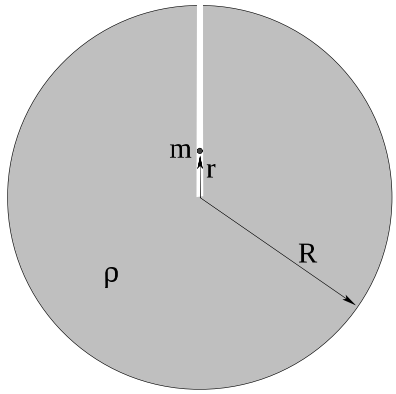
If you want to have fun or "check" your algebra, try evaluating this expression for $v_f = 40$ m/sec (nearly 90 mph) and $\rho_p = 10^4$ kg/m^3. I get around 17 km, making the planet just about 10 miles in radius. The same expression could be used to find the largest planet you could *jump off of* (assuming you have a vertical leap of 1 meter on Earth).

Problem 544. problems-1/gravitation-pr-geosync-orbit.tex

The Duke Communications company wants to put a satellite into a circular geosynchronous orbit over the equator (this is a satellite whose period is exactly one day, so that it stays over the same point of the rotating Earth).

Ignoring perturbations like the Moon and the Sun, find the radius R_g of such an orbit as a multiple of the radius of the Earth R_e. Although *as always* you should *solve for the result algebraically first* you may wish to know some of the following data: The radius of the Moon's orbit is $R_m = 384,000$ kilometers, or $R_m = 60R_e$. The period of the Moon is $T_m = 27.3$ days compared to $T_g = 1$ day. $R_e = 6400$ kilometers. $M_e = 6 \times 10^{24}$ kilograms. One day contains 86400 seconds.

Problem 545. problems-1/gravitation-pr-half-tunnel-escape.tex



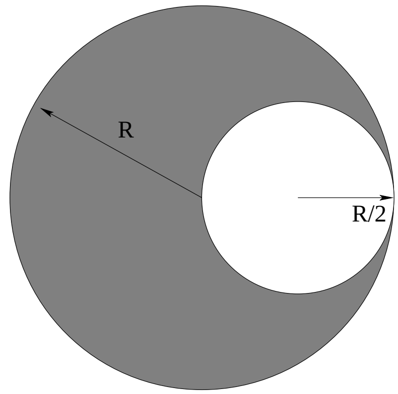
In the figure above a spherical planet of uniform density ρ and total radius R is shown. A small tunnel is drilled from the surface to the center.

a) Find the magnitude of the gravitational field $g(r)$ in the tunnel as a function of r.

b) How much work is required to lift a mass m at a constant speed from the center of this planet to the surface?

c) Suppose the mass m has reached the surface of the planet and is at rest. What upward-directed speed must you give the mass m at the surface so that the mass escapes from the planet altogether?

Problem 546. problems-1/gravitation-pr-planet-with-spherical-hole.tex



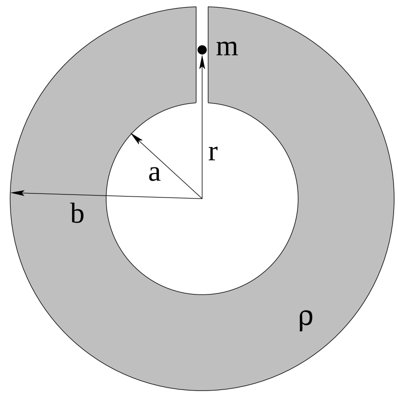
Above is pictured a spherical mass with radius R and mass density (mass per unit volume) ρ. It has a spherical hole cut out of it of radius $R/2$ as shown. Find the gravitational field in the hole in terms of G, R, and ρ, proving that it is uniform and points to the left.

Problem 547. problems-1/gravitation-pr-spherical-cow.tex

There is an old physics joke involving cows, and you will need to use its punchline to solve this problem.

A cow is standing in the middle of an open, flat field. A plumb bob with a mass of 1 kg is suspended via an unstretchable string 10 meters long so that it is hanging down roughly 2 meters away from the center of mass of the cow. Making any reasonable assumptions you like or need to, *estimate* the angle of deflection of the plumb bob from vertical due to the gravitational field of the cow.

Problem 548. problems-1/gravitation-pr-thick-shell-force.tex



A "thick" shell of mass with uniform mass density ρ, inner radius a, and outer radius b is shown. A small (frictionless) hole has been drilled at the top along the z axis, and a mass m is at a distance r from the center of the shell along the z axis so that it can be moved vertically up or down from outside of the shell to the inside by means of the tunnel.

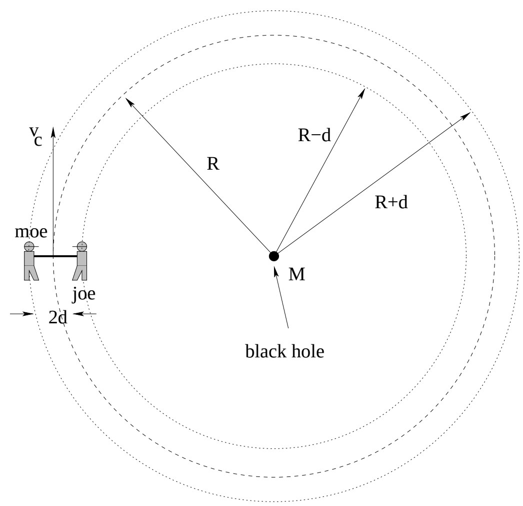
Find an expression for the magnitude of the radial force F_r acting on m when the mass is:

a) Outside of the shell of mass entirely, at some $r > b$.

b) In the tunnel, where $a < r < b$.

c) Inside the shell, at some point $r < a$.

Problem 549. problems-1/gravitation-pr-tides-moe-and-joe.tex



Moe and Joe, who have identical masses m, are in a circular orbit around a ***black hole*** about the size of a marble, which contains roughly the same mass M as the earth, in the orientation shown above. The radius of the orbit of their center of mass is R (which we'll assume is much larger than the BH). Moe and Joe and tied with a very strong rope $2d$ meters long (with $d \ll R$) that keeps them moving around the Black Hole at the *same* angular speed as their center of mass. Alas, this means that neither Joe nor Moe are actually in orbit (free fall) so the rope has to exert a force to keep them moving with their center of mass. Find:

a) The speed v_c of their center of mass in the circular orbit, as well as its angular speed ω_c, as a function of G, M, and R. This is just an ordinary circular orbit problem, don't make it overcomplicated.

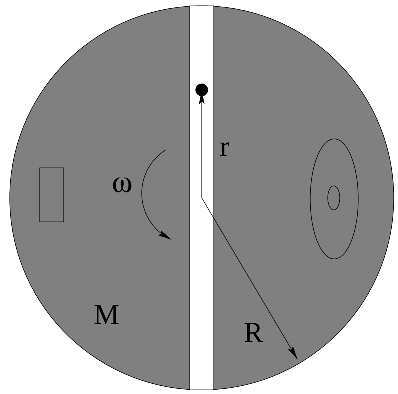
b) If Joe (closer to the BH) is moving in a circular trajectory with radius $R - d$ and the *same* angular velocity that you obtained in a) as the orbital angular velocity correct for radius R, what is the *net* force that must be exerted on Joe by the BH and the rope together?

c) What is the force exerted on Joe by the BH alone at this radius?

d) Therefore, what must the tension T be in the rope (still as a function of G, M, m, R and d)?

This "force" (opposed by the tension T) is the ***tide***.

Problem 550. problems-1/gravitation-pr-tunnel-through-death-star-orbit.tex



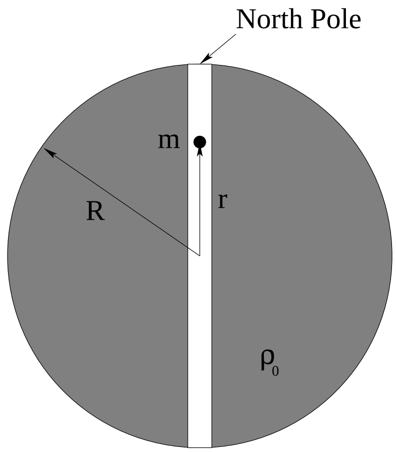
A straight, smooth (frictionless) transit tunnel is dug through a spherical asteroid of radius R and mass M that has been converted into Darth Vader's *death star*. The tunnel is in the equatorial plane and passes through the center of the death star. The death star moves about in a hard vacuum, of course, and the tunnel is open so there are no drag forces acting on masses moving through it.

a) Find the force acting on a car of mass m a distance $r < R$ from the center of the death star.

b) You are commanded to find the precise rotational frequency of the death star ω such that objects in the tunnel will orbit *at* that frequency and hence will appear to *remain at rest* relative to the tunnel at any point along it. That way Darth can Use the Dark Side to move himself along it almost without straining his midichlorians. In the meantime, he is reaching his crooked fingers towards you and you feel a choking sensation, so better start to work.

c) Which of Kepler's laws does your orbit satisfy, and why?

Problem 551. problems-1/gravitation-pr-tunnel-through-planet-oscillator.tex



A straight, smooth (frictionless) transit tunnel is dug through a planet of radius R whose mass density ρ_0 is constant. The tunnel passes through the center of the planet and is lined up with its axis of rotation (so that the planet's rotation is **irrelevant** to this problem). All the air is evacuated from the tunnel to eliminate drag forces.

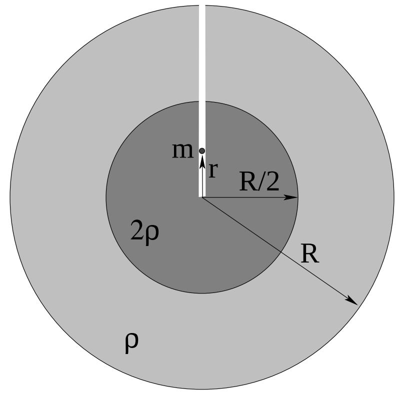
a) Find the force acting on a car of mass m a distance $r < R$ from the center of the planet.

b) Write Newton's second law for the car, and extract the differential equation of motion. From this find $r(t)$ for the car, assuming that it starts at $r_0 = R$ on the North Pole at time $t = 0$.

c) How long does it take the car to get to the center of the planet starting from rest at the North Pole? How long does it take if one starts half way down to the center? Comment.

All answers should be given in terms of G, ρ_0, R and m (or in terms of quantities you've already defined in terms of these quantities, such as ω).

Problem 552. problems-1/gravitation-pr-two-densities-difficult.tex

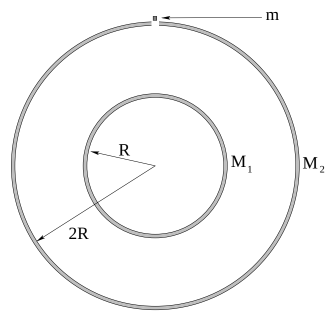


In the figure above a spherical planet of total radius R is shown that has a spherical iron core with radius $R/2$ and density 2ρ surrounded by a (liquid) rock mantle with density ρ.

a) Find the gravitational field $\vec{g}(r)$ as a function of the distance from the center.

b) Suppose a small, well-insulated tunnel were drilled all the way to the center. How much work is required to lift a mass m from the center to the surface?

Problem 553. problems-1/gravitation-pr-two-spherical-shells.tex



A hollow spherical mass shell of mass M_1 and radius R is inside another hollow spherical mass shell of mass M_2 and radius $2R$. The shells are concentric and of negligible thickness.

a) A small mass m is placed on the outer surface of the bigger shell M_2. Calculate its acceleration due to gravity g_2 in terms of the shell masses M_1 and M_2, G and R.

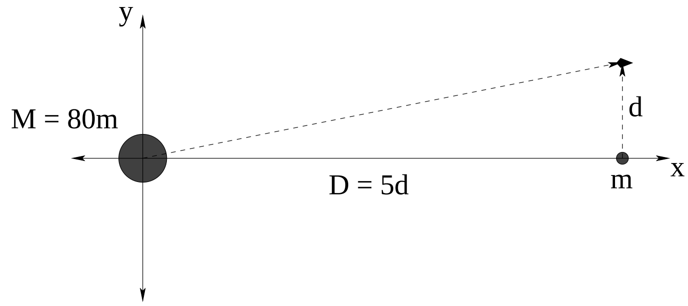
b) The mass m is placed on the outer surface of the smaller shell M_1. Its acceleration due to gravity g_1 is measured and found to be the same as the value of g_2 from part (a). Use this equality of g_1 and g_2 to express M_2 in terms of M_1, G and R.

c) With the relationship you have just derived between M_1 and M_2, compute the gravitational potential energy P_1 of a mass m on the outer surface of the bigger shell. Express P_1 in terms of G, m, M_1 and R, using the convention that the gravitation potential is defined as zero at infinite radius.

d) Compute the *change* in gravitational potential energy ΔP as the mass m moves from its position on the outer surface of M_2 to a position on the outer surface of of M_1 (being lowered through the small hole in the outer shell). Is the potential energy larger (more positive) at R or $2R$?

e) If an object is *dropped from rest* through the hole in the bigger shell, what is its speed when it hits the smaller shell? You may give this answer in terms of ΔP so that you can get it right even if you get (d) wrong.

Problem 554. problems-1/gravitation-pr-vector-field-two-masses.tex



The large mass above is the Earth, the smaller mass the Moon. Find an expression for the **vector gravitational field** acting on the spaceship on its way from Earth to Mars (swinging past the Moon at the instant drawn) in the picture above, in terms of M, m and d. Remember, magnitude *and* direction!