Review Problems
for
Introductory Physics 2

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Robert G. Brown, Instructor
Duke University Physics Department
Durham, NC 27708-0305
rgb@phy.duke.edu

Contents

Chapter 1

Preface

The problems in this review guide are provided as is ***without any guarantee of being correct!*** That's not to suggest that they are all broken – on the contrary, most of them are well-tested and have been used as homework, quiz and exam problems for decades if not centuries. It is to suggest that they have typos in them, errors of other sorts, bad figures, and one or two of them are really too difficult for this course but haven't been sorted out or altered to make them doable.

Leaving these in just adds to the fun. Physics problems are *not* all cut and dried; physics itself isn't. One thing you should be building up as you work is an appreciation for what is easy, what is difficult, what is correct and what is incorrect. If you find an error and bring it to my attention, I'll do my best to correct it, of course, but in the meantime, be warned!

A *few* of the problems have rather detailed solutions (due to Prof. Ronen Plesser and myself), provided as examples of how a really good solution might develop, with considerable annotation. However, most problems do not have included solutions and *never will have.* I am actually philosophically opposed to providing students with solutions that they are then immediately tempted to memorize. This guide is provided so that you can learn to solve problems and work sufficiently carefully that they can trust the solutions.

Students invariably then ask: "But how are we to know if we've solved the problems correctly?"

The answer is simple. The same way you would *in the real world!* Work on them in groups and check your algebra, your approach, and your answers against one another's. Build a consensus. Solve them with mentoring (course TAs, professors, former students, tutors all are *happy* to help you). Find answers through research on the web or in the literature.

To be honest, almost any of the ways that involve *hard work on your part* are good ways to learn to solve physics problems. The only *bad* way to (try to) learn is to have the material all laid out, cut and dried, so that you don't have to *struggle* to learn, so that you don't have to *work hard* and thereby *permanently* imprint the knowledge on your brain as you go. Physics requires engagement and investment of time and energy like no subject you have ever taken, if only because it is one of the most difficult subjects you've ever tried to learn (at the same time it is remarkably simple, paradoxically enough).

In any event, to use this guide most effectively, first skim through the whole thing to see what is there, then start in at the beginning and work through it, again and again, reviewing repeatedly all of the problems and material you've covered so far as you go on to what you are working on currently in class and on the homework and for the upcoming exam(s). Don't be afraid to solve problems more than once, or even more than three or four times.

And *work in groups!* Seriously! With pizza and beer...

Chapter 2

Short Math Review Problems

The problems below are a diagnostic for what you are likely to need in order to work physics problems. There aren't really enough of them to constitute "practice", but if you have difficulty with *any* of them, you should probably find a math review (there is usually one in almost any introductory physics text and there are a number available online) and work through it.

Weakness in geometry, trigonometry, algebra, calculus, solving simultaneous equations, or general visualization and graphing will all negatively impact your physics performance and, if uncorrected, your grade.

Short Problem 1.

problems/short-math-binomial-expansion.tex

Write down the **binomial expansion** for the following expressions, given the conditions indicated. FYI, the binomial expansion is:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

where x can be positive or negative and where n is any real number and only converges if $|x| < 1$. Write at least the first *three non-zero* terms in the expansion:

a) For $x > a$:
$$\frac{1}{(x+a)^2}$$

b) For $x > a$:
$$\frac{1}{(x+a)^{3/2}}$$

c) For $x > a$:
$$(x+a)^{1/2}$$

d) For $x > a$:
$$\frac{1}{(x+a)^{1/2}} - \frac{1}{(x-a)^{1/2}}$$

e) For $r > a$:
$$\frac{1}{(r^2 + a^2 - 2ar\cos(\theta))^1/2}$$

Short Problem 2.

problems/short-math-differentiate-expressions.tex

Evaluate the following expressions:

a)
$$\frac{d}{dt}(at^5 + be^{-ct^2} + \sin(dt)) =$$

b)
$$\frac{d}{dt}e^{\alpha t} =$$

c)
$$\frac{d}{dt}e^{\alpha t^2} =$$

d)
$$\frac{d}{dt}\tan(\omega t) =$$

Short Problem 3.

problems/short-math-elliptical-trajectory.tex

The position of a particle as a function of time is given by:

$$\vec{x}(t) = x_0 \cos(\omega t)\hat{x} + y_0 \sin(\omega t)\hat{y}$$

where $x_0 > y_0$.

a) What is $\vec{v}(t)$ for this particle?

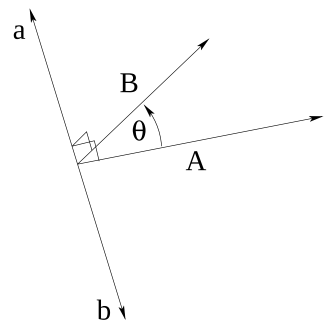
b) What is $\vec{a}(t)$ for this particle?

c) Draw a generic plot of the trajectory function for the particle. What kind of shape is this? In what direction/sense is the particle moving (indicate with arrow on trajectory)?

d) Draw separate plots of $x(t)$ and $y(t)$ on the same axes.

Short Problem 4.

problems/short-math-evaluate-vector-products.tex



Evaluate the following vector expressions.

a) Express the dot product in terms of its Cartesian components e.g. $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}$:

$$\vec{A} \cdot \vec{B} =$$

b) Express the dot product in terms of the magnitudes A, B and θ:

$$\vec{A} \cdot \vec{B} =$$

c) Express the cross product in terms of its Cartesian components e.g. $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}$ (this has a lot of terms):

$$\vec{A} \times \vec{B} =$$

d) Express the *magnitude* of cross product in terms of the magnitudes A, B and θ and indicate its direction:

$$\vec{A} \times \vec{B} =$$

Short Problem 5.

problems/short-math-integrate-expressions.tex

Evaluate the following (indefinite) integrals. Don't forget the constant of integration!

a)
$$\int \sin(\theta) d\theta =$$

b)
$$\int \cos(\theta) d\theta =$$

c)
$$\int \sin^3(\theta) d\theta =$$

d)
$$\int e^{i\omega t} dt =$$

e)
$$\int \cos(\omega t) dt =$$

f)
$$\int x^n dx =$$

g)
$$\int \frac{1}{x} dx =$$

h)
$$v(t) = \int -g dt =$$

i)
$$x(t) = \int (-\frac{1}{2} g t + v_0) dt =$$

Short Problem 6.

problems/short-math-solve-simple-equations.tex

Solve for t:

a)
$$v_0 t - x_0 = 0$$

b)
$$-\frac{1}{2}gt^2 + v_0 t = 0$$

c)
$$-\frac{1}{2}gt^2 + v_0 t + x_0 = 0$$

Short Problem 7.

problems/short-math-solve-simultaneous-equations.tex

Solve the following system of simultaneous equations for x, y and z:

$$5x + 5y = 10$$

$$5x - 2z = 4$$

$$2z - y = 0$$

Short Problem 8.

problems/short-math-sum-two-vectors-1.tex

(3 points) Vector $\vec{\boldsymbol{A}} = 3\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$. Vector $\vec{\boldsymbol{B}} = -7\hat{\boldsymbol{x}} - 3\hat{\boldsymbol{y}}$. The vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$:

a) is in the first quadrant (x+,y+) and has magnitude 7.

b) is in the second quadrant (x-,y+) and has magnitude 7.

c) is in the second quadrant (x-,y+) and has magnitude 5.

d) is in the fourth quadrant (x+,y-) and has magnitude 5.

e) is in the third quadrant (x-,y-) and has magnitude 6.

Short Problem 9.

problems/short-math-sum-two-vectors.tex

(3 points) Vector $\vec{\boldsymbol{A}} = -4\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$. Vector $\vec{\boldsymbol{B}} = 9\hat{\boldsymbol{x}} + 6\hat{\boldsymbol{y}}$. The vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$:

a) is in the first quadrant (x+,y+) and has magnitude 17.

b) is in the fourth quadrant (x+,y-) and has magnitude 12.

c) is in the first quadrant (x+,y+) and has magnitude 13.

d) is in the second quadrant (x-,y+) and has magnitude 17.

e) is in the third quadrant (x-,y-) and has magnitude 13.

Short Problem 10.

problems/short-math-taylor-series.tex

Evaluate the first *three nonzero terms* for the **Taylor series** for the following expressions. Expand about the indicated point:

a) Expand about $x = 0$:
$$(1 + x)^n \approx$$

b) Expand about $x = 0$:
$$\sin(x) \approx$$

c) Expand about $x = 0$:
$$\cos(x) \approx$$

d) Expand about $x = 0$:
$$e^x \approx$$

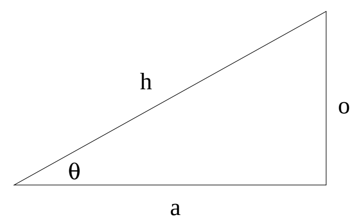
e) Expand about $x = 0$ (note: $i^2 = -1$):
$$e^{ix} \approx$$

Verify that the expansions of both sides of the following expression match:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Short Problem 11.

problems/short-math-trig-diagnostic.tex



Fill in the following in terms of the marked sides:

a)
$$\sin(\theta) =$$

b)
$$\cos(\theta) =$$

c)
$$\tan(\theta) =$$

Chapter 3

Essential Laws, Theorems, and Principles

The questions below guide you through basic physical laws and concepts. They are the stuff that one way or another you should know" going into any exam or quiz following the lecture in which they are covered. Note that there aren't really all that many of them, and a lot of them are actually easily derived from the most important ones.

There is absolutely no point in memorizing solutions to all of the problems in this guide. In fact, for all but truly prodigious memories, memorizing them all would be impossible (presuming that one could work out all of the solutions into an even *larger* book to memorize!). However, *every student* should memorize, internalize, learn, *know* the principles, laws, and and theorems covered in this section (and perhaps a few that haven't yet been added). These are things upon which all the rest of the solutions are based.

Short Problem 1.

problems/true-facts-ALMDC.tex

What is Ampere's Law **with** the Maxwell displacement current? (Equation and figure, please; circle the displacement current *only*.)

Short Problem 2.

problems/true-facts-AL.tex

What is Ampere's Law? (Equation and figure please. Your answer does not need to have the Maxwell Displacement Current in it yet, as it has not yet been covered in class, but if you put it in (correctly) anyway, you may have two points of extra credit.)

Short Problem 3.

problems/true-facts-Biot-Savart-law.tex

What is the Biot-Savart Law? (Equation and figure, please.)

Short Problem 4.

problems/true-facts-Coulombs-Law.tex

What is Coulomb's Law? (Equation and figure ok, or in words.)

Short Problem 5.

problems/true-facts-critical-angle.tex

What is the formula for the critical angle (of total internal reflection) for a beam of light going from medium 1 to medium 2 with $n_1 > n_2$? (Equation and figure, please.)

Short Problem 6.

problems/true-facts-definition-of-capacitance.tex

What is the defining relation for the capacitance of an arrangement of charge

Q stored at a potential V? (Equation.)

Short Problem 7.

problems/true-facts-energy-density-of-b-field.tex

What is the energy density η_M of the magnetic field $\vec{B}$? (Equation.)

Short Problem 8.

problems/true-facts-energy-density-of-e-field.tex

What is the energy density η_E of the electric field $\vec{E}$? (Equation.)

Short Problem 9.

problems/true-facts-energy-in-capacitor.tex

Give three equivalent expressions for the total energy stored on a capacitor in terms of any two of Q, C, V (at a time).

Short Problem 10.

problems/true-facts-fermat-principle.tex

What is the Fermat Principle (from which e.g. Snell's Law follows)? (Statement in words, possible with illustrating figure.)

Short Problem 11.

problems/true-facts-FL.tex

What is Faraday's Law? Circle Lenz's Law within. (Equation and figure, please.)

Short Problem 12.

problems/true-facts-force-on-dipole-in-uniform-electric-field.tex

What is the force $\vec{F}$ acting on an electric dipole $\vec{p}$ in a *uniform* electric field $\vec{E}$? (Equation and figure, please.)

Short Problem 13.

problems/true-facts-fraunhofer-diffraction.tex

What is Fraunhofer diffraction? (Short statement in words, plus picture.)

Short Problem 14.

problems/true-facts-fresnel-diffraction.tex

What is Fresnel diffraction? (Short statement in words, plus picture.)

Short Problem 15.

problems/true-facts-GLE.tex

What is Gauss's Law for Electricity? (Equation and figure, please.)

Short Problem 16.

problems/true-facts-GLM.tex

What is Gauss's Law for Magnetism? What important experimental result does it represent? (Equation and figure, please.)

Short Problem 17.

problems/true-facts-index-of-refraction-dispersion.tex

What is the "dispersion" of refracted light passing through some medium? (Short statement, but please relate the phenomenon in some way to a particular property of the **index of refraction** of the medium.)

Short Problem 18.

problems/true-facts-intensity-of-dipole-source.tex

What is the formula for the intensity of radiation from an oscillating dipole oriented along the $\hat{\boldsymbol{z}}$-direction, as a function of r and θ, in the limit that one is far from the dipole? (Equation and figure, please.)

Short Problem 19.

problems/true-facts-kirchoffs-rules.tex

What are Kirchoff's Rules? Also (and still for credit!) what physical principle does each rule corresponds to?

a)

b)

Short Problem 20.

problems/true-facts-lensmakers-formula.tex

What is the "lensmakers formula" that predicts the focal length of a lens made of a material with index of refraction n in air (with index of refraction approximately equal to 1), given the radii of curvature r_1 and r_2 of its two refracting surfaces? (Equation and figure, please.)

Short Problem 21.

problems/true-facts-lorentz-force-law.tex

What is the "Lorentz Force Law" (the law that relates the electromagnetic force on a charged particle q moving at velocity $\vec{v}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$? (Give equation and draw a figure with vE, $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{v}}$ to illustrate the relative directions).

Short Problem 22.

problems/true-facts-magnetic-force-charge-particle.tex

What is the net force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$? (Give equation and draw a figure to illustrate its relative direction.)

Short Problem 23.

problems/true-facts-malus-law.tex

If vertically polarized light of intensity I_0 strikes a polarizing filter with a transmission axis at an angle θ with respect to the vertical, what is the intensity of the transmitted light (Malus' Law)? (Equation and figure, please.)

Short Problem 24.

problems/true-facts-near-point.tex

What is the near point distance of the human eye (both what *is* it and what is its presumed/approximate value)? (Answer in words and of course a length in appropriate units.)

Short Problem 25.

problems/true-facts-ohms-law.tex

What is Ohm's Law? (Equation.)

Short Problem 26.

problems/true-facts-paraxial-mirror-equation.tex

What is the equation one uses to locate the image of an object a distance s in front of a spherical mirror with radius r (in the limit that the rays are all paraxial, that is the object height $y \ll r$ for an object on the axis)? (Equation and figure, please.)

Short Problem 27.

problems/true-facts-potential-energy-dipole-b-field.tex

What is the potential energy of a magnetic dipole $\vec{m}$ in a magnetic field $\vec{B}$? (Equation and figure, please.)

Short Problem 28.

problems/true-facts-potential-energy-dipole-e-field.tex

What is the potential energy of an electric dipole $\vec{p}$ in an electric field $\vec{E}$? (Equation and figure, please.)

Short Problem 29.

problems/true-facts-poynting-vector-intensity.tex

What is the Poynting vector? (Equation and figure, please.) What do we call its absolute magnitude? (Single word.)

Short Problem 30.

problems/true-facts-rayleigh-criterion-for-resolution.tex

What is the Rayleigh criterion for resolution (of, say, two diffraction limited images of stars seen through a telescope or viruses seen through a microscope)? (Equation and figure, please.)

Short Problem 31.

problems/true-facts-snells-law.tex

What is Snell's Law? (Equation and figure, please.)

Short Problem 32.

problems/true-facts-speed-of-light-formula.tex

What is the speed of light in terms of $\epsilon - 0$ and μ_0? (Equation.)

Short Problem 33.

problems/true-facts-thin-lens-equation.tex

What is the "thin lens equation"? (Equation and figure, please.)

Short Problem 34.

problems/true-facts-torque-on-dipole-in-electric-field.tex

What is the torque $\vec{\tau}$ acting on an electric dipole $\vec{p}$ in an electric field $\vec{E}$? (Equation and figure, please.)

Short Problem 35.

problems/true-facts-transformer.tex

A transformer has N_1 turns in its primary winding and N_2 turns in its secondary winding. If it has an alternating voltage V_1 applied across its primary, what would one expect to measure for V_2 across its secondary? (Equation and schematic/figure, please.)

Short Problem 36.

problems/true-facts-transverse-wave-E-and-B.tex

What is the relation between $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ in a transverse electromagnetic wave? (Equation relating amplitudes, figure indicating directions including direction of wave propagation.)

Short Problem 37.

problems/true-facts-waves-in-medium.tex

If a beam of light has speed c, frequency f, and wavelength λ in a vaccuum, what are its speed, frequency and wavelength in a medium with index of refraction n? (Equations for v_n, f_n and λ_n.)

Chapter 4

Problem Solving

The following problems are, at last, the meat of the matter: serious, moderately to extremely difficult physics problems. An A" student would be able to construct *beautiful solutions*, or almost all, of these problems.

Note well the phrase beautiful solutions". In no case is the answer" to these problems an equation, or a number (or set of equations or numbers). It is a *process.* Skillful physics involves a systematic progression that involves:

- Visualization and conceptualization. What's going on? What will happen?
- Drawing figures and graphs and pictures to help with the process of determining what physics principles to use and how to use them. The paper should be an extension of your brain, helping you associate coordinates and quantities with the problem and working out a solution strategy. For example: drawing a free body diagram" in a problem where there are various forces acting on various bodies in various directions will usually help you break a large, complex problem into much smaller and more manageable pieces.
- Identifying (on the basis of these first two steps) the *physical principles* to use in solving the problem. These are almost invariably things from the Laws, Theorems and Principles chapter above, and with practice, you will get to where you can easily identify a Newtons Second Law" problem (or part of a problem) or an Energy Conservation" (part of) a problem.
- Once these principles are identified (and identifying them *by name* is a good practice, especially at first!) one can proceed to *formulate* the solution. Often this involves translating your figures into equations using the laws and principles, for example creating a free body diagram and trans-

lating it into Newton's Second Law for each mass and coordinate direction separately.

- At this point, believe it or not, the hard part is usually done (and most of the *credit* for the problem is *already secured*). What's left is using *algebra* and other mathematical techniques (e.g. trigonometry, differentiation, integration, solution of simultaneous equations that combine the results from different laws or principles into a single answer) to obtain a *completely algebraic* (symbolic) expression or set of expressions that answer the question(s).

- At this point you should *check your units!* One of several good reasons to solve the problem algebraically is that all the symbols one uses carry *implicit* units, so usually it is a simple matter to check whether or not your answer has the right ones. If it does, that's good! It means you *probably* didn't make any trivial algebra mistakes like dividing instead of multiplying, as that sort of thing would have led to the wrong units.

  Remember, an answer with the right units may be wrong, but it's not crazy and will probably get lots of credit if the reasoning process is clear. On the other hand, an answer with the *wrong* units isn't just mistaken, its *crazy* mistaken, impossible, silly. Even if you can't see your error, if you check your answer and get the wrong units *say so*; your instructor can then give you a few points for being diligent and checking and knowing that you are wrong, and can usually quickly help you find your mistake and permanently correct it.

- Finally, *at the very end*, substitute any numbers given for the algebraic symbols, do the arithmetic, and determine the final numerical answers.

Most of the problems below won't have any numbers in them at all to emphasize how unimportant this last step is *in learning physics!* Sure, you should learn to be careful in your doing of arithmetic, but anybody can (with practice) learn to punch numbers into a calculator or enter them into a computer that will do all of the arithmetic flawlessly no matter what. It is the process of determining *how* to punch those numbers or program the computer to evaluate a *correct formula* that is what physics is all about. Indeed, with skill and practice (especially practice at estimation and conceptual problem solving) you will usually be able to at least *approximate* an answer and fully understand what is going on and what will happen even without doing any arithmetic at all, or doing only arithmetic you can do in your head.

As with all things, practice makes perfect, wax on, wax off, and the more *fun* you have while doing, the more you will learn. Work in groups, with friends, over pizza and beer. Learning physics should not be punishment, it should be a pleasure. And the ultimate reward is seeing the entire world around you with different eyes...

Chapter 5

The Electric Field (discrete and continuous distributions)

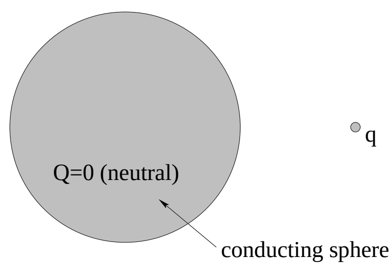
The problems in this chapter are intended to help you practice using ***Coulomb's Law*** (the fundamental equation governing electrostatic force between charged particles) and the closely derived ideas of the ***electric field*** of charged particles and continuous charge distributions.

5.1 Electric Field

5.1.1 Multiple Choice

Problem 1.

problems/efield-mc-force-between-conductor-and-charge.tex



In the figure above, a point charge q is brought near a neutral conducting sphere.

Is the force of between the sphere and the charge:

a) Zero.

b) Attractive.

c) Repulsive.

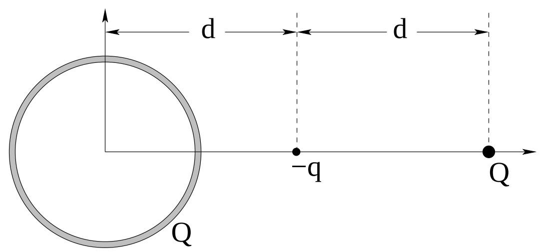
d) Out of the page.

e) Into the page.

Draw a representation of the charge distribution on the conductor that explains your answer.

Problem 2.

problems/efield-mc-force-between-two-charges.tex



I release a negatively charged particle with charge q at a point P centered midway between the centers of an insulating spherical shell of ***uniformly distributed*** charge Q and a ***point charge*** Q as shown. The charged particle:

a) accelerates to the right and the electric potential decreases.

b) accelerates to the left and the electric potential increases.

c) does not accelerate.

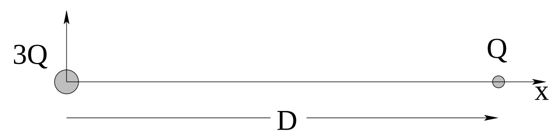
d) accelerates to the right and the electric potential increases.

e) accelerates to the left and the electric potential decreases.

5.1.2 Short Answer/Concept

Problem 3.

problems/efield-sa-find-equilibrium-point-charges.tex



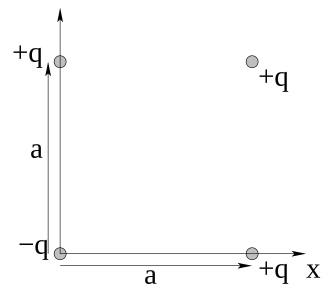
In the figure above, a charge of $3Q$ is at the origin and a charge of Q is at the position $x = D$ on the x-axis.

a) Find the point x_e where the total field vanishes and mark it in on the figure (drawn approximately to scale).

b) If a bead is threaded on a wire stretched between the two charges (so that movement in the y or z direction is opposed by the wire) what is the *sign* of the charge we need to give the bead so that x_e is a point of *stable force equilibrium* for it?

Problem 4.

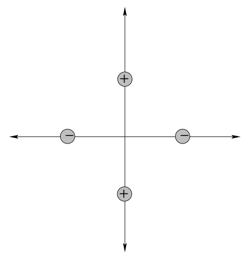
problems/efield-sa-force-on-fourth-charge.tex



In the figure above, four charges are located at at the corners of a square of side length a. Find the force on the upper right hand charge.

Problem 5.

problems/efield-co-draw-quadrupole-field.tex

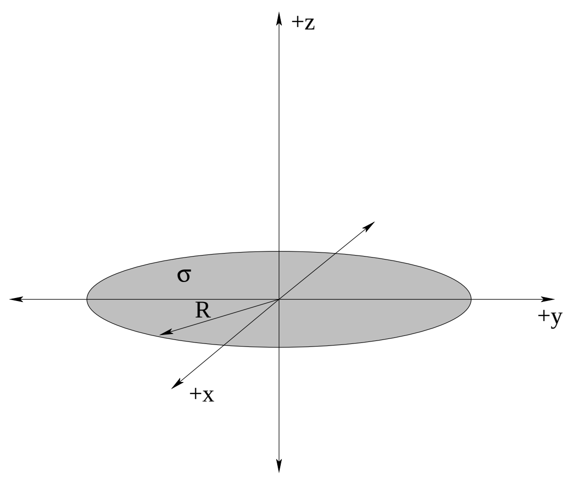


(5 points) In the figure above, four charges of equal magnitude are arranged in a square as shown. *Sketch* the field lines you might expect to result from this arrangement in the plane of the figure. Hint: Remember that field lines flow out from positive charges, flow into negative charges, and cannot cross (the field is well defined in direction at all points in space). What kind of field is this?

5.1.3 Long Problems

Problem 6.

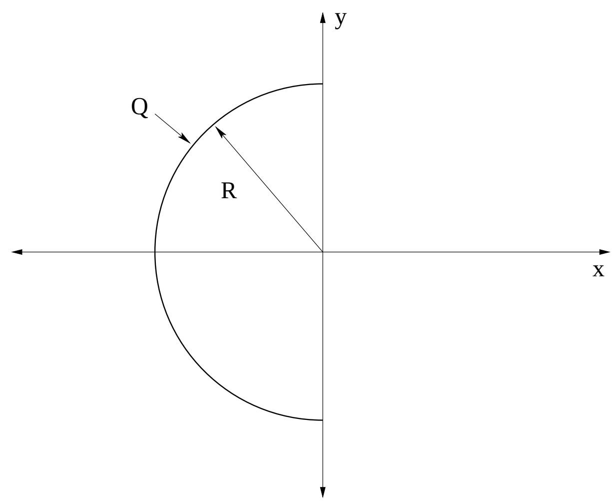
problems/efield-pr-axis-of-disk.tex



Find the electric field on the (z) axis of a disk of charge of radius R with uniform surface charge distribution σ by direct integration.

Problem 7.

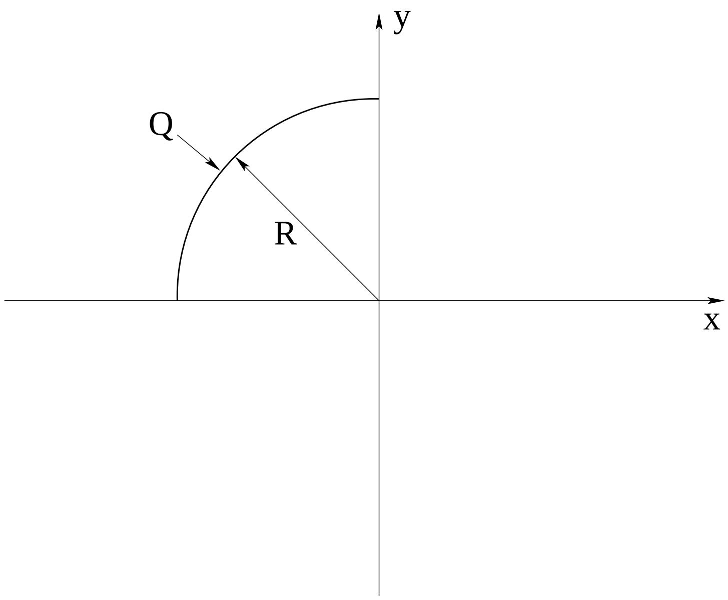
problems/efield-pr-center-half-circle-charge.tex



A half-ring of total charge Q and radius R sits symmetrically across the x-axis around the origin as shown in the figure above. Find the electric field at the origin (magnitude and direction) from direct integration.

Problem 8.

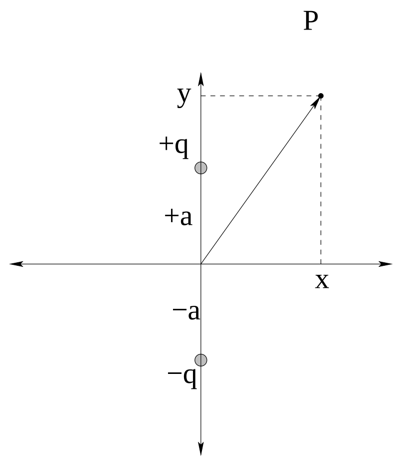
problems/efield-pr-center-quarter-circle-charge.tex



A quarter of a ring of total charge Q and radius R is oriented as shown in the figure above. Find the electric field at the origin (***magnitude and direction***) from direct integration. Show all work.

Problem 9.

problems/efield-pr-dipole-cartesian.tex



In the figure above an electric dipole is shown consisting of two equal and opposite charges $\pm q$ separated by a distance $2a$ lined up with the y-axis and centered on the origin. A point P with arbitrary coordinates (x, y) is shown.

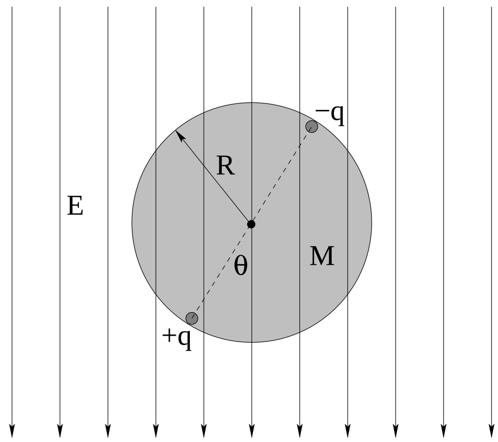
a) Find an expression for the vector field $\vec{\boldsymbol{E}}$ at the point P in *Cartesian* coordinates. Recall that a vector is a magnitude and a direction, and can be specified by e.g. E_x and E_y, by $|\vec{\boldsymbol{E}}|$ and θ

b) Draw a proportionate picture of the resultant electric field vector at P (showing its approximately correct direction for reasonable representations of the relative field strengths for each charge).

c) Show that in the limit $x \gg a$, $x \gg y$, the field *near* the x-axis is roughly $E_x \approx -k_e p/x^3$, $E_y \approx 0$ (where p is the magnitude of the dipole moment).

Problem 10.

problems/efield-pr-dipole-pendulum.tex



In the figure above a neutral insulating disk of mass M and radius R is pivoted at the center. A small charge $+q$ is fixed to one edge and a charge of $-q$ is fixed to the opposite edge so that the line between the two charges makes a ***small*** angle θ with an applied, uniform electric field $\vec{\boldsymbol{E}}$ as shown.

a) Find the torque on the disk.

b) Write the equation of motion for the disk.

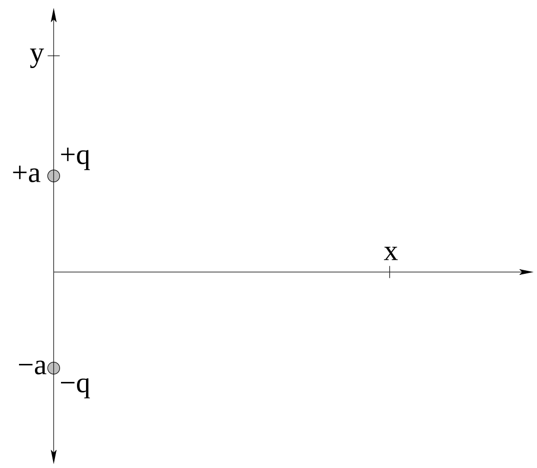
c) Find the angular frequency with which the disk oscillates.

d) If we start the disk at an initial angle θ_0 at $t = 0$, what is $\theta(t)$?

e) What is the potential energy of the disk in the field? Assume that zero potential energy is at $\theta = 0$.

Problem 11.

problems/efield-pr-dipole.tex



In the figure above we see an ***electric dipole*** consisting of equal and opposite charges separated by a (vector) distance.

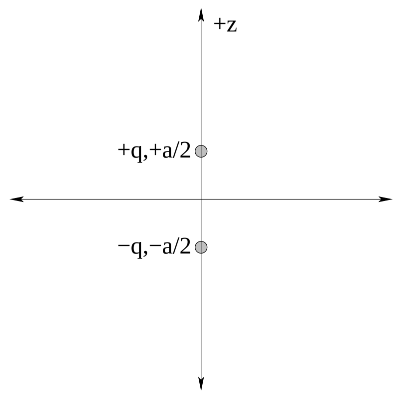
a) Find the electric field at an arbitrary point (for example, the one drawn) on the y-axis. Remember, the field is a ***vector*** so you must (somehow) specify ***both magnitude and direction***.

b) Find the electric field at an arbitrary point on the x-axis.

c) What are the ***asymptotic*** forms of the electric fields on the x and y axes in the limits that x or y is much, much larger than a?

Problem 12.

problems/efield-pr-dipole-z-only.tex



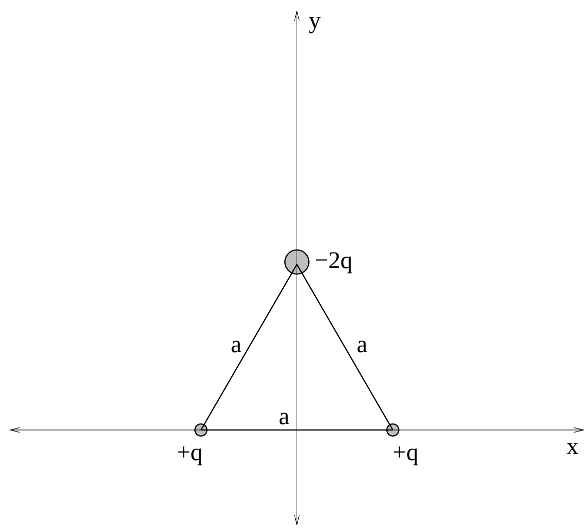
Charges of $\pm q$ are located at both $z = \pm a/2$, respectively. This arrangement forms a electric dipole.

a) Find both the electric potential and electric field (magnitude and direction) at an arbitrary point $z > a/2$ on the z-axis.

b) What is the first *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $z \gg a/2$?

Problem 13.

problems/efield-pr-equilateral-quadrupole.tex



Charges of $+q$ are located at the two bottom corners of an equilateral triangle with sides of length a. A charge of $-2q$ is at that top corner. This arrangement of charge can be considered two dipoles oriented at 60° with respect to one another.

a) Find the electric field (magnitude and direction) at an arbitrary point on the y-axis above/outside the triangle.

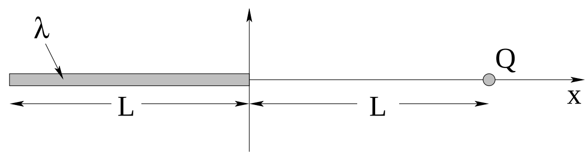
b) What are the first two surviving terms in the binomial-theorem-derived series for the electric field evaluated *far* from the charges, i.e. – for $y >> a$[1]?

---

[1]Hint: since the net charge balances (=0), we expect no monopolar part (like $1/x^2$). Since the dipoles do not quite balance, we might see a dipolar part (like $1/x^3$). However, since the dipoles are not parallel we might expect to see a significant *quadrupolar* term that varies like $1/x^4$ as well.

Problem 14.

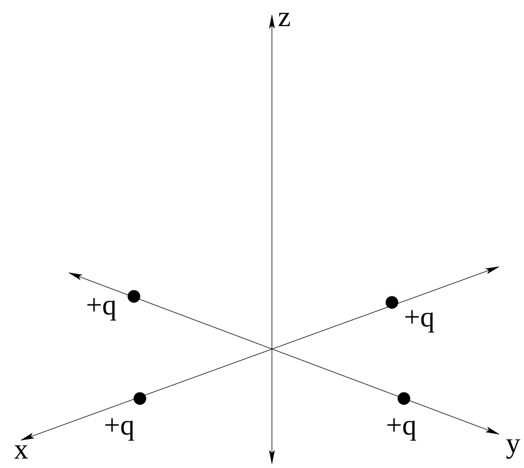
problems/efield-pr-force-rod-on-charge.tex



A rod with uniform charge per unit length $\lambda = Q/L$ and length L is located on the negative x-axis with one end at the origin. A charge Q is located a distance L from the end of the rod as shown. Find the ***total force*** acting on the charge Q due to the rod (magnitude and direction).

Problem 15.

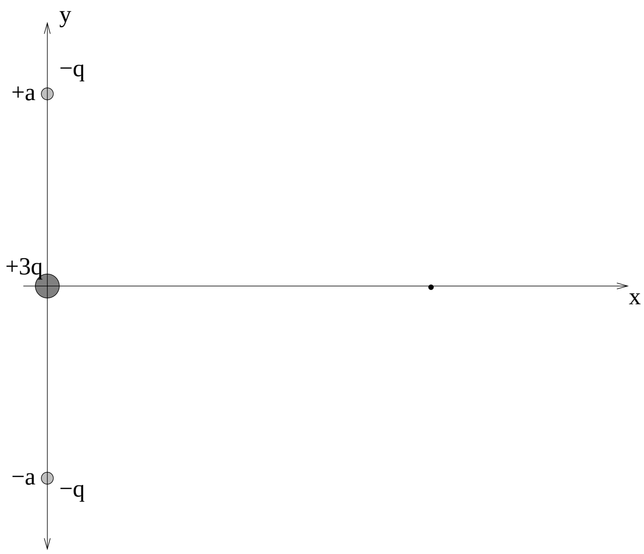
problems/efield-pr-four-charges-on-z-axis.tex



Four positive charges of magnitude $+q$ are located on at positions $(0, a, 0)$, $(0, -a, 0)$, $(a, 0, 0)$, $(-a, 0, 0)$ on the $x - y$ plane as shown. Find the electrostatic field at an arbitrary point on the z axis.

Problem 16.

problems/efield-pr-inline-monopole-quadrupole.tex



Charges of $-q$ are located at both $y = a$ and $y = -a$, and a charge of $+3q$ is located at $y = 0$ on the y-axis. This arrangement of charge can be visualized as two opposing dipoles plus a charge at the center.

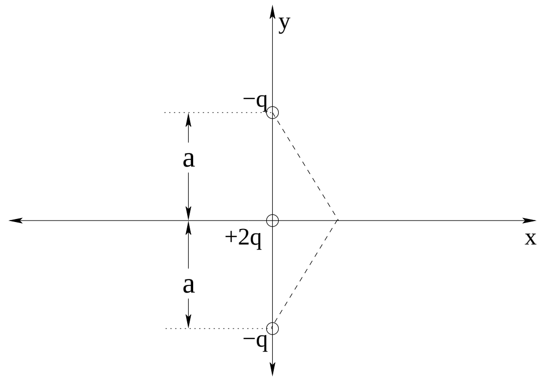
a) Find the electric field (magnitude and direction) at an arbitrary point on the x-axis.

b) What are the first two *nonzero* terms in the electric field evaluated *far* from the charges, i.e. – for $x >> a$? Your answer should be a series of terms in inverse powers of x.

c) What is the total potential energy of this collection of charges?

Problem 17.

problems/efield-pr-inline-quadrupole-x.tex



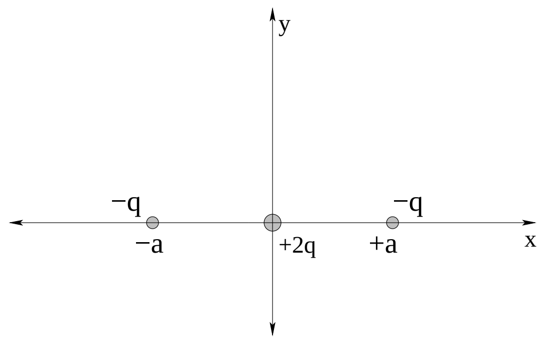
Charges of $-q$ are located at both $y = a$ and $y = -a$, and a charge of $+2q$ is located at $y = 0$ on the y-axis. This arrangement of charge can be visualized as two opposing dipoles.

a) Find the electric field (magnitude and direction) at an arbitrary point on the x-axis.

b) What is *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $x >> a$? Your answer should be a series of terms in inverse powers of x.

Problem 18.

problems/efield-pr-inline-quadrupole-y.tex



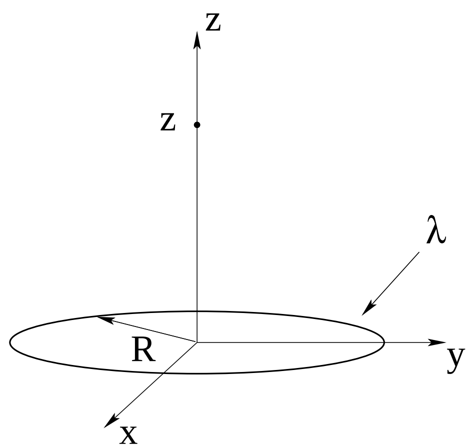
Charges of $-q$ are located at both $x = a$ and $x = -a$, and a charge of $+2q$ is located at $x = 0$ on the x-axis. This arrangement of charge can be visualized as two opposing dipoles.

a) Find the electric field (magnitude and direction) at an arbitrary point on the y-axis.

b) What is *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $y >> a$? Your answer should be a series of terms in inverse powers of y.

Problem 19.

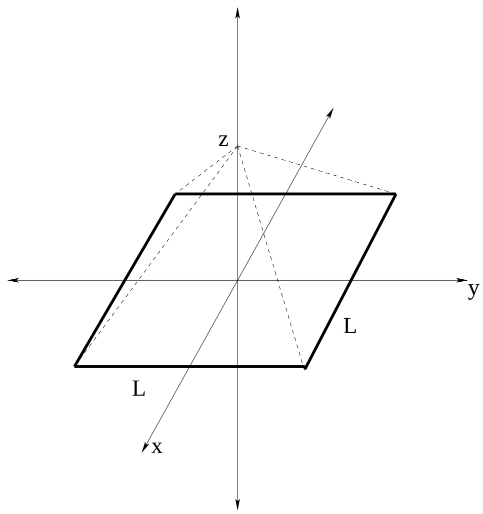
problems/efield-pr-ring-on-axis.tex



Find the electric field at an arbitrary point on the z axis for the ring of charge of radius R with charge per unit length λ above.

Problem 20.

problems/efield-pr-square-on-axis.tex

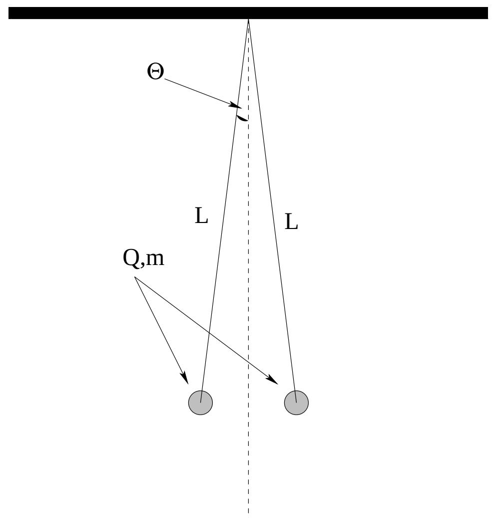


Four lines of uniform linear charge density λ and length L form a square in the xy-plane centered symmetrically on the z-axis as shown. Find the electric field at an arbitrary point on the z-axis. You may use $E_\perp = \frac{k_e \lambda}{y}(\sin\theta_2 - \sin\theta_1)$ for a line segment of charge without deriving it.

Hint: The arbitrary point and corners of the square form a square pyramid. If you use the pythagorean theorem a couple of times you can find the perpendicular height of one face of the pyramid and the length of an edge of that face. A bit of trig using the triangles involved will tell you the sines and cosines of the angles that you might need to find the answer.

Problem 21.

problems/efield-pr-two-pith-balls.tex



Two positively charged pith balls of mass m each have a charge Q and are suspended by insulating (massless) lines of length L from a common point as shown. Assume that L is long enough that θ at the top is a small angle. Find θ such that the pith balls are in static equilibrium in terms of k, Q, m, L and of course g.

Chapter 6

Gauss's Law for Electrostatics

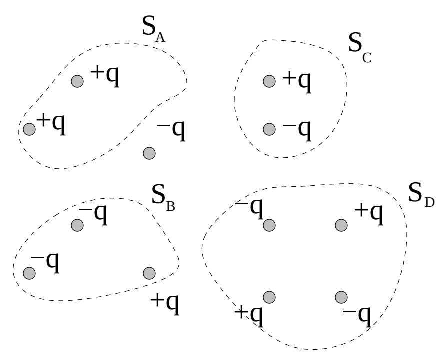
Gauss's Law is our first one of ***Maxwell's Equations***, and is considered to be (part of) the ***Laws of Nature*** that describe the electromagnetic field. We will find a surprising richness of uses for it – an easy way to find the electric field of many sufficiently symmetric charge distributions, a way of understanding and deriving many results for conductors, a way of conceptually visualizing what the field/flux of electric charge looks like. Future physics majors or students of mathematics, however, may find our treatment most useful as a stepping stone to a partial differential treatment that is more useful still (beyond the scope of these practice problems, but represented by an advanced problem in the book).

6.1 Gauss's Law and Flux

6.1.1 Ranking/Scaling

Problem 22.

problems/gausslaw-ra-flux.tex



Rank the net outward directed flux through the four surfaces $S_{A,B,C,D}$ above, from the least (most negative) to the most (most positive). Your answer could look like (but probably isn't) $A = B < C < D$.

6.1.2 Short Answer/Concept

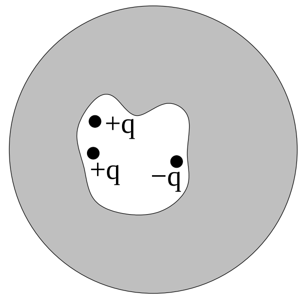
Problem 23.

problems/gausslaw-sa-e-perp-sigma.tex

What is the electric field just *outside* of a conductor at electrostatic equilibrium in terms of its surface charge density σ? Note well that the electric field is a vector quantity, so you'll have to give at least two generic components.

Problem 24.

problems/gausslaw-co-charge-in-hollow-conductor.tex



Two charges of magnitude $+q$ and one of charge $-q$ are arranged inside a spherical conductor with a hollow irregular cavity as shown so that they do not touch the conductor. The total charge on the ***outside*** surface of the spherical conductor is:

a) $+q$, with positive charge on the left side of the sphere and negative charge on the right.

b) $-q$, with negative charge on the left side of the sphere and positive on the right.

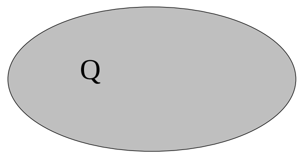
c) $+q$, uniformly distributed.

d) $-q$, uniformly distributed.

e) zero.

Problem 25.

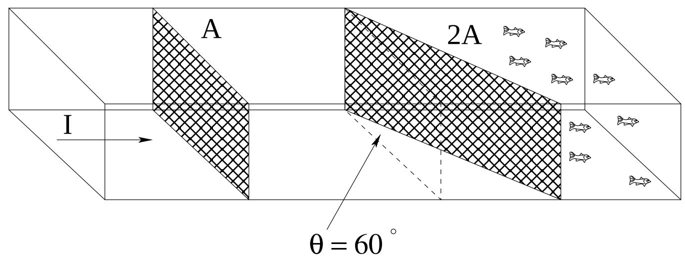
problems/gausslaw-co-draw-field-of-charged-conducting-ellipse.tex



(5 points) In the figure above, a solid conducting ellipsoid of revolution is shown that was charged up with a total charge Q and then left for a moment to come to equilibrium. Draw a *qualitative* picture of the field lines (in the plane of the paper only) you might expect to "see" (if you could see field lines) both inside and outside of the surface of the ellipsoid.

Problem 26.

problems/gausslaw-co-flow-through-two-nets.tex



Larry stretches a fishing net of area A straight across a river that is flowing at a rate of $I = 100$ cubic meters per second to the sea. "I'm going to catch a lot of fish!" he tells Curly.

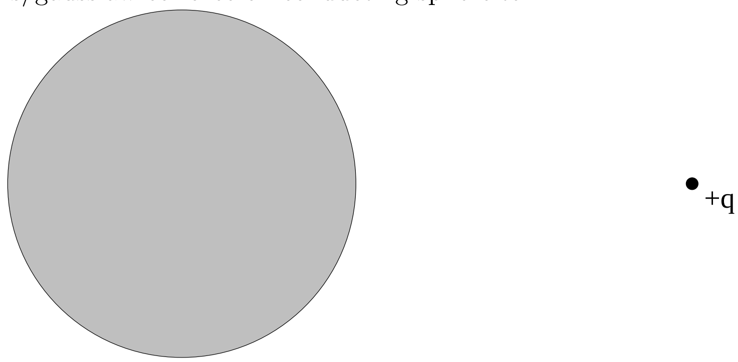
"N'yuk n'yuk, I've got a net that is ***twice*** as long," says Curly, "so I'll catch all the fish in ***twice*** as much water!"

"You knucklehead!" says Moe to Curly, whapping him upside the head. "You won't catch all the fish in twice as much water."

Aside from the fact that all of the fish are downstream from their nets, why is Moe correct? Show explicitly that the ***flux*** of river water through the two nets is the total current of the river independent of the size or angle (or even shape), as long as they both stretch across the river, using the idea of a flux integral. You will probably need to define a "current density" of river water: $\vec{\boldsymbol{J}} = (I/A)\hat{\boldsymbol{x}}$ to do this properly, although many heuristic arguments might also work (they worked for Moe).

Problem 27.

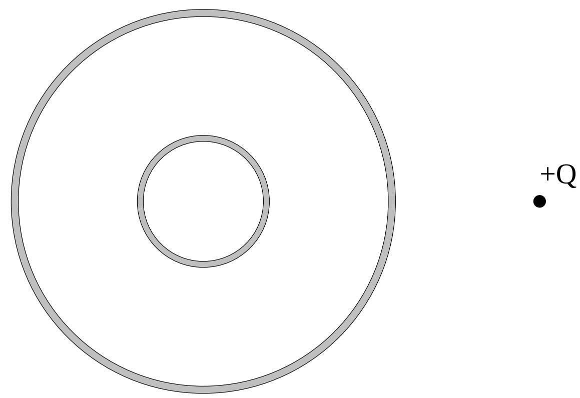
problems/gausslaw-co-force-on-conducting-sphere.tex



A charge q is sitting a short distance away from an uncharged conducting sphere. Is there a force acting on this charge? If so, in what direction (towards the sphere or away from the sphere) does the force point? Indicate *why* you answer what you answer, dressing up the picture above to illustrate what happens.

Problem 28.

problems/gausslaw-co-inner-sphere-outer-charge.tex

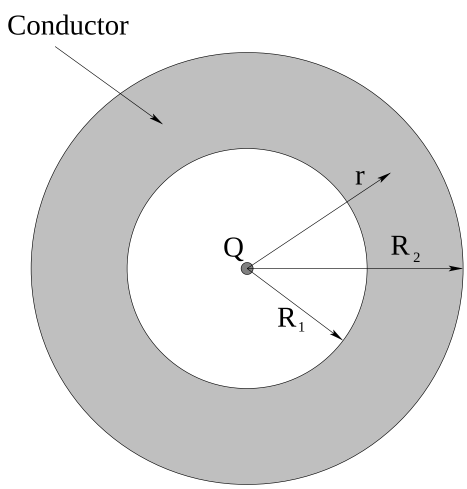


In the picture above, a charge Q sits close to an uncharged conducting sphere concentric with a second uncharged conducting sphere as shown. Is there a nonzero electrostatic force between the *inner* sphere and the *external* charge Q? (Explain your answer – an answer such as "yes" or "no" is not sufficient.)

6.1.3 Long Problems

Problem 29.

problems/gausslaw-pr-charge-in-conducting-shell.tex



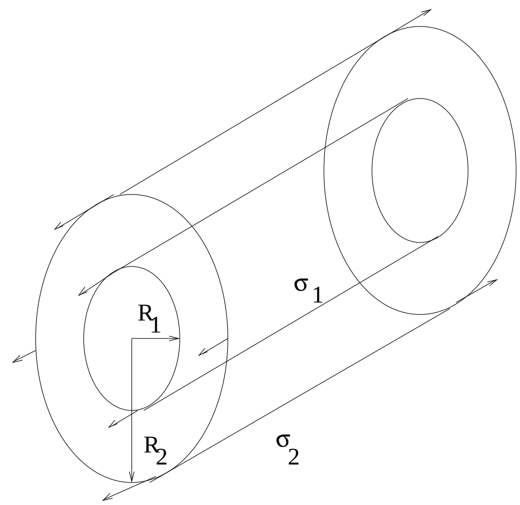
A conducting shell concentrically surrounds a point charge of magnitude Q located at the origin. The inner radius of the shell is R_1 and the outer radius R_2.

a) Find the electric field $\vec{E}$ at all points in space (you should have three answers for three distinct regions).

b) Find the surface charge density σ on the inner surface of the conductor. Justify your answer with Gauss's law.

Problem 30.

problems/gausslaw-pr-concentric-cylinders.tex



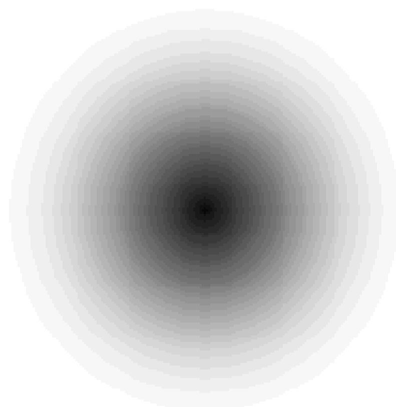
Two infinitely long, cylindrical conducting shells are concentrically arranged as shown above. The inner shell has a radius R_1 and the outer shell the radius R_2. The inner shell has a charge per unit area σ_1, and the outer shell a charge per unit area σ_2.

a) Find the electric field $\vec{E}$ at all points in space (you should have three answers for three distinct regions).

b) Find the surface charge density σ_2 (in terms of σ_1, R_1, R_2, etc.) that causes the field to vanish everywhere but in between the two shells. *Justify your answer with Gauss's law.*

Problem 31.

problems/gausslaw-pr-hydrogen-atom.tex



Find the electric field at all points in space of a spherical charge distribution with radial charge density:

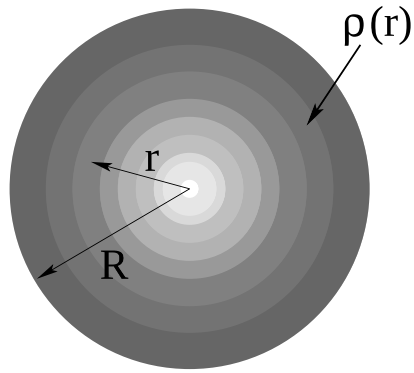
$$\rho(r) = \rho_0 \frac{e^{-r/2a}}{r^2}$$

For extra credit, determine ρ_0 such that the total charge Q in the distribution is $-e$.

This is the charge distribution of the electron cloud about a hydrogen atom in the ground state. Remember, if you can't quite see how to do the integral (which is actually pretty easy) set the problem up, systematically – following the steps outlined in class – until all that is LEFT is doing the integral, to end up with most of the credit.

Problem 32.

problems/gausslaw-pr-linear-charge-density-sphere.tex

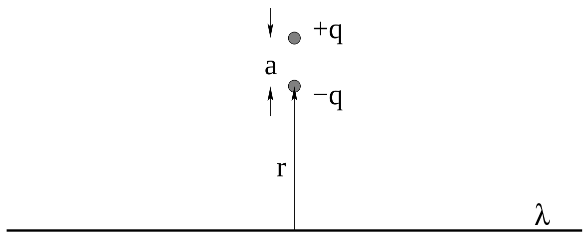


Find the electric field and electric potential at all points in space of a sphere with radial charge density:

$$\rho(r) = \rho_0 \frac{r}{R} \qquad r \leq R$$
$$\rho(r) = 0 \qquad r > R$$

Problem 33.

problems/gausslaw-pr-line-charge-force-on-dipole.tex



The electric field of an infinite straight line of charge is given by:

$$\vec{\boldsymbol{E}} = \frac{2k\lambda}{r}\hat{\boldsymbol{r}}$$

(where λ is the charge per unit length and $\vec{\boldsymbol{r}}$ is in cylindrical coordinates).

A dipole consisting of a charges $\pm q$ separated by a (small) distance a is located with the charge $-q$ a distance r away from the line and the direction of the dipole parallel to vr as shown.

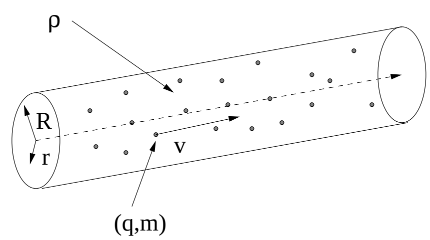
Find:

a) The next force on the dipole.

b) The net force on the dipole in the limit that $a \to 0$ while the magnitude of the diple moment $p = qa$ is held constant. This is the force on a "point dipole" in the field of the line of charge.

Problem 34.

problems/gausslaw-pr-particle-beam-electrostatics.tex

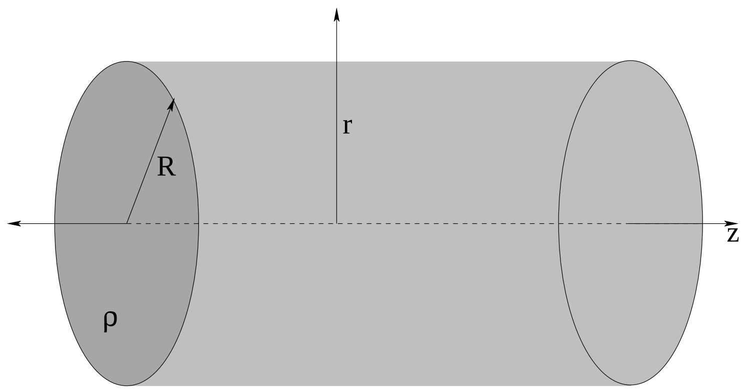


Beam Electrostatics: A cylindrical beam of particles each with charge q and mass m has a uniform initial (charge) density ρ and radius R. Each particle in the beam is initially travelling with velocity v parallel to the beam's axis. Consider the stability of this beam by examining the forces on a particle travelling in the beam at a distance $r < R$ from the axis (the center of the cylinder).

Find the approximate force on a particle at radius r caused by the other particles in the beam. You will need to use Gauss's law to calculate the electric field at radius r. Describe your work, and do not skip steps; show that you understand Gauss's law. Make a sketch as needed. .

Problem 35.

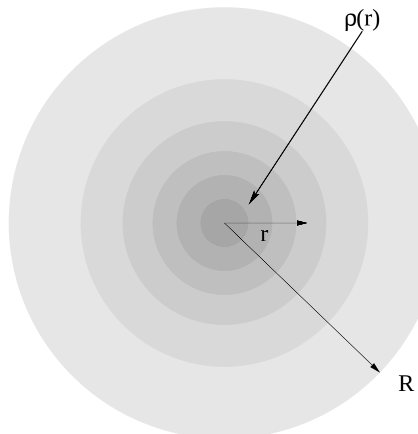
problems/gausslaw-pr-solid-cylinder.tex



A segment of an "infinitely long" cylinder of uniform charge density ρ and radius R is pictured above. Find the electric field at all points in space.

Problem 36.

problems/gausslaw-pr-sphere-1-over-r-density.tex

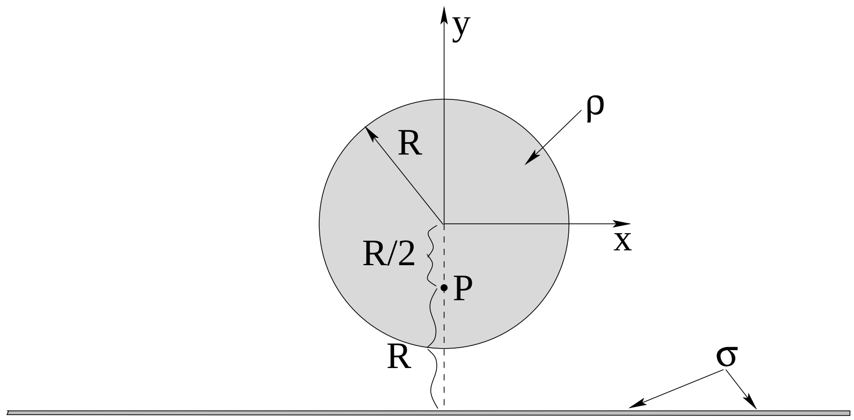


Find the electric field at all points in space of a sphere with radial charge density:

$$\rho(r) = \frac{\rho_0 R}{r} \qquad r \leq R$$
$$\rho(r) = 0 \qquad r > R$$

Problem 37.

problems/gausslaw-pr-sphere-and-plane.tex



A ***uniform insulating ball of charge*** with charge density ρ and radius R is suspended with its center a distance $3R/2$ above a ***uniform insulating plane of charge*** with charge per unit area σ. The origin of a set of xyz coordinates is located at the center of the sphere with z pointing out of the page, and a point P is located at $x = z = 0$, $y = -R/2$ inside the sphere as shown above.

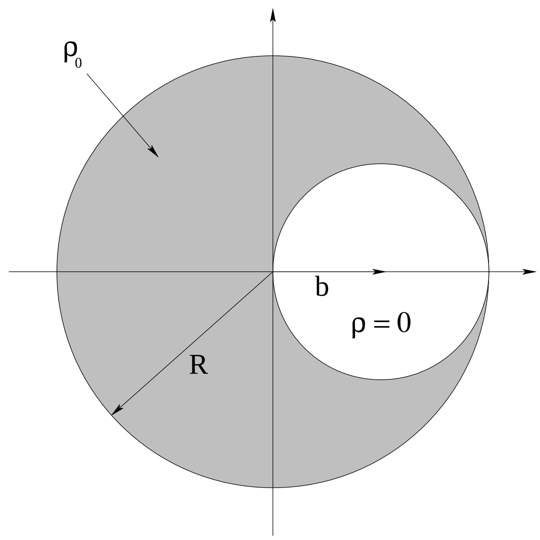
a) ***Using Gauss's Law,*** find the electric field (magnitude and direction) due to the ***plane of charge only*** at the point P.

b) ***Using Gauss's Law,*** find the electric field (magnitude and direction) due to the ***ball of charge only*** at point P.

c) Find the ***total electric field*** at point P.

Problem 38.

problems/gausslaw-pr-sphere-with-hole.tex

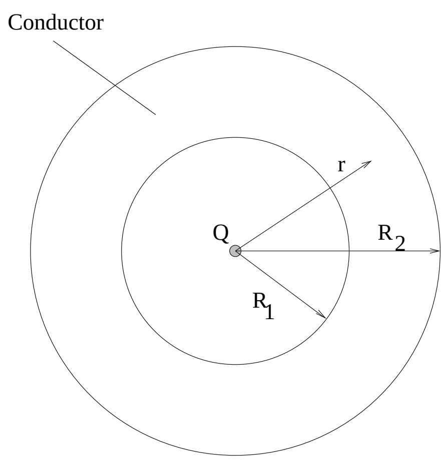


A sphere of uniform charge density ρ_0 has a hole of radius $b = R/2$ centered on $x = b$ cut out of it as shown in the figure.

a) Find the electric field **vector** inside the hole.

b) What do you *expect* to be the first two terms in the electric field expansion at an arbitrary point $x \gg R$ on the x axis? Note that you can *either* guess this answer based on what you know of multipolar fields *or* you can evaluate the field exactly (which is pretty easy) and do a binomial expansion through two surviving terms.

Hint: Remember, you can think of this as a superposition problem for two spheres, one with uniform charge density ρ_0 and one with uniform charge density $-\rho_0$.



Problem 39.

problems/gausslaw-pr-thick-spherical-conductor-surrounds-Q.tex

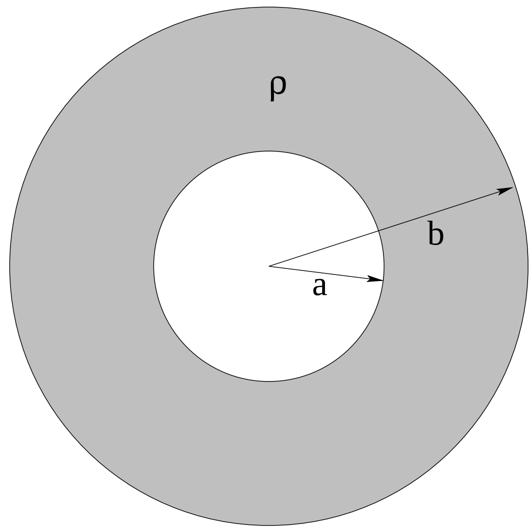
A conducting shell concentrically surrounds a point charge of magnitude Q located at the origin. The inner radius of the shell is R_1 and the outer radius R_2.

a) Find the electric field $\vec{E}$ at all points in space (you should have three answers for three distinct regions).

b) Find the surface charge density σ on the inner surface of the conductor. Justify your answer with Gauss's law.

Problem 40.

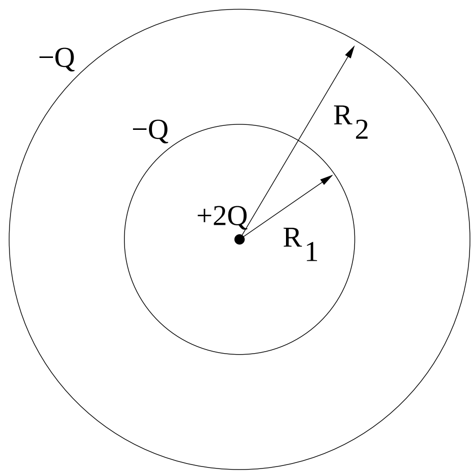
problems/gausslaw-pr-thick-spherical-shell.tex



Find the ***electric field at all points in space*** for a spherical shell of constant charge density ρ that has inner radius a and outer radius b. Express your answer in terms of ρ, not Q.

Problem 41.

problems/gausslaw-pr-two-spheres-one-charge.tex



Two spherical shells with radii R_1 and R_2 respectively concentrically surround a point charge. The central point charge has magnitude $2Q$. Both the spherical shells have a charge of $-Q$ (each) distributed uniformly upon the shells. ***Find the electric field at all points in space.***

Chapter 7

Potential and Potential Energy

At this point, you should be pretty good at finding the electric field of various charge distributions and/or solving problems involving electrostatic force, as well as familiar with some of the most important properties of conductors in electrostatic equilibrium. But one feature of the electric field that ***sucks*** (from the point of view of the problem solver) is that one has to solve 2-3 problems per problem, not just one, because the electric field is a ***vector quantity***.

Fortunately, the electrostatic field is ***conservative***, so we define the electrostatic potential energy of two charges from Coulomb's Law and the electrostatic ***potential*** from a similar treatment of the electric field. Suddenly we have ***lots*** of ways of evaluating the electrostatic potential (or potential difference) as a ***scalar*** function, and can (more) easily find the vector field by differentiating the potential than we could have by directly integrating its components.

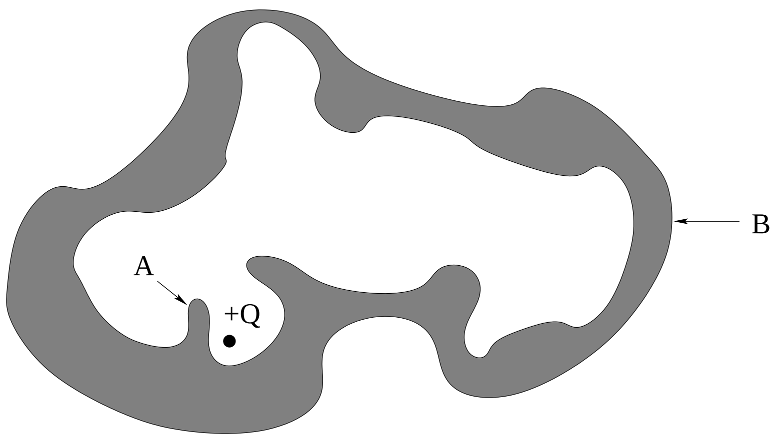
Knowing that conductors in electrostatic equilibrium are ***equipotential*** also helps us understand things like charge sharing and dielectric breakdown and the corona effect near sharp conducting points. This conceptual understanding is often tested with simple questions or problems.

7.1 Electrostatic Potential

7.1.1 Multiple Choice

Problem 42.

problems/potential-mc-conducting-blob.tex



A charge Q sits close to the inner surface of a hollow conductor in electrostatic equilibrium as shown above. Is the potential at A:

a) Greater than the potential at B.

b) Equal to the potential at B.

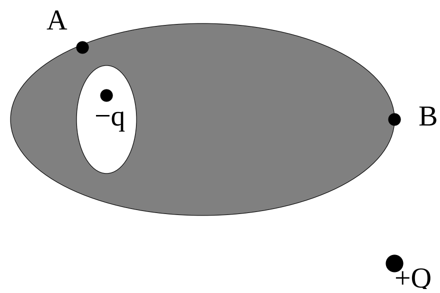
c) Less than the potential at B.

d) Zero.

e) Negative relative to infinity.

Problem 43.

problems/potential-mc-conducting-ovoid.tex



The conducting ovoid above is in ***electrostatic equilibrium*** in an external electric field generated by a charge Q outside and a charge -q inside a hollow in the conductor as shown. Two points, A and B, are labelled on the accompanying figure.

Is the potential at A

a) less than

b) greater than

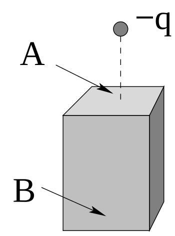
c) equal to

d) impossible to determine relative to

the potential at B?

Problem 44.

problems/potential-mc-equipotential-conductor.tex



A negative charge $-q$ is held at rest above a silver bar (which is a good conductor) as shown. The points A and B are on the surface of the conductor itself. The electric potential is:

a) Greater at point A than at point B.

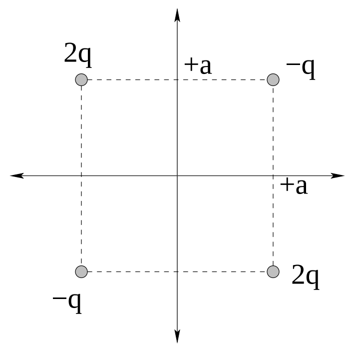
b) Greater at point B than at point A.

c) Equal at points A and B.

d) Zero at points A and B.

Problem 45.

problems/potential-mc-four-charges-at-center.tex



What is the potential at the center of the square of four charges shown (at the origin)? Note that the square has sides of length $2a$.

a) $V = k_e q/a$

b) $V = 2k_e q/a$

c) $V = \sqrt{2}k_e q/a$

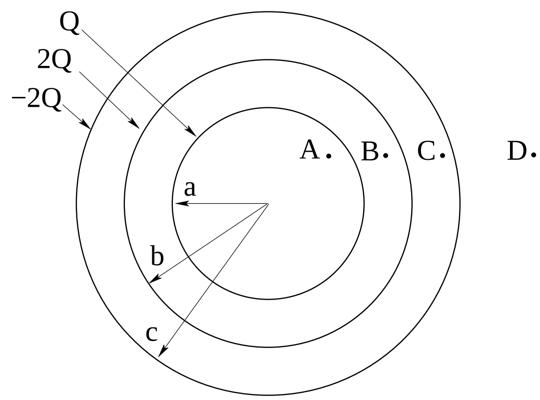
d) $V = k_e q/2a$

e) $V = k_e q/\sqrt{2}a$

7.1.2 Ranking/Scaling

Problem 46.

problems/potential-ra-three-spheres.tex

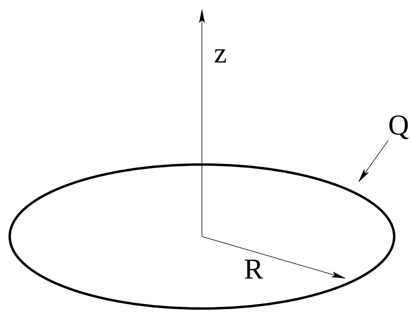


Rank the ***electrostatic potential*** at the four points $\boldsymbol{A, B, C, D}$ shown, from highest to lowest, with equality a possibility.

7.1.3 Short Answer/Concept

Problem 47.

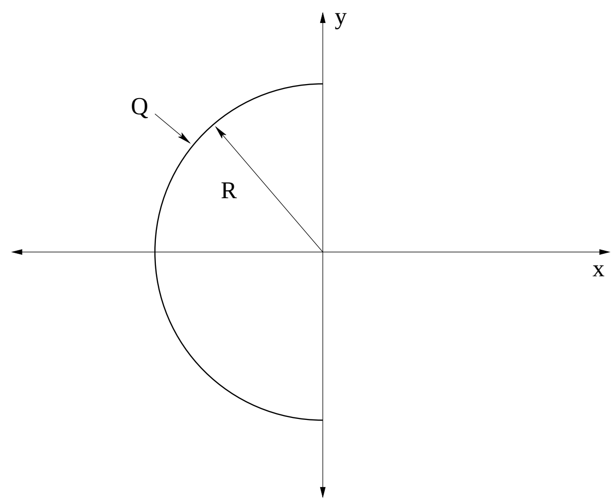
problems/potential-sa-center-circle.tex



What is the potential at the origin/center of the *ring* of total charge Q and radius R shown?

Problem 48.

problems/potential-sa-center-half-circle.tex



A half-ring of total charge Q and radius R sits symmetrically across the x-axis around the origin as shown in the figure above. What is the electric potential at the origin?

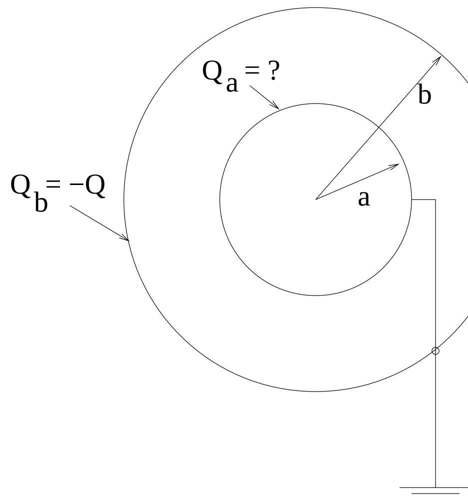
Problem 49.

problems/potential-sa-jump-or-die.tex

Oops! You ran into a power pole in your car, and a 14,000 volt primary supply line is resting on its hood, sparking occasionally as your car loses built up charge to the air. Unfortunately, you smell gasoline. Staying in your car doesn't seem like the good idea that it might otherwise be. What do you do (and why)? [Physics as a survival skill. Who would have imagined!]

Problem 50.

problems/potential-sa-two-spheres-inner-grounded-find-charge.tex

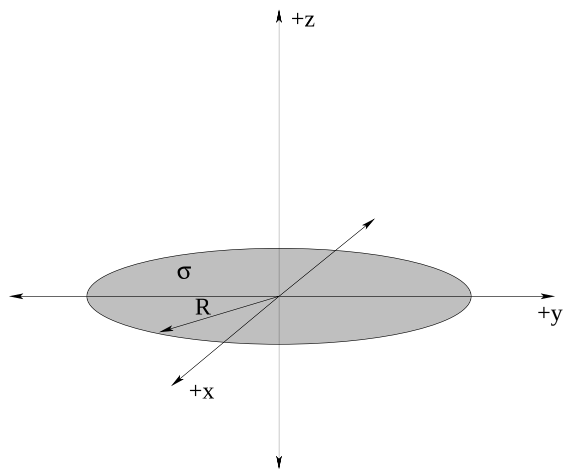


Two spherical conducting shells are drawn above. The outer shell, at a radius b, has a total charge of $Q_b = -Q$. The inner shell is grounded (has potential zero). What is the charge Q_a on the inner shell?

7.1.4 Long Problems

Problem 51.

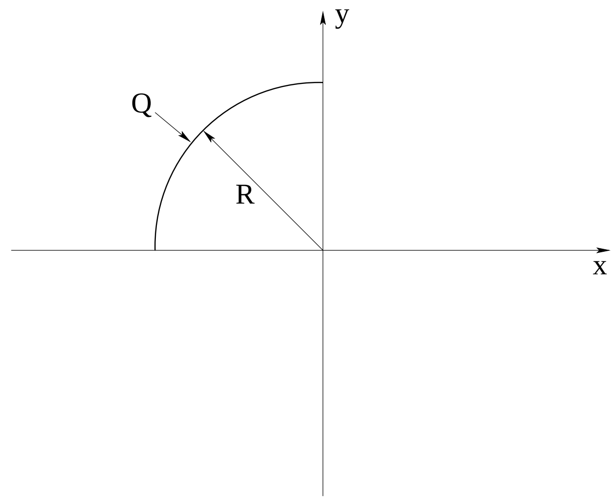
problems/potential-pr-axis-of-disk.tex



Find the electric potential on the (z) axis of a disk of charge of radius R with uniform surface charge distribution σ.

Problem 52.

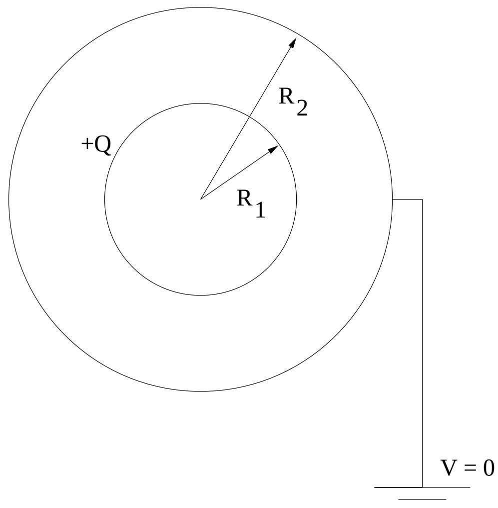
problems/potential-pr-center-quarter-circle-charge.tex



A quarter of a ring of total charge Q and radius R is oriented as shown in the figure above. Find the electric potential at the origin. Indicate your reasoning if you just write the answer down, or show all work if you integrate to find it.

Problem 53.

problems/potential-pr-concentric-spherical-shells.tex

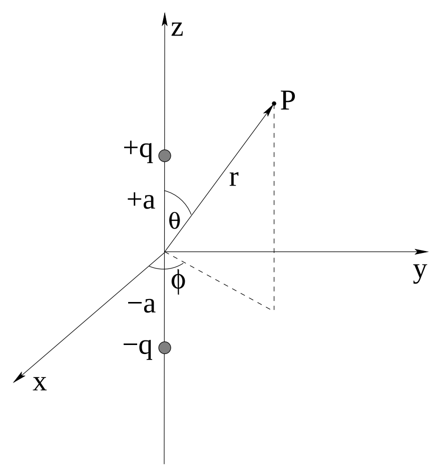


Two concentric spherical conducting shells of radii R_1 and R_2 are arranged as shown. The inner shell is given a total charge $+Q$. The outer shell is grounded (connected to a conductor at zero potential) as shown.

Find the ***potential*** and the ***electric field*** at all points in space. Show all work – don't just write down answers even if you can "see" what the answers must be. Don't forget what ***kind*** of quantity each thing is!

Problem 54.

problems/potential-pr-dipole-all-space.tex



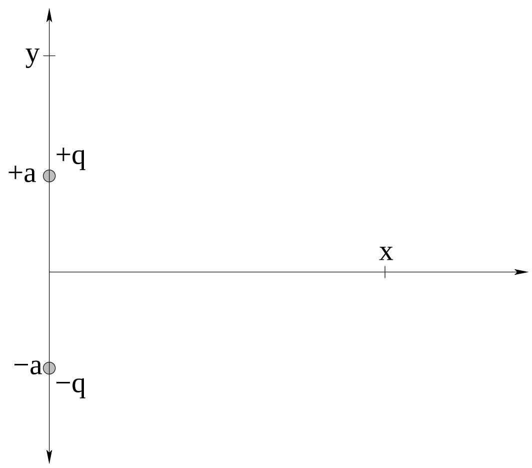
A point charge of $-q$ is located at $z = -a$ on the z-axis and a point charge of $+q$ is located at $z = +a$.

a) Write down the ***potential*** at an arbitrary point P in space in spherical coordinates: $V(r, \theta)$. Note that the problem has azimuthal symmetry (has no ϕ dependence) so your answer shouldn't either.

b) What is the leading term in the expansion of the potential for $r \gg a$, expressed in terms of the dipole moment p_z (and the coordinates)?

Problem 55.

problems/potential-pr-dipole-axes.tex



In the figure above we see an ***electric dipole*** consisting of equal and opposite charges separated by a (vector) distance.

a) What is the dipole moment of this pair of charges?

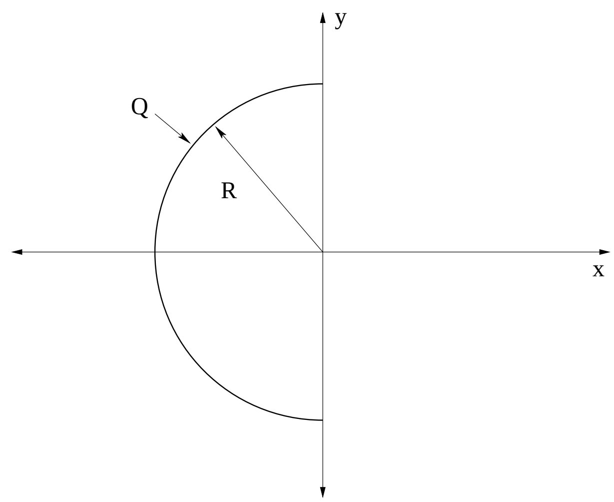
b) Find the electric potential at an arbitrary point (for example, the one drawn) on the y-axis in terms of the dipole moment.

c) Find the electric potential at an arbitrary point on the x-axis in terms of the dipole moment.

d) What are the ***asymptotic*** forms of the electric potentials on the x and y axes in the limits that x or y is much, much larger than a?

Problem 56.

problems/potential-pr-half-circle-charge-and-field.tex



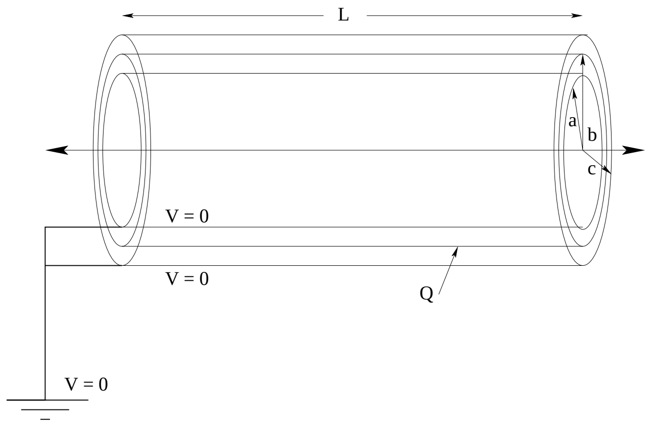
A half-ring of total charge Q and radius R sits symmetrically across the x-axis around the origin as shown in the figure above.

a) Find the electric field at the origin (magnitude and direction) from direct integration.

b) What is the electric potential at the origin?

Problem 57.

problems/potential-pr-three-cylinders-two-grounded.tex



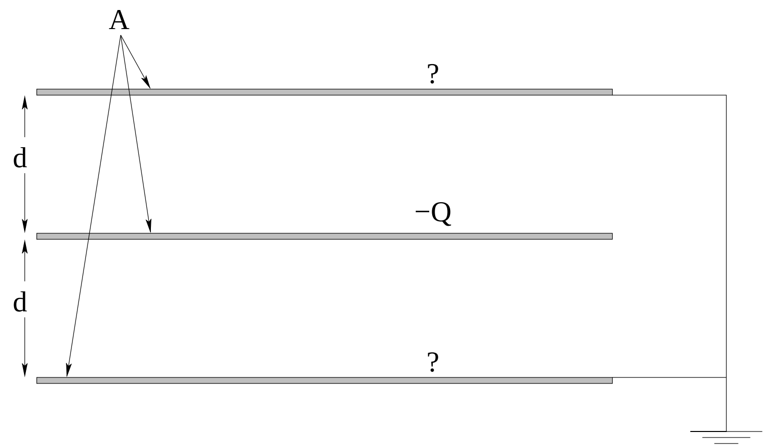
(10 points) Three cylindrical conducting shells of radii $a < b < c$ and of length $L \gg c$ are placed in a concentric configuration as shown. The middle shell is given a total charge Q, and both the inner and outer shells are grounded (connected by a thin wire to each other and to something at a potential of "0"). Find:

a) The total charge on the *inner* shell, in terms of a, b, c, L, Q and k.

b) The potential on the middle shell. In what direction does the field point in between a and b and in between b and c?

Problem 58.

problems/potential-pr-three-plates-two-grounded.tex



Three parallel plates with area A and separation d are shown in the figure above. The middle plate has a charge $-Q$, and the bottom plate and top plates are grounded (at $V = 0$). Find:

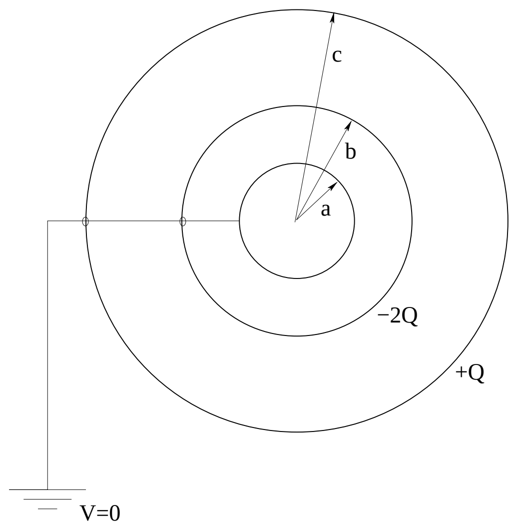
a) The charge on the top and bottom plates.

b) The electric field at all points in space.

c) The potential of the middle plate.

Problem 59.

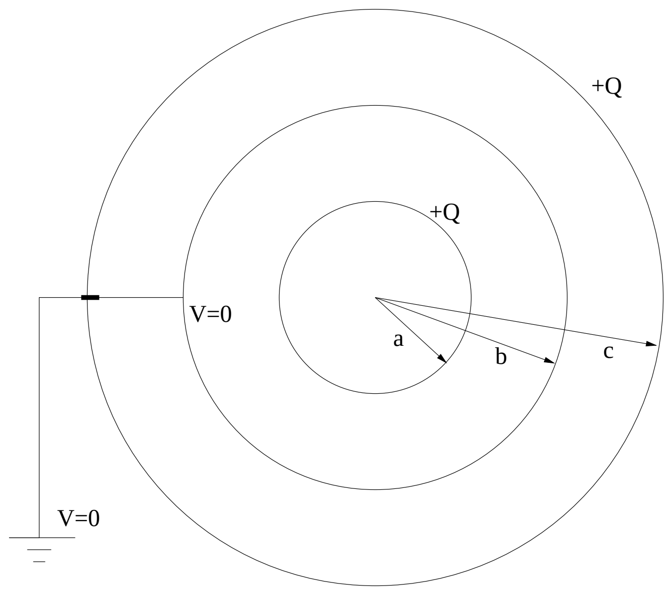
problems/potential-pr-three-spheres-inner-grounded.tex



Three concentric conducting spheres of radii a, b, c are drawn above. The outer shell has a charge Q. The middle shell has a charge $-2Q$. The inner shell is grounded. Find the charge on the inner shell, and the potential at all points in space.

Problem 60.

problems/potential-pr-three-spheres-middle-grounded.tex



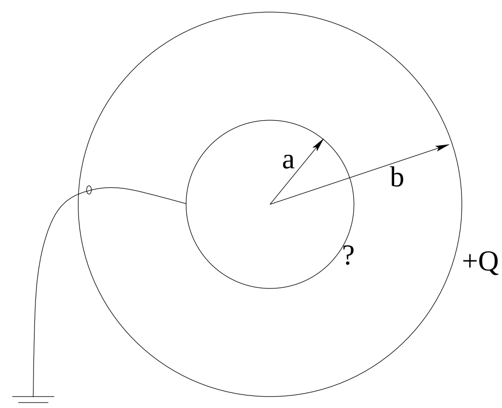
A charge of $+Q$ is placed on the innermost and outermost of three concentric conducting spherical shells. The middle shell is grounded via a thin wire that passes through an insulated hole in the outer shell and hence has a potential (relative to ∞) of 0.

a) Find the charge Q_s on the middle shell in terms of k, Q, and the given radii a, b and c.

b) Find the potential at all points in space (in each region where there is a distinct field). You may express your answers algebraically in terms of Q_s to make life a bit simpler (and independent of your answer to part a).

Problem 61.

problems/potential-pr-two-spheres-inner-grounded.tex

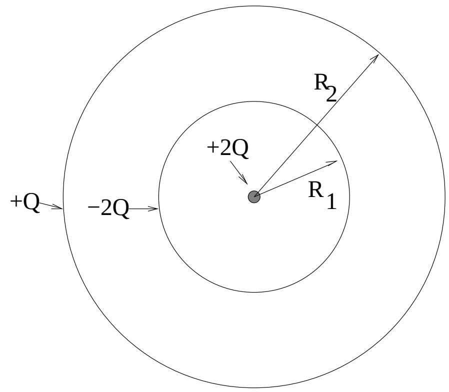


Two spherical conducting shells have radius a and b respectively. The outer shell has a total charge $+Q$ on it. The inner shell is *grounded* by means of a thin wire through a tiny hole in the outer shell as shown, and therefore is at potential $V_a = 0$. Find:

a) The total charge Q_a on the *inner* shell, in terms of k_e, Q, a, b.

b) The electric field at all points in space.

c) The electric potential at all points in space.



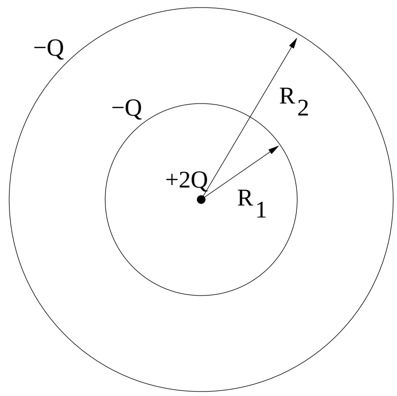
Problem 62.

problems/potential-pr-two-spheres-one-charge-2.tex

Find the potential $V(r)$ at all points in space for the arrangement of charge pictured above, where there is a point charge $+2Q$ at the origin, a charge uniformly distributed $-2Q$ on the inner shell (radius R_1), and a charge $+Q$ uniformly distributed on the outer shell (radius R_2). You will need three different answers for the three distinct regions of space.

Problem 63.

problems/potential-pr-two-spheres-one-charge.tex

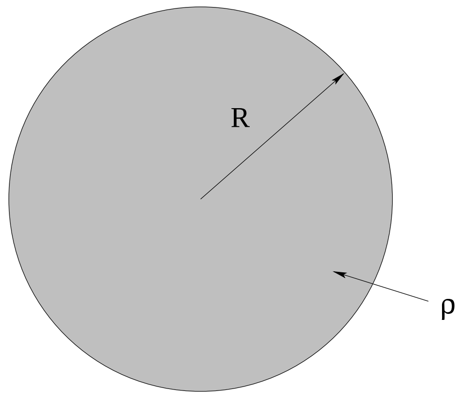


Two spherical shells with radii R_1 and R_2 respectively concentrically surround a point charge. The central point charge has magnitude $2Q$. Both the spherical shells have a charge of $-Q$ (each) distributed uniformly upon the shells.

Find the field and potential at all points in space. Show your work – even if you can just write the answer(s) down for each region, briefly sketch the methodology used to get the answers.

Problem 64.

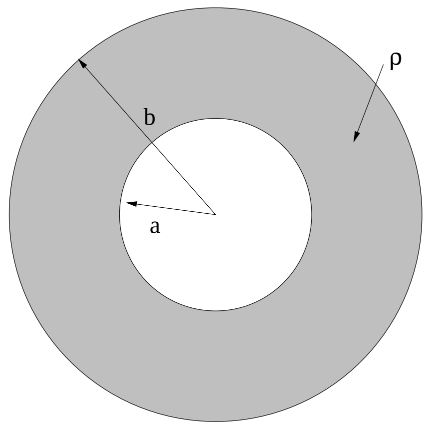
problems/potential-pr-uniform-sphere.tex



Find the electric field and electric potential at all points in space for a solid sphere with a constant/uniform charge density ρ and radius R. Show all your work, step by step.

Problem 65.

problems/potential-pr-uniform-spherical-shell-thick.tex



A spherical shell of inner radius a and outer radius b contains a uniform distribution of charge with charge density ρ.

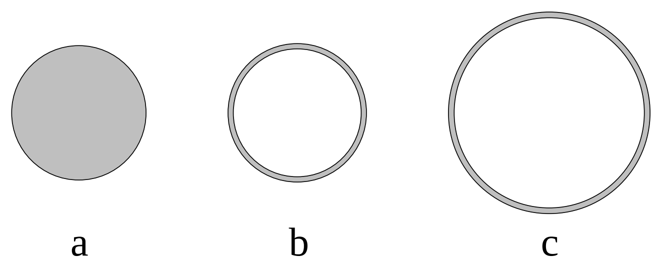
Find the field and potential at all points in space.

7.2 Electrostatic Potential Energy

7.2.1 Ranking/Scaling

Problem 66.

problems/potential-energy-ra-three-spheres-same-charge.tex

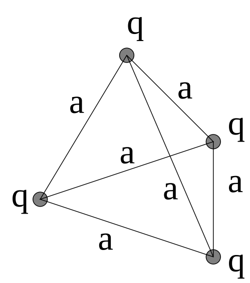


In the figure above, a charge Q is uniformly distributed in the grey region of each sphere. The relative sizes of the spheres are as shown. Rank the distributions from the ***least*** potential energy to the ***most*** potential energy with equality a possibility. That is, your answer could be (but probably isn't) $A = B < C$.

7.2.2 Short Answer/Concept

Problem 67.

problems/potential-energy-sa-tetrahedron-of-charges.tex



Four identical charges q are arranged in a tetrahedron with identical distance a between any pair of charges.

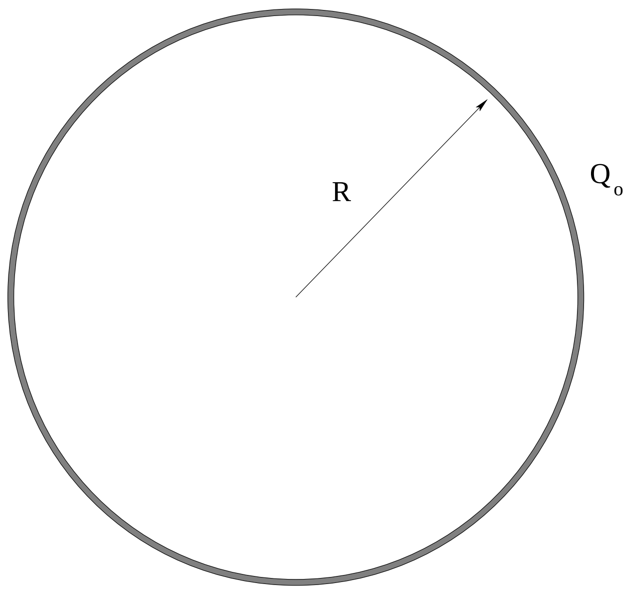
a) What is the ***total*** potential energy of this arrangement?

b) The top charge is released from rest and pushed away by the other charges in free space (no gravity or other objects nearby). What is its speed when it has travelled to where it is very far away?

7.2.3 Long Problems

Problem 68.

problems/potential-energy-pr-spherical-shell-charge.tex



A spherical conducting shell of radius R is pictured above. It is charged up to a total charge Q_0. Find the potential energy of the charged sphere.

(Note: There are two ways to do this problem, one very easy and one a bit harder. *Both* should be within your capabilities.)

Chapter 8

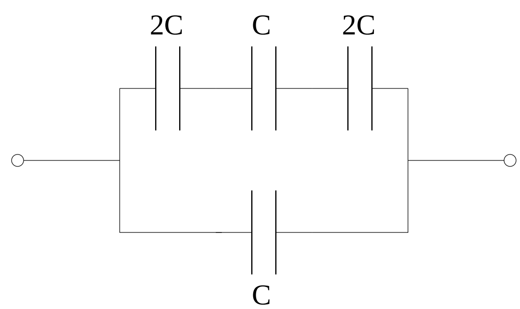
Capacitance and Dielectrics

8.1 Capacitance

8.1.1 Multiple Choice

Problem 69.

problems/capacitance-mc-four-capacitors.tex



What is the total capacitance of the arrangement of four capacitors above?

a) C

b) $2C$

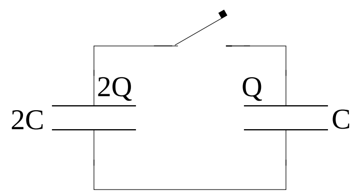
c) $3C/2$

d) $2C/3$

e) $5C$

Problem 70.

problems/capacitance-mc-two-capacitor-equilibrium.tex



In the figure above, the capacitor on the left has capacitance $2C$ and is charged with a battery to a total charge $2Q$. The capacitor on the right has capacitance C and is charged up to a total charge Q. The two capacitors are then connected as shown (still charged) and the switch is thrown. The charge on the ***left*** capacitor ($2C$) then:

a) Decreases

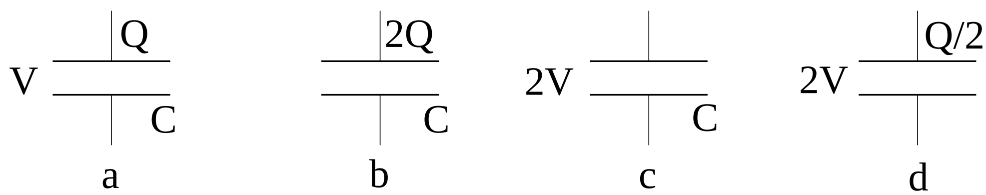
b) Increases

c) Remains the same.

8.1.2 Ranking/Scaling

Problem 71.

problems/capacitance-ra-energy-of-capacitor.tex



In the figure above four capacitors are shown. The first one (a) is a capacitor with capacitance C, charge Q, and has a voltage across the plates of V. The others have capacitance, charge, and/or voltages as shown, given in terms of C, Q, V from capacitor (a). Rank the ***energy stored*** on the capacitors from ***least to most***, with permitted equality. A possible answer might be (but probably isn't) $a = b < c < d$.

8.1.3 Short Answer/Concept

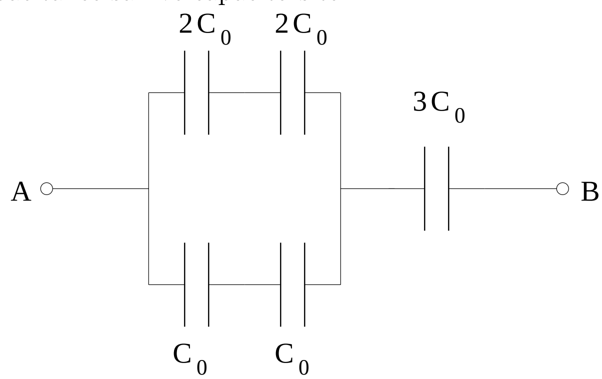
Problem 72.

problems/capacitance-sa-4-Cs-to-C.tex

You are given four capacitors, each with a capcitance of C. Discover an arrangement of these capacitors in series and/or parallel combinations that has a *net* capacitance of C.

Problem 73.

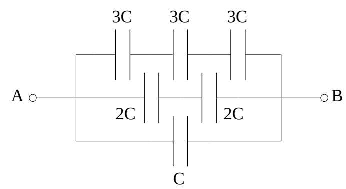
problems/capacitance-sa-five-capacitors.tex



Five capacitors, each with the capacitance shown (in terms of a reference capacitance C_0, are arranged in the circuit above. Find the total capacitance between the points A and B.

Problem 74.

problems/capacitance-sa-six-capacitors.tex



A network of capacitors with various values given as multiples of a reference capacitance C is drawn above. Find the total capacitance between points A and B in terms of C.

Problem 75.

problems/capacitance-sa-three-reasons-for-dielectrics.tex

Dielectric materials are generally used when designing capacitors for three excellent reasons. What are they?

a)

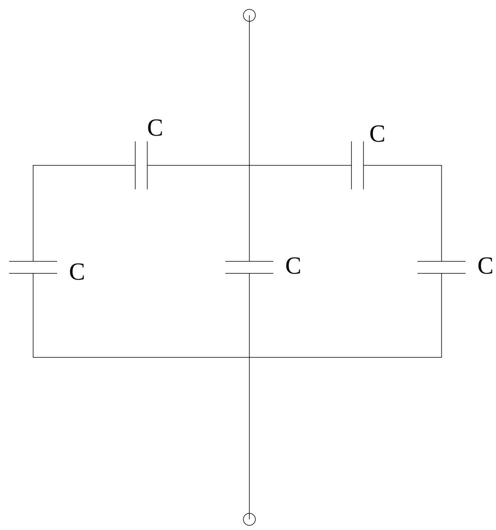
b)

c)

8.1.4 Long Problems

Problem 76.

problems/capacitance-pr-five-capacitors.tex

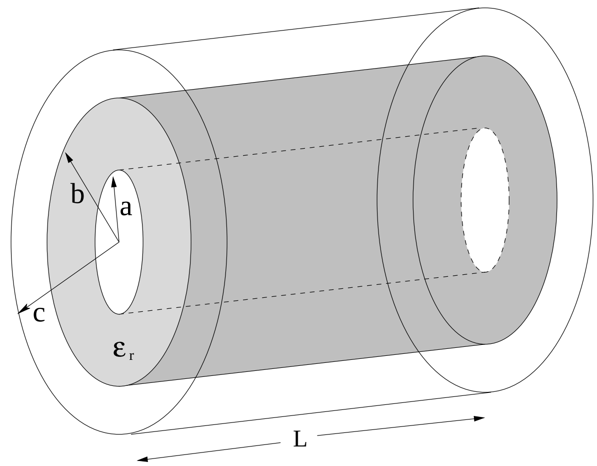


a) Find the total effective capacitance between the two contacts (the round circles at the top and the bottom) of the arrangement of capacitors drawn above. Naturally, show all work.

b) If a potential V is connected across the contacts, indicate the *relative* size of the charge on each capacitor. (It will probably be easiest if you give your answers in terms of $Q = CV$.)

Problem 77.

problems/capacitance-pr-half-filled-cylindrical.tex

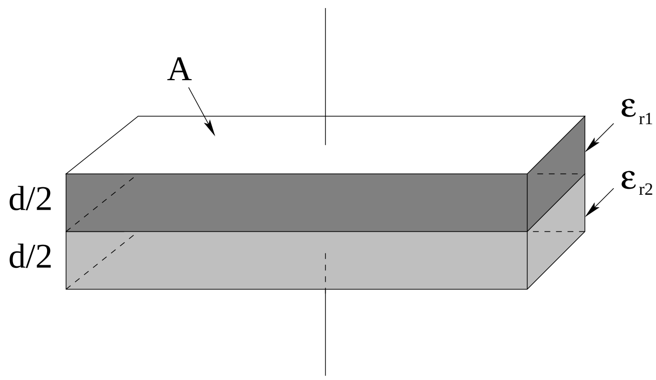


Derive C for the cylindrical capacitor drawn above. This capacitor has length L and consists of a conducting shell with radius a, a relative permittivity ϵ_r from radius a to radius b, and empty space from radius b to radius c, the outer conductor.

Show all work! You ***must*** follow the progression $\vec{E} \rightarrow \Delta V \rightarrow C$, inserting or using the important property of the dielectric where it is appropriate or necessary.

Problem 78.

problems/capacitance-pr-half-filled-dielectric-series.tex

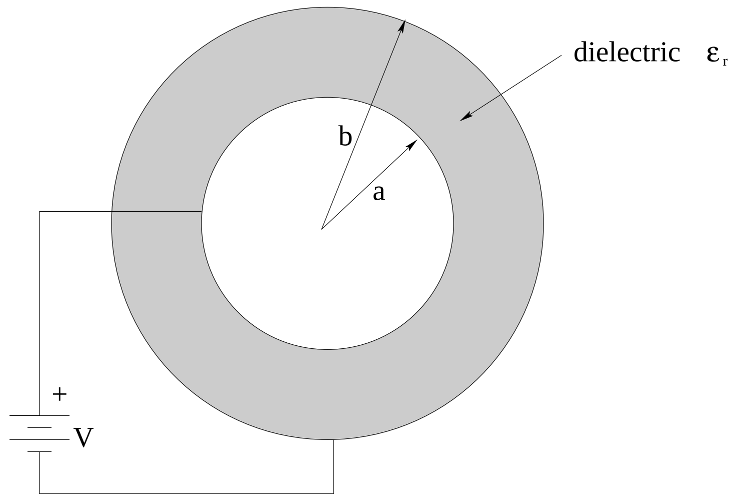


A parallel plate capacitor has cross sectional area A and separation d. A dielectric material with relative permittivity ϵ_r of thickness $d/2$ and area A half fills the space in between as shown.

Find the capacitance of this arrangement. *Show all work!*.

Problem 79.

problems/capacitance-pr-spherical-dielectric.tex

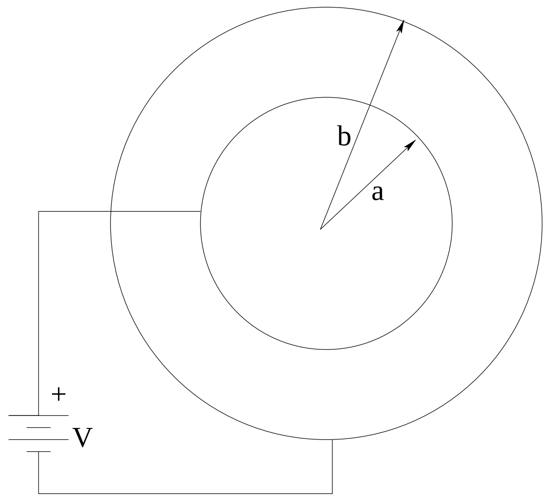


A spherical capacitor has inner radius a and outer radius b. The space between the spheres is filled with a dielectric of relative permittivity ϵ_r.

- Find the capacitance of this arrangement, including the effect of the dielectric ϵ_r. ***Show All Reasoning and Work!***
- Show that when $b = a + \delta$ with $\delta \ll a$ the capacitance has the limiting form $C = \epsilon_0 A/\delta$ (parallel plate result) where A is the area of the inner sphere and δ is the separation of the shells.

Problem 80.

problems/capacitance-pr-spherical.tex



A spherical capacitor has inner radius a and outer radius b

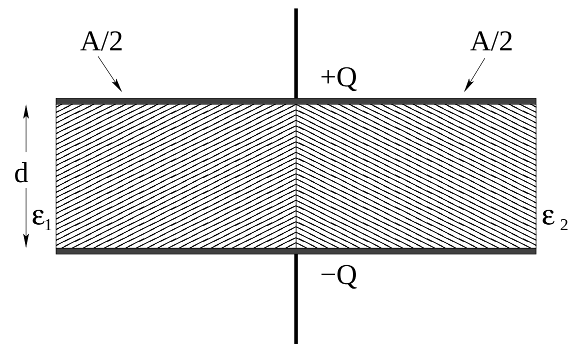
- Find the capacitance of this arrangement. ***Show All Work!***
- Show that when $b = a + \delta$ with $\delta \ll a$ the capacitance has the limiting form $C = \epsilon_0 A/\delta$ (parallel plate result) where A is the area of the inner sphere and δ is the separation of the shells.

8.2 Dielectrics

8.2.1 Long Problems

Problem 81.

problems/dielectrics-pr-capacitor-two-dielectrics-parallel.tex

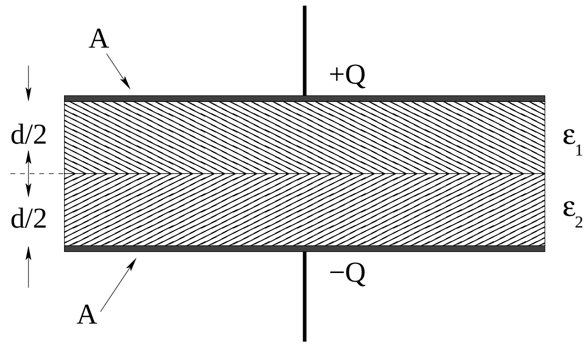


In the figure above, a parallel plate capacitor with cross-sectional area A and plate separation d is drawn. The space between the plates is filled with *two* dielectrics with relative permittivities $\epsilon_r = \epsilon_1$ and $\epsilon_r = \epsilon_2$ of thickness d and *area* $A/2$ as shown. A (free) charge Q is placed on the upper plate and $-Q$ is similarly placed on the lower plate.

Find the capacitance of this arrangement any way you like.

Problem 82.

problems/dielectrics-pr-capacitor-two-dielectrics-series.tex

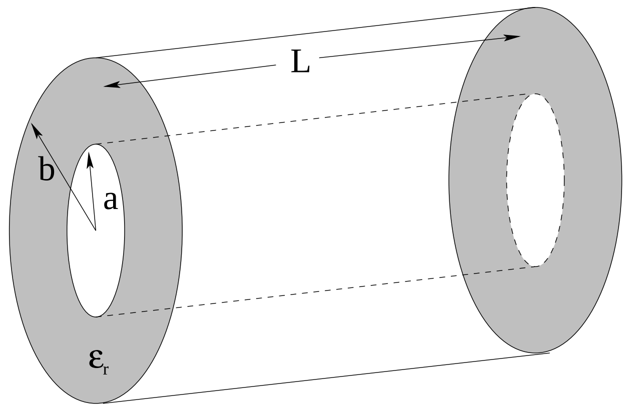


In the figure above, a parallel plate capacitor with cross-sectional area A and plate separation d is drawn. The space between the plates is filled with *two* dielectrics with relative permittivities $\epsilon_r = \epsilon_1$ and $\epsilon_r = \epsilon_2$ of equal thickness $d/2$ as shown. A (free) charge Q is placed on the upper plate and $-Q$ is similarly placed on the lower plate.

Find the capacitance of this arrangement any way you like.

Problem 83.

problems/dielectrics-pr-cylindrical-capacitor.tex

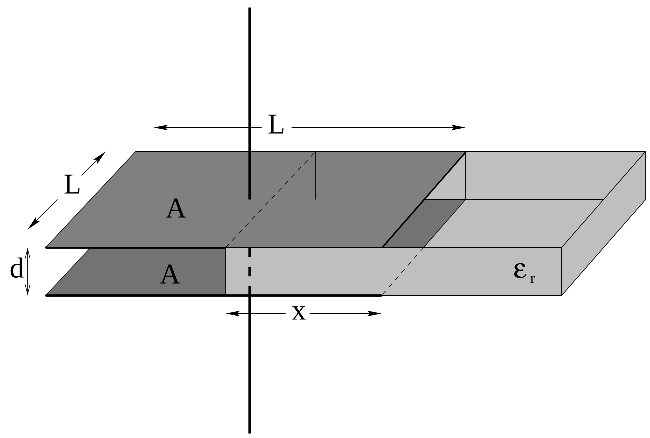


Derive C for the cylindrical capacitor drawn above, with inner radius a, outer radius b, length L, filled with a dielectric with relative permittivity ϵ_r.

Show all work! You ***must*** follow the progression $\vec{E} \rightarrow \Delta V \rightarrow C$, inserting or using the relevant important property of the dielectric medium either at the beginning or the end.

Problem 84.

problems/dielectrics-pr-force-constant-Q-half-filled.tex



A parallel plate capacitor is constructed from two square conducting plates with side length L and an area of $A = L^2$, separated by a distance of d. An insulating slab of thickness d and with a relative permittivity ϵ_r is inserted to a distance $x = L/2$ so that it ***half-fills*** the space between the plates as shown. Find:

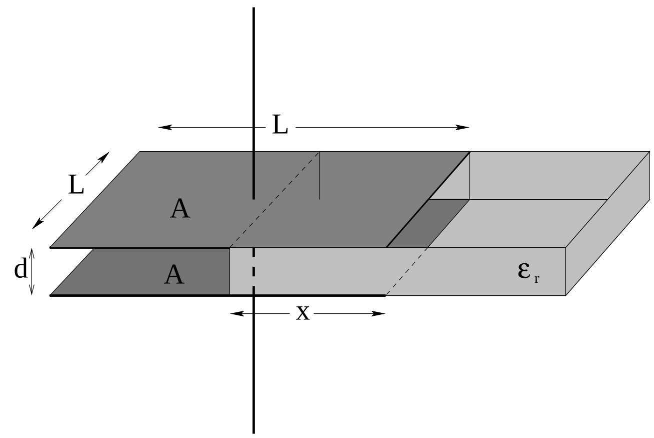
a) The capacitance of this arrangement.

b) When a constant charge $\pm Q$ is placed on the capacitor does the electric field pull the dielectric in between the plates or push it out from between them?

c) What ***is*** the force on the capacitor in this case? Hint: Consider how the energy stored on the capacitor varies with x.

Problem 85.

problems/dielectrics-pr-force-constant-V-half-filled.tex



A parallel plate capacitor is constructed from two square conducting plates with side length L and an area of $A = L^2$, separated by a distance of d. An insulating slab of thickness d and with a relative permittivity ϵ_r is inserted to a distance $x = L/2$ so that it ***half-fills*** the space between the plates as shown. Find:

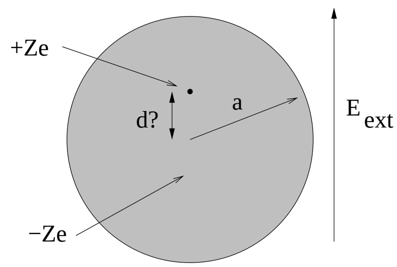
a) The capacitance of this arrangement.

b) When a potential V is maintained across the capacitor does the electric field pull the dielectric in between the plates or push it out from between them?

c) **Challenge Problem:** What ***is*** the force on the capacitor in this case? Note well: ***Do not forget the work done by the battery as the slab's position is varied!*** This is why it is difficult!

Problem 86.

problems/dielectrics-pr-lorentz-atom-polarizability.tex



A simple model for an atom has a tiny (point-like) nucleus with charge $+Ze$ is located at the center of a uniform sphere of charge with radius a and total charge -Ze. The atom is placed in a uniform electric field which displaces the nucleus as shown. Find:

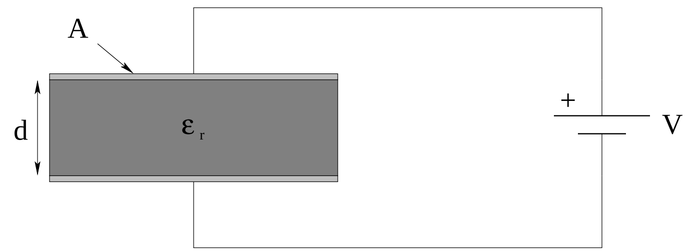
a) The equilibrium separation d of the nucleus from the center of the spherical electron cloud;

b) The *average* polarization density $\vec{\boldsymbol{P}}$ (dipole moment per unit volume) of a solid made up of these atoms, assuming that the atoms are arranged so that they "touch" in a simple cubic lattice (a three dimensional array where atoms sit at positions (x, y, z) where all three coordinates are integer multiples of $2R$).

Remember, this polarization density can be related to the total electric field inside the conductor by a relation like $\vec{\boldsymbol{P}} = \chi_e \epsilon_0 \vec{\boldsymbol{E}}$ where χ_e is called the ***electric susceptibility*** of the material. This in turn lets you completely understand $\epsilon_r = 1 + \chi_e$, the ***relative permittivity*** of the model material.

Problem 87.

problems/dielectrics-pr-parallel-plate-capacitor.tex



A parallel plate capacitor has cross sectional area A and separation d. A dielectric material with relative permittivity ϵ_r, thickness d, and area A fills the space in between as shown. A potential V is connected across this capacitor. Find:

a) The electric field inside the dielectric.

b) The free charge on the plates.

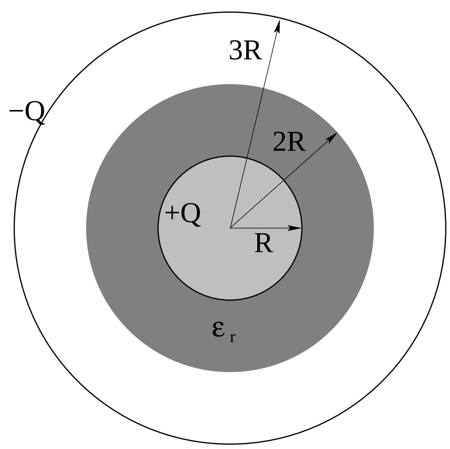
c) The capacitance.

d) The bound charge on the surface of the dielectric.

Show all work!

Problem 88.

problems/dielectrics-pr-spherical-capacitor-half-dielectric-series.tex



The inner conductor of a spherical capacitor of inner radius R is surrounded by a dielectric shell of thickness R and relative permittivity ϵ_r. The outer conductor of the capacitor is at $3R$.

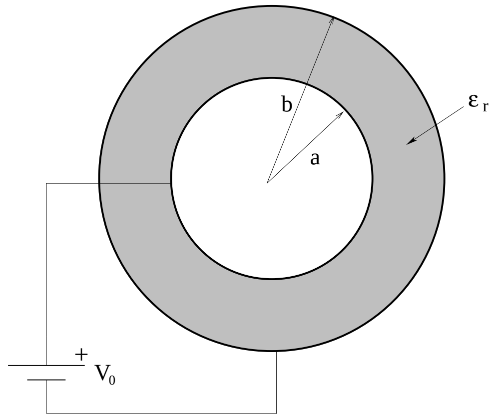
a) If the inner conducting sphere has a charge $+Q$ on it and the outer conducting sphere has a charge $-Q$ on it, what is the electric field in all space?

b) What is the potential difference between the inner shell and the outer one (just the magnitude is fine, but indicate which shell is at the higher potential on the figure).

c) Find the capacitance of this arrangement.

Problem 89.

problems/dielectrics-pr-spherical-capacitor.tex



A spherical capacitor with inner radius a and outer radius b has the space in between filled with a dielectric with relative permittivity ϵ_r. A potential V_0 is connected across it as shown. Find:

a) The field between the shells

b) The charge on the inner and outer shells

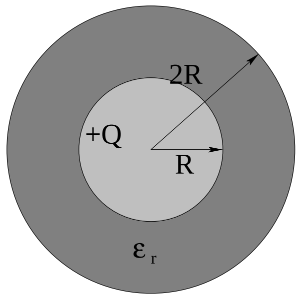
c) The capacitance.

d) The total energy of the charged capacitor

Show All Work!

Problem 90.

problems/dielectrics-pr-spherical-conductor-with-dielectric-shell.tex



An isolated conducting sphere of radius R is surrounded by a dielectric shell of thickness R with relative permittivity ϵ_r.

a) If the sphere has a charge Q on it, what is the electric field in all space?

b) What is the potential of the sphere?

c) Find its capacitance.

d) What is the total energy of the charged sphere?

Chapter 9

Resistance and DC Circuits

9.1 Resistance

9.1.1 Multiple Choice

Problem 91.

problems/resistance-mc-electric-field-in-current-carrying-wire.tex

A conducting wire of resistivity ρ, length L and cross sectional area A is carrying current I.

The wire has no electric field inside.

a) **T**

b) **F**

(Briefly explain your answer.)

Problem 92.

problems/resistance-mc-is-wire-with-current-equipotential.tex

A conducting wire of resistivity ρ, length L and cross sectional area A is carrying current I.

The wire is equipotential.

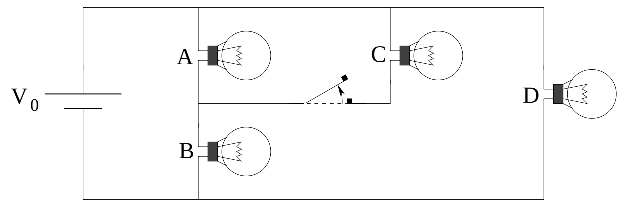
a) **T**

b) **F**

(Briefly explain your answer.)

Problem 93.

problems/resistance-mc-power-light-bulbs.tex



Four identical light bulbs are connected to a battery as shown. Initially the switch is closed and the circuit draws a total initial power from the battery P_i. At a certain time, the switch is opened, breaking the circuit through bulb C. P_f is the power drawn by the circuit after the switch is opened. At that time:

a) $P_f = P_i$ and bulb D is the brightest.

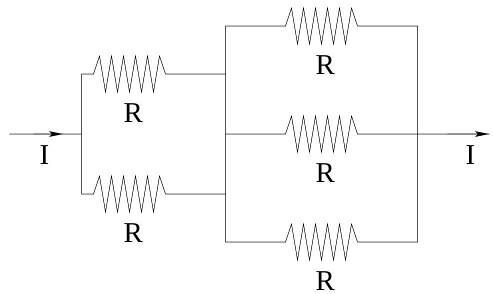
b) $P_f < P_i$ and bulb D is the brightest.

c) $P_f > P_i$ and bulb D is the dimmest.

d) $P_f = P_i$ and bulb D is the dimmest.

Problem 94.

problems/resistance-mc-power-resistance-network.tex



The total power dissipated in the circuit above is:

a) $3/5I^2R$

b) $2/3I^2R$

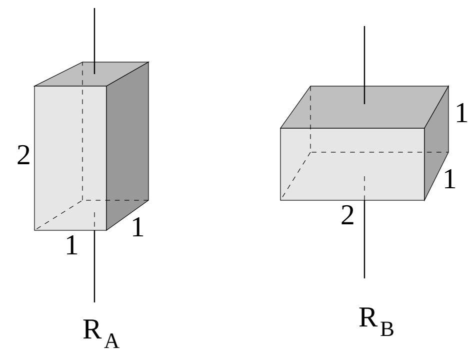
c) $4/5I^2R$

d) $5/6I^2R$

e) I^2R

Problem 95.

problems/resistance-mc-resistance-scaling-1.tex



Two resistors are made out of the same conducting material. In R_A, current runs across a length 2 in between two ***square*** faces that are 1×1 in size (in any units you like). In R_B, current runs across a length 1 in between two ***rectangular*** faces that are 2×1 in size.

The ratio R_A/R_B is:

a) $\frac{R_A}{R_B} = 1/4$

b) $\frac{R_A}{R_B} = 1/2$

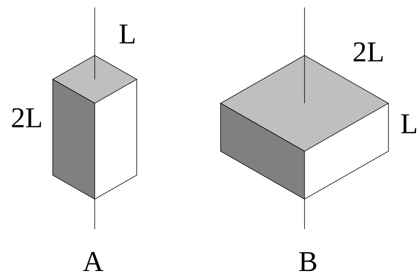
c) $\frac{R_A}{R_B} = 1$

d) $\frac{R_A}{R_B} = 2$

e) $\frac{R_A}{R_B} = 4$

Problem 96.

problems/resistance-mc-resistance-scaling-2.tex



Two box-shaped resistors with ***square*** end caps are made from the ***same*** material are shown, with their relative dimensions, above (the perfectly conducting leads are the vertical whiskers).

What is the ratio R_A/R_B?

a) $\frac{R_A}{R_B} = 1/4$

b) $\frac{R_A}{R_B} = 1/2$

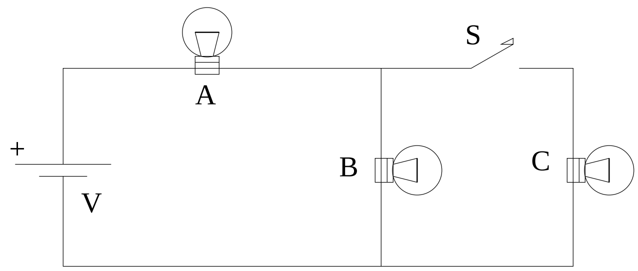
c) $\frac{R_A}{R_B} = 2$

d) $\frac{R_A}{R_B} = 4$

e) $\frac{R_A}{R_B} = 8$

Problem 97.

problems/resistance-mc-three-bulbs-1.tex



Three identical light bulbs are arranged in a simple DC circuit as drawn. At a certain time the switch S is closed. Does the brightness of bulb $\boldsymbol{A}$:

a) increase?

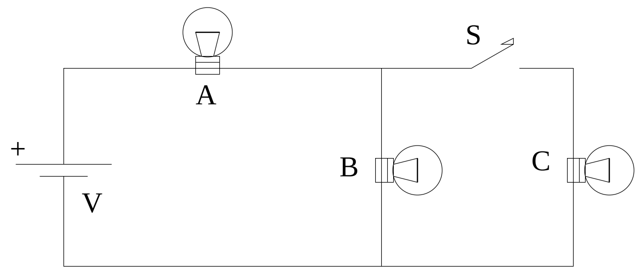
b) decrease?

c) stay the same?

Assume that the circuit is powered by an ideal battery with no internal resistance.

Problem 98.

problems/resistance-mc-three-bulbs-2.tex

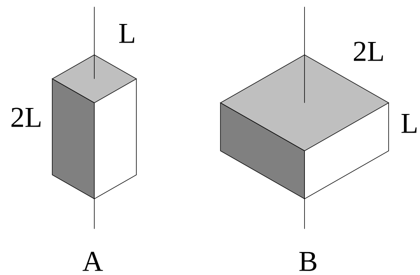


Three identical light bulbs are arranged in a simple DC circuit as drawn. At a certain time the switch S is closed. Does the brightness of bulb B increase, decrease, or remain the same? Assume that the circuit is powered by an ideal battery with no internal resistance.

9.1.2 Ranking/Scaling

Problem 99.

problems/resistance-ra-resistivity.tex



Two box-shaped resistors with square end caps are made from two ***different*** materials with their relative dimensions shown above (the perfectly conducting leads are the vertical whiskers). Their ***resistance is equal***. Rank the ***resistivity*** ρ of the materials (he two possible answers are $\rho_A < \rho_B$ or vice versa) and ***indicate why you made this choice.***

9.1.3 Short Answer/Concept

Problem 100.

problems/resistance-sa-kirchoffs-rules.tex

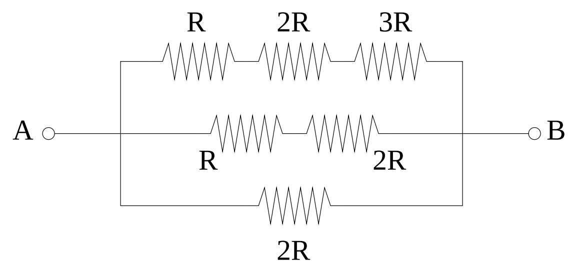
What are Kirchoff's Rules? Also (and still for credit!) what physical principle does each rule corresponds to?

a)

b)

Problem 101.

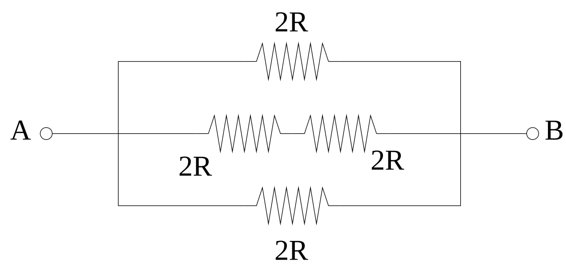
problems/resistance-sa-network-1.tex



A network of resistors with various values given as multiples of a reference resistance R is drawn above. Find the total resistance between points A and B in terms of R.

Problem 102.

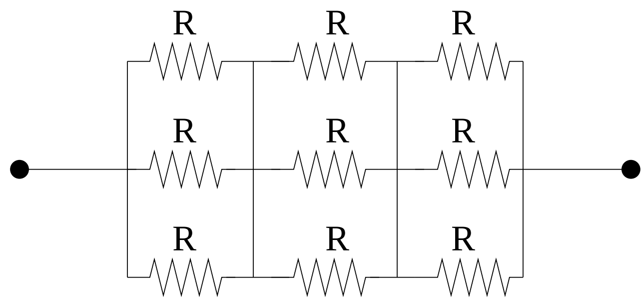
problems/resistance-sa-network-2.tex



A network of resistors with various values given as multiples of a reference resistance R is drawn above. Find the total resistance between points A and B in terms of R.

Problem 103.

problems/resistance-sa-network-3.tex

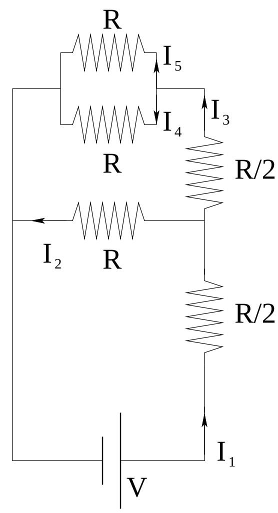


A network of resistors with various values given as multiples of a reference resistance R is drawn above. Find the total resistance between points A and B in terms of R.

9.1.4 Long Problems

Problem 104.

problems/resistance-pr-five-resistors-one-voltage.tex

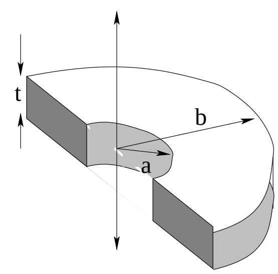


In the figure above, a $V = 8$ Volt battery is applied across the resistance network, where $R = 1$ Ohm. Find the currents I_1, I_2, I_3, I_4 and I_5, **and** indicate the total *power* delivered by the battery.

Your answer can be given algebraically in terms of V and R or numerically in terms of Amps and Watts. The arithmetic should be easy enough to do in your head should you wish to do the latter.

Problem 105.

problems/resistance-pr-half-annulus.tex



A printed circuit board contains a resistor formed of a semicircular bend of resistive material (resistivity ρ) of thickness t, inner radius a and outer radius b as shown in the figure above. Copper traces maintain a constant voltage V across the semicircular resistor. Find (in terms of the givens):

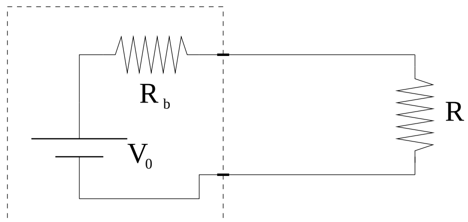
a) the resistance R of the resistor – be sure to indicate the basic formulae you are starting from for partial credit in case you can't quite get the integral right (it is like the integral in a homework problem);

b) the energy dissipated as heat in the resistor in s seconds. Again, if you cannot find R in terms of the givens (or have little confidence in your answer), you may use R and other given quantities to answer b) for at least partial credit.

Check the Units of Your Answers!

Problem 106.

problems/resistance-pr-impedance-matching.tex



In the circuit above, the dashed box represents a battery with an internal "pole voltage" of V_0 that has a signficant internal resistance R_b. R_l represents a "load" resistance (such as a light bulb or motor).

a) What is the current I in this circuit.

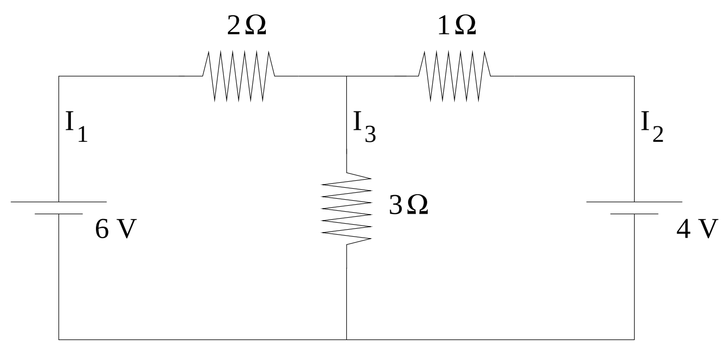
b) What is the power P_l delivered to the load resistance?

c) Prove that the power delivered to the load resistance is maximum when $R_l = R_b$.

This is called ***impedance matching*** – to get the most power out of a circuit element with some resistance/impedance, one matches it to the internal resistance of the part of the circuit providing the power.

Problem 107.

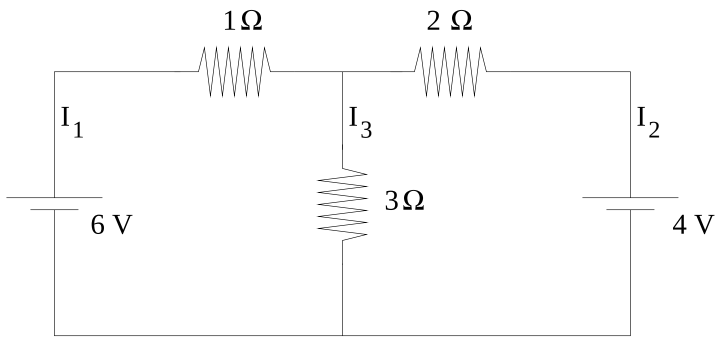
problems/resistance-pr-three-resistors-two-voltages-1.tex



Find the three currents I_1, I_2 and I_3 in the figure above, and clearly indicate their direction on the figure. Note that you'll likely have to assume a direction for each current in order to solve the problem, so go back and put your *final* direction(s) back in on the figure when you are done!

Problem 108.

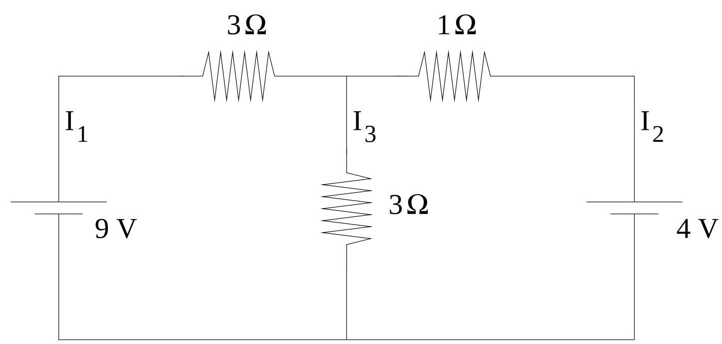
problems/resistance-pr-three-resistors-two-voltages-2.tex



Find the three currents I_1, I_2 and I_3 in the figure above, and clearly indicate their direction on the figure. Note that you'll likely have to assume a direction for each current in order to solve the problem, so go back and put your *final* direction(s) back in on the figure when you are done!

Problem 109.

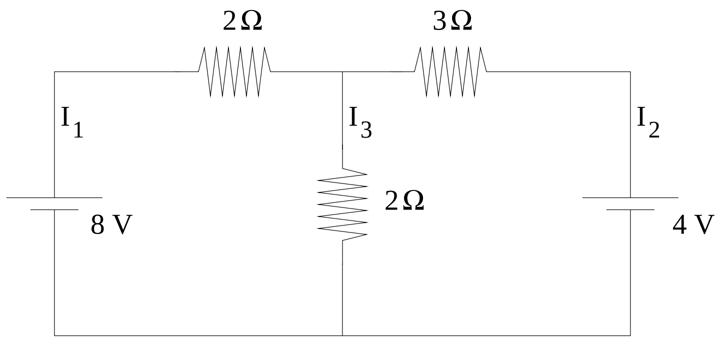
problems/resistance-pr-three-resistors-two-voltages-3.tex



Find the three currents I_1, I_2 and I_3 in the figure above, and clearly indicate their direction on the figure. Note that you'll likely have to assume a direction for each current in order to solve the problem, so go back and put your *final* direction(s) back in on the figure when you are done!

Problem 110.

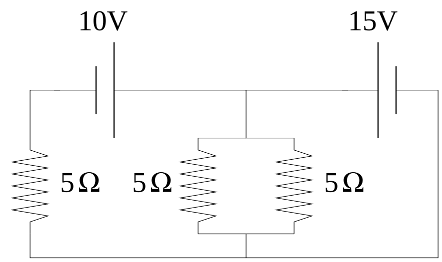
problems/resistance-pr-three-resistors-two-voltages-4.tex



Find the three currents I_1, I_2 and I_3 in the figure above, and clearly indicate their direction on the figure. Note that you'll likely have to assume a direction for each current in order to solve the problem, so go back and put your *final* direction(s) back in on the figure when you are done!

Problem 111.

problems/resistance-pr-three-resistors-two-voltages-5.tex

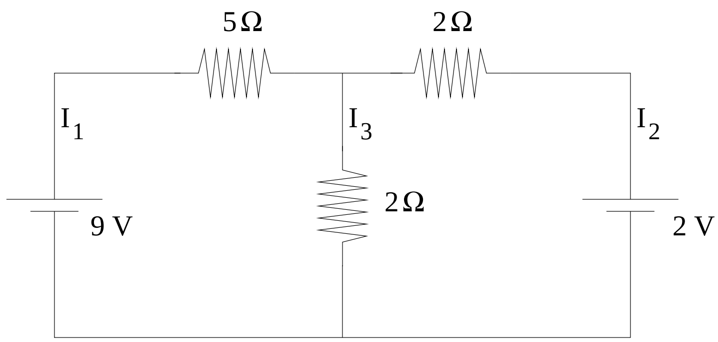


a) Find the magnitude and direction of the currents flowing in each resistor.

b) Find the combined power dissipated in the resistors.

Problem 112.

problems/resistance-pr-three-resistors-two-voltages-6.tex



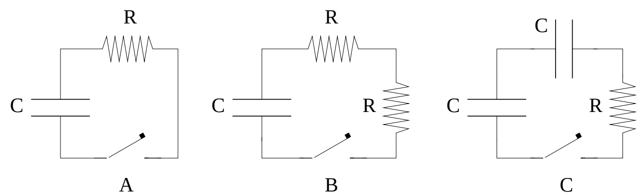
Find the three currents I_1, I_2 and I_3 in the figure above, and clearly indicate their direction on the figure. Note that you'll likely have to assume a direction for each current in order to solve the problem, so go back and put your *final* direction(s) back in on the figure when you are done!

9.2 RC Circuits

9.2.1 Ranking/Scaling

Problem 113.

problems/rc-circuits-ra-three-circuits.tex

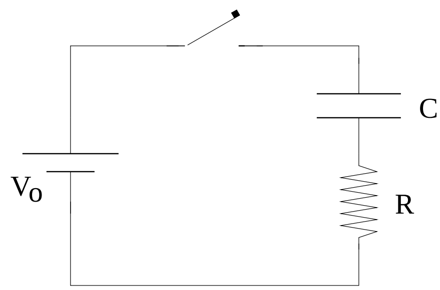


In the figure above, three RC circuits are shown, built out of equal resistances R and capacitances C. The circuits are all charged to a total charge Q and then discharged through the switch at the same time. Rank the circuits in the order of *fastest* discharge, where equality is possible (so e.g. $A = B > C$ is a possible answer).

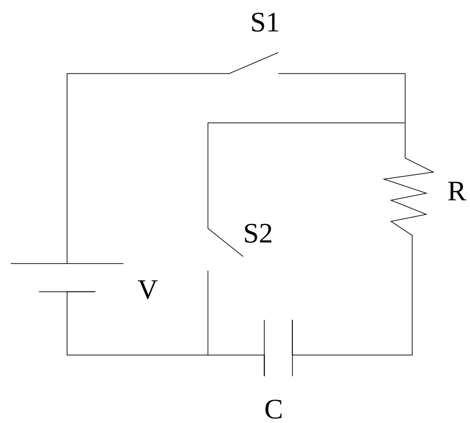
9.2.2 Short Answer/Concept

Problem 114.

problems/rc-circuits-sa-initial-current-charging.tex



In the figure above, the switch is closed at time $t = 0$ and the *uncharged* capacitor starts to charge. What is the *initial current* in the circuit *immediately* after the switch is closed?



9.2.3 Long Problems

Problem 115.

problems/rc-circuits-pr-charge-and-discharge-numbers.tex

In the circuit above, R = 100Ω and C = 1μFarad, and V = 10 volts. The capacitor is initially uncharged. To simplify arithmetic to the finger and toe level, answers given algebraically in terms of powers of e are acceptable – no calculators should be strictly necessary although you can smoke 'em if you got 'em.

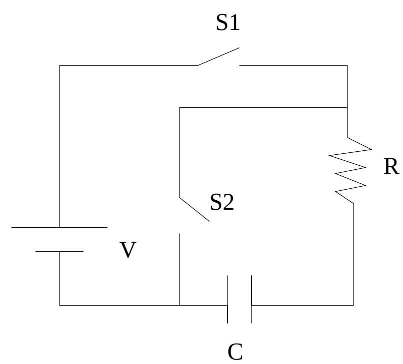
a) At time $t = 0$, switch 1 is closed. What is the charge on the capacitor as a function of time?

b) At time $t = 300$ microseconds, switch 1 is opened and switch 2 is closed. What is the voltage across the capacitor as a function of time.

c) At time $t = 500$ microseconds (from $t = 0$ in part a) switch 2 is opened. How much energy is stored in the capacitor at that time?

Problem 116.

problems/rc-circuits-pr-charge-and-discharge.tex



In the circuit above, assume R, C, and V are given. Derive all your answers for up to five bonus points, but you may just give the answers below for full credit. All answers may be expressed in terms of powers of e.

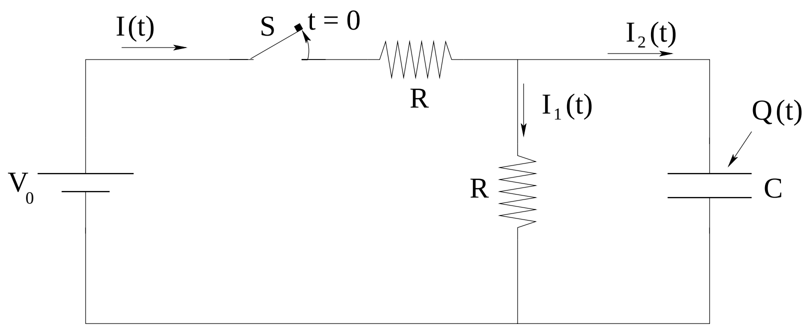
a) At time $t = 0$, switch 1 is closed. What is the *charge* on the capacitor as a function of time in terms of the given quantities.

b) After a very long time ($\gg RC$) switch 1 is opened and switch 2 is closed. What is the *voltage* across the capacitor as a function of time.

c) At time $t = 2RC$ switch 2 is opened. How much energy is stored in the capacitor at that time?

Problem 117.

problems/rc-circuits-pr-charging-discharging-two-R-paths.tex



In the circuit above, the switch S has been closed for a ***very long time already*** at $t = 0$. Find:

a) The initial currents $I_1(0)$ and $I_2(0)$.

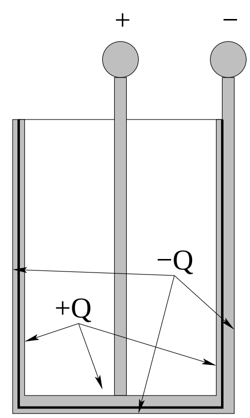
b) The initial charge $Q(0)$ on the capacitor.

c) At $t = 0$ the switch is opened. Find the charge on the capacitor $Q(t)$ for all $t > 0$.

Note that you can solve parts b) and c) independently by assuming that you know $Q(0)$ for part c).

Problem 118.

problems/rc-circuits-pr-leyden-jar-discharge-time.tex

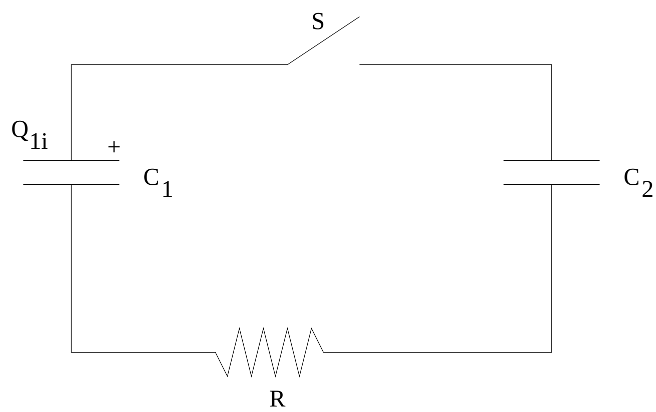


A large Leyden jar (capacitor) is surrounded by dry air so that the net resistance between its charged and grounded terminal is approximately 10^{10} Ω. It is charged up to 50,000 volts by a Wimshurst generator (at which time it contains 0.005 Coulombs of charge). It is then disconnected and left there by a negligent physics instructor. 33 1/3 minutes later, an astrophysics professor comes into the room and, seeking to move the jar, grabs the ungrounded, charged, central terminal. How much charge seeks ground through this hapless soul's body? How much stored energy is dissipated in the process? (You can solve this algebraically if you have no calculator handy.)

Ouch! These are not unrealistic parameters. Leyden jars or other large capacitors can be very, very dangerous for hours after they are charged up.

Problem 119.

problems/rc-circuits-pr-two-capacitors-one-resistor.tex



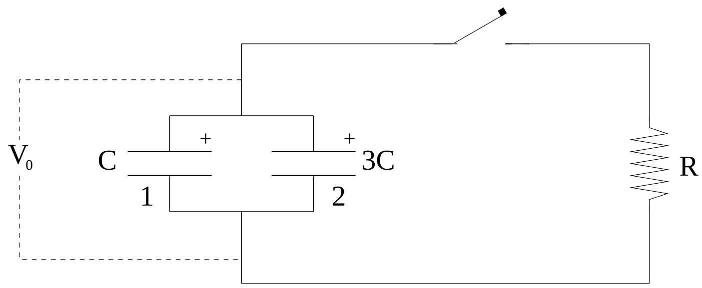
A pair of capacitors C_1 and C_2 is connected as shown, with a resistance R in between them. Initially, the first capacitor carries a total charge Q_{1i} and the second one is uncharged, $Q_{2i} = 0$. At $t = 0$ the switch is closed. Find:

a) The equilibrium ($t = \infty$) charges on the two capacitors, Q_{1f} and Q_{2f}.

b) Using Kirchoff's laws for this arrangement, find the time constant for the equilibration process. Note that you do NOT have to solve the DE, just formulate it with dt and some arrangement of R, C_1, and C_2 on the other side.

Problem 120.

problems/rc-circuits-pr-two-capacitors.tex



In the figure above, a voltage V_0 is briefly applied across two capacitors in parallel to charge them to this potential difference as shown (higher potential on the upper plate) *with the switch open.* At time $t = 0$ the switch is closed. Find:

a) The initial charges on the two capacitors, Q_1 and Q_2, given that the capacitances are C and $3C$ respectively.

b) The initial energy stored on the capacitors.

c) The time constant of the discharge process in terms of R and C.

d) Write Kirchoff's Loop Rule for this circuit at an arbitrary time t after the switch is closed and turn it into a first order, linear, homogeneous ordinary differential equation (of motion) for the total charge on the capacitors.

e) Write down and sketch a graph of $Q(t)$, the total charge on the capacitors as a function of time. If you don't remember it, you are welcome to derive it, and if you do derive it you might get a few extra credit points. Be sure to mark τ, the exponential time constant of the discharge process, on your graph to set its scale.

Chapter 10

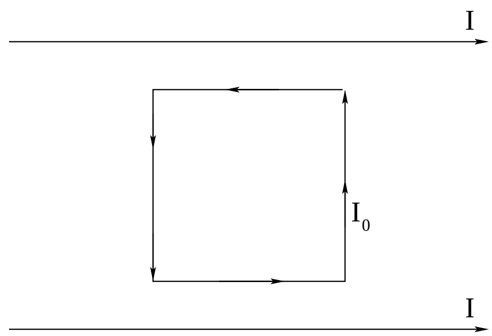
Magnetic Force

10.1 Magnetic Force

10.1.1 Multiple Choice

Problem 121.

problems/magnetic-force-mc-loop-between-two-wires.tex



A square conducting loop carries a current I_0 in the direction shown. It is sitting symmetrically between two wires each carrying a current I to the right. The net force on the rectangular loop is:

a) up.

b) down.

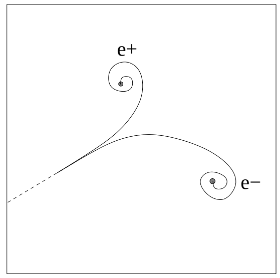
c) left.

d) right.

e) zero.

Problem 122.

problems/magnetic-force-mc-positron-electron-cloud-chamber.tex



The figure above portrays the tracks left by a ***positron*** and an ***electron*** (labelled) in a cloud chamber. Is the magnetic field that bends their tracks (predominantly):

a) into

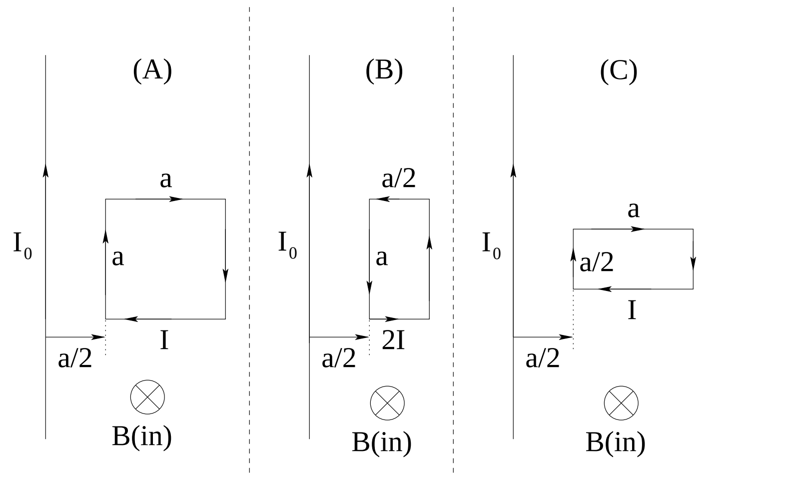
b) out of

the paper? ***Circle a or b.***

10.1.2 Ranking/Scaling

Problem 123.

problems/magnetic-force-ra-magnetic-force-rectangular-loops.tex



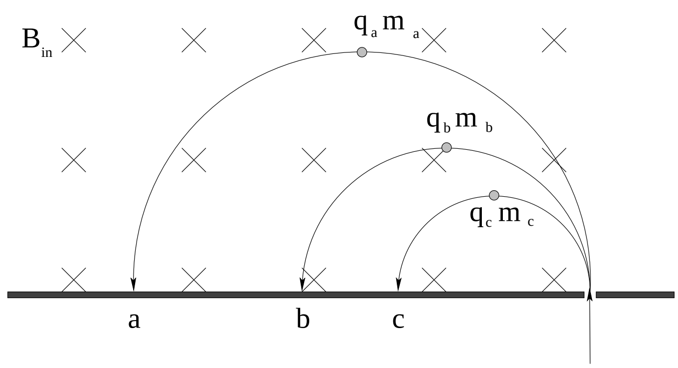
In the figures above, the long straight wire to the left produces a field of varying magnitude:

$$B = \frac{\mu_0 I_0}{2\pi r}$$

where r is the perpendicular distance of the point of observation from the wire that is ***into the page*** at the location of the wire loops on the right (which have the dimensions and carry the currents shown). The near wire of all of the loops is a distance $a/2$ from and parallel to the long straight wire. For each example, rank the *magnitude* of the net magnetic force on the loop compared to the others, and indicate the *direction* of the net force on the diagram with an arrow. Equality is permitted, so your answer for the ranking might be $A = B > C$.

Problem 124.

problems/magnetic-force-ra-three-curved-trajectories.tex



In the figure about, particles a, b and c enter a magnetic field travelling in a straight line as shown. All three particles have the same *velocity* as they enter. Circle all of the statements below that *could* be consistent with the observed trajectories of the particles.

a) All the particle have the same mass m, and $q_a > q_b > q_c$.

b) All of the particles have the same charge q, and have mass $m_a < m_b < m_c$.

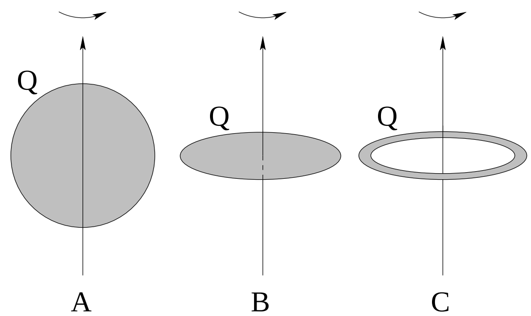
c) All of the particles have the same mass, and $q_c > q_b > q_a$.

d) All of the particles have the same charge q, and have mass $m_c < m_b < m_a$.

e) The particles have $q_c > q_b > q_a$ *and* $m_c < m_b < m_a$.

Problem 125.

problems/magnetic-force-ra-three-magnetic-moments.tex

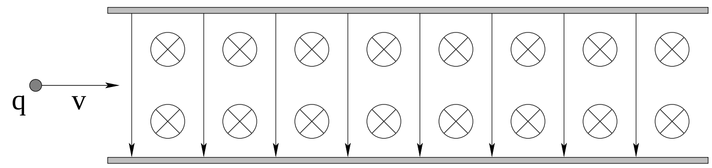


Three objects that each have a total charge Q uniformly distributed in the grey region are pictured above, rotating about the z-axis (which is an axis of symmetry for the object). The *horizontal dimension* (width) of each object is the same, as shown. Rank the magnetic moments of the objects from smallest to largest, where equality is a possible answer. That is an answer might be $A < B = C$.

10.1.3 Short Answer/Concept

Problem 126.

problems/magnetic-force-sa-region-crossed-fields.tex



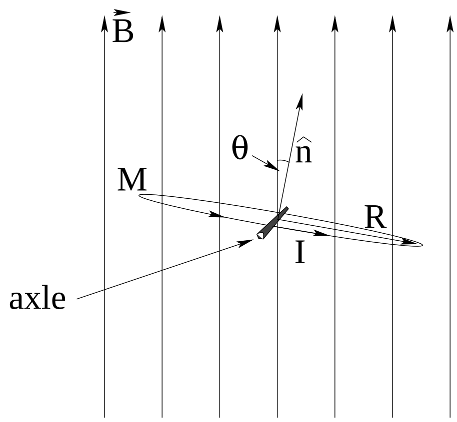
E down, B in

Above is portrayed a region of crossed $\vec{E}$ and $\vec{B}$ fields. If one shoots a charged particle q through the region from the left, what must its *speed* be for it to pass through the region undeflected? Neglect fringe fields.

10.1.4 Long Problems

Problem 127.

problems/magnetic-force-pr-circular-loop-oscillator.tex

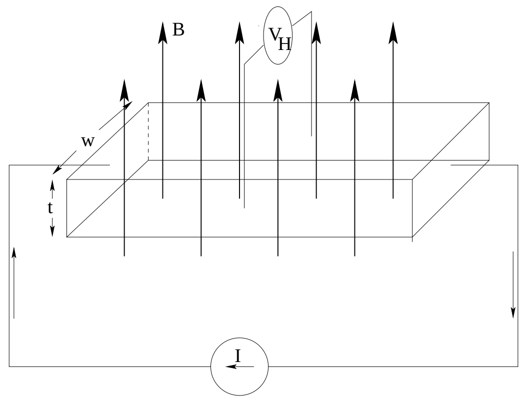


In the figure above, a circular loop of wire of radius R carries a current I and has mass M. The loop is pivoted so it can turn freely about an axle through its center (more or less perpendicular to the plane of the paper as shown). A uniform magnetic field $\vec{B}$ in the z-direction surrounds the loop. The loop is twisted through a small angle θ relative to the applied field and released to ***oscillate***.

Find the angular frequency of oscillation. Note that the moment of inertia of a circular ring of mass M and radius R about the axis shown is $I_{\rm rot} = \frac{1}{2}MR^2$.

Problem 128.

problems/magnetic-force-pr-hall-effect-2.tex

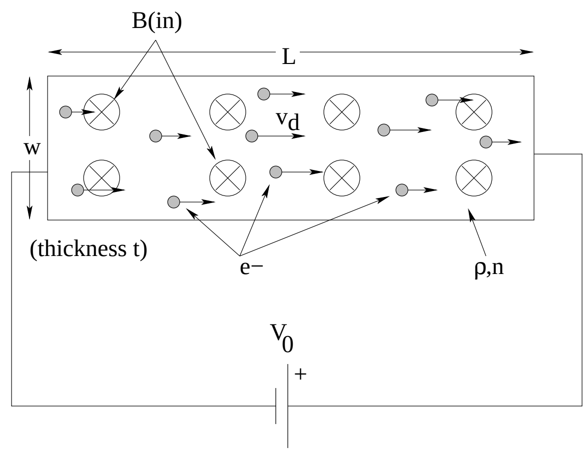


Voltage V_H measured across w
Magnetic Field B perpendicular to strip as shown
Current I measured through strip in direction shown

The apparatus for measuring the Hall effect is shown above. Consider a charge carrier q (to keep you from having to mess with the negative charge on the real charge carriers – electrons) moving through the apparatus in a material with an unknown n charge carriers per unit volume. **Derive** an expression for n, given I, V_H, t, B, w and q. Note that I'd have to consider you moderately insane to have memorized this result (I certainly haven't) but by considering the strip to be a region of self-maintaining crossed fields and relating the current to the drift velocity you should be able to get it fairly easily.

Problem 129.

problems/magnetic-force-pr-hall-effect.tex



A rectangular metal strip of length L, width w, and thickness t sits in a uniform magnetic field B perpendicular to the strip and into the page as shown. The material has resistivity ρ and a free (conducting) *electron* (charge $q = -e$) density of n. A voltage V_0 is connected across the strip so that the *electrons* travel from left to right as shown.

a) What is the resistance of the strip in terms of ρ and its dimensions?

b) Find an expression for the Hall potential (the potential difference across the strip from top to bottom) in terms of the given quantities.

c) Is the top of the strip at *higher* or *lower* potential than the bottom?

Hints: you'll have to start by determining I (the current in the strip) in terms of the givens, and then translate that into a form involving the drift velocity v_d. From v_d and your knowledge of magnetic forces you should be able to determine the electric field and then the potential across the strip in the steady state.

Problem 130.

problems/magnetic-force-pr-helical-motion-point-charge.tex

A beam of particles with velocity $\vec{\boldsymbol{v}}$ enters a region of uniform magnetic field $\vec{B}$ such that $\vec{\boldsymbol{v}}$ makes a small angle $\theta < \pi/2$ with $\vec{B}$. Show that after a particle moves a distance

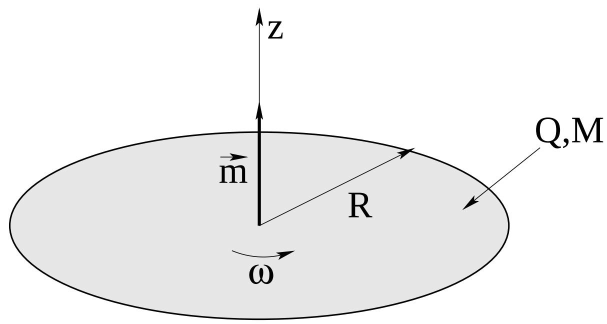
$$D = \frac{2\pi m}{qB} v \cos(\theta)$$

measured ***along the direction of*** $\vec{B}$, the velocity of the particle is in the same direction as it was when it entered the field.

Hint: The magnetic force does not alter the component of the velocity in the same direction as the field.

Problem 131.

problems/magnetic-force-pr-magnetic-moment-rotating-disk-2.tex



A flat disk of radius R and mass M with uniform surface charge density

$$\sigma = \frac{Q}{\pi R^2}$$

is rotating at angular velocity ω about the z-axis as shown.

a) Find its magnetic moment $\vec{\boldsymbol{m}}$.

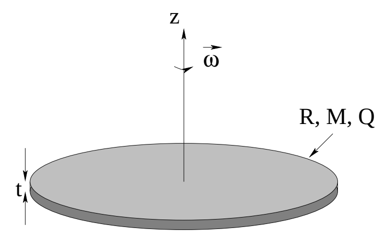
b) Show that:

$$\vec{\boldsymbol{m}} = \frac{Q}{2M}\vec{\boldsymbol{L}}$$

where $\vec{\boldsymbol{L}}$ is the angular momentum of the disk.

Problem 132.

problems/magnetic-force-pr-magnetic-moment-rotating-disk.tex



A disk of radius R and thickness t, with uniform charge density ρ_q and uniform mass density ρ_m is rotating at angular velocity $\vec{\omega} = \omega\hat{z}$.

Consider a tiny differential chunk of the disk's volume $dV = dA\ t$ located at r, θ in cylindrical polar coordinates. Note that this chunk is orbiting the z-axis at angular frequency ω in a circular path.

a) Find the magnetic moment dm_z of this chunk in terms of ρ_q, ω, dV and its coordinates.

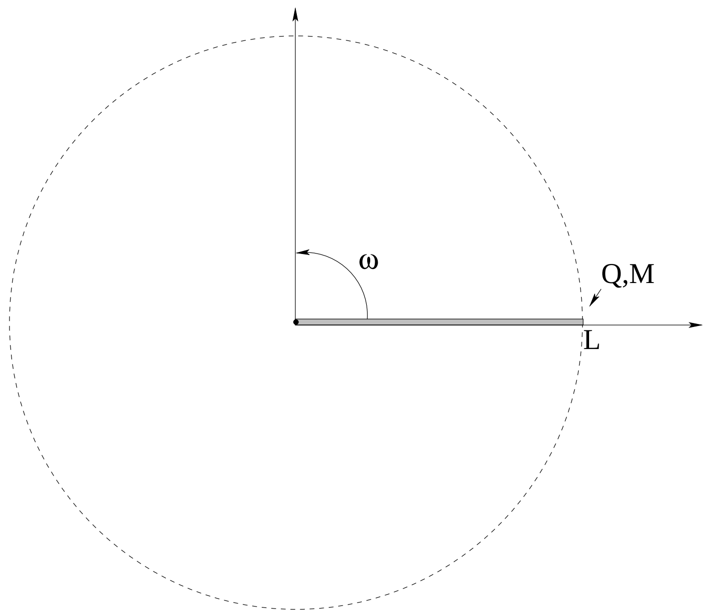
b) Find the angular momentum dL_z of this chunk in terms of ρ_m, ω, dV and its coordinates.

c) Doing the two (simple) integrals, express them in terms of the total charge and total mass of the disk, respectively, and show that the magnetic moment of the disk is given by $\vec{m} = \mu_B \vec{L}$, where $\mu_B = \frac{Q}{2M}$.

d) What do you expect the magnetic *field* of this disk to look like on the z axis for $z \gg R$? (Answer in terms of $\vec{\boldsymbol{m}}$ is fine.)

Problem 133.

problems/magnetic-force-pr-magnetic-moment-rotating-rod.tex



A rod of mass M and length L is uniformly charged with a total charge Q and pivoted around one end. It is rotating in a plane at angular velocity ω. Find:

a) The magnetic moment of the rod in the direction of its angular velocity (axis of rotation). Is it into or out of the page as drawn? (You will have to do a simple integral for this – what is the average "current" of a small chunk of the rod a distance x from the pivot?)

b) The angular momentum of the rod. Note that its moment of inertia is:

$$I = \frac{1}{3}ML^2$$

c) **From** your answers to a) and b), show that

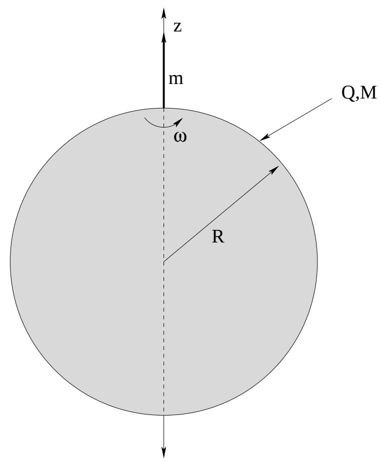
$$\vec{\boldsymbol{m}} = \mu_B \vec{\boldsymbol{L}}$$

and find μ_B. If you cannot make it work out but know the answer, be sure to put it down (but *getting* it to work out is an important check that will give you confidence in your answers to a) and b).

Show all work.

Problem 134.

problems/magnetic-force-pr-magnetic-moment-rotating-sphere.tex



A sphere of radius R with uniform charge density $\rho_q = \frac{3Q}{4\pi R^3}$ and uniform mass density $\rho_m = \frac{3M}{4\pi R^3}$ is rotating at angular velocity $\vec{\omega} = \omega\hat{z}$.

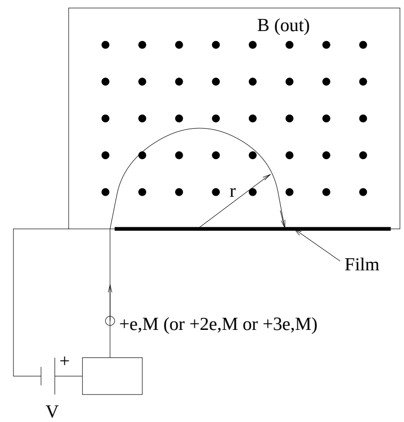
Consider a tiny differential chunk of the sphere's volume dV located at r, θ, ϕ in spherical polar coordinates. Note that this chunk is orbiting the z-axis at angular frequency ω in a circular path.

a) Find the magnetic moment dm_z of this chunk in terms of ρ_q, ω, dV, and its coordinates. You do not need to express dV in coordinates – leave it as dV.

b) Find the angular momentum dL_z of this chunk in terms of ρ_m, ω, dV, and its coordinates.

c) Expressing the two integrals *without doing them in actual coordinates*, show that the magnetic moment of the sphere is given by $\vec{m} = \mu_B \vec{L}$, where $\mu_B = \frac{Q}{2M}$.

d) For extra credit: Despite its generality, the conclusion is not true for *any* shape with a uniform mass and charge density rotating about the z-axis at a constant angular velocity $\vec{\omega}$. Why not? Give a specific example of a (simple) distribution for which it is not true, and/or the condition for it to be true.



Problem 135.

problems/magnetic-force-pr-mass-spectrograph.tex

In the mass spectrograph above, the goo in the source chamber contains molecules of mass M that are ionized to have charges of $+e$, $+2e$ or $+3e$ at the source. The particles then fall through a potential of V and enter the uniform B field in the box.

a) Derive an expression for the radius r at which a fragment of charge-to-mass ratio of m/q hits.

b) Use this expression to find r for each of the three possible ionization charges, and draw a picture of the bars produced on the film to a reasonable scale.

Problem 136.

problems/magnetic-force-pr-motion-of-point-charge.tex

A particle of charge q and mass m has momentum (magnitude) $p = mv$ and kinetic energy $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$. If the particle moves in a circular orbit of radius r perpendicular to a uniform magnetic field of magnitude B, show that:

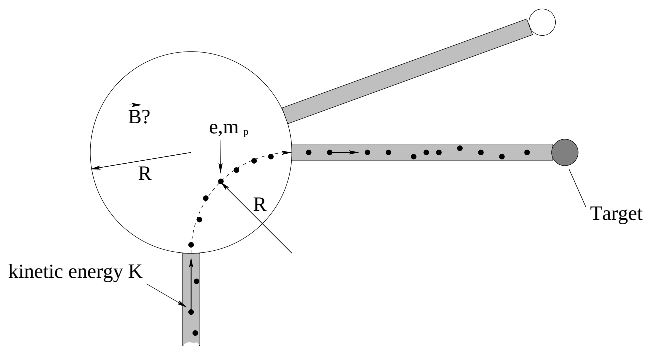
a) The magnitude of the momentum is $p = Bqr$

b) The kinetic energy is $K = \frac{B^2q^2r^2}{2m}$

c) The magnitude of the angular momentum is $L = Bqr^2$.

Problem 137.

problems/magnetic-force-pr-particle-beam-bending-magnet.tex



A beam of protons (charge $+e$, mass m_p) with kinetic energy K produced by an accelerator moving inside a beam pipe is incident on a circular bending magnet that creates a uniform field in the circular region drawn above. We wish to bend it through $\pi/2$ so it emerges from the magnet in the indicated pipe to hit the target at the end. Assume that the magnetic field cuts off sharply at the edge of the magnet.

a) In order to bend the beam as shown, is $\vec{B}$ into the page or out of the page?

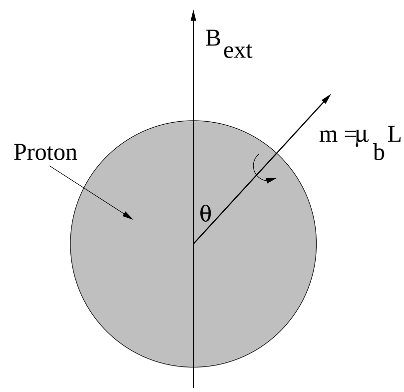
b) What is the speed of a proton in the beam in terms of K and m_p?

c) What is the ***magnetic force*** acting on the proton indicated in terms of the givens? Draw the direction in on the figure.

d) What should we set the magnitude of the magnetic field B to be as a function of K, m_p, e and R in order to put the beam of particles on target as shown?

Problem 138.

problems/magnetic-force-pr-precession-of-proton-advanced.tex



Challenge problem for physics majors: A proton (charge $+e$) with mass m_p has an intrisic angular momentum given by $\vec{L}$ and a magnetic moment given by $\vec{m} = \mu_B \vec{L}$. When the proton is placed in a uniform magnetic field of strength $\vec{\boldsymbol{B}} = B_0 \hat{\boldsymbol{z}}$ so that $\vec{L}$ makes an angle of θ with $\vec{\boldsymbol{B}}$, the angular momentum precesses around $\vec{\boldsymbol{B}}$.

a) ***Using calculus, derive*** the angular frequency ω_p with which the angular momentum precesses. That is, from the equations of motion for the x and y components angular momentum, show that L_x and L_y oscillate harmonically and determine their oscillation frequency.

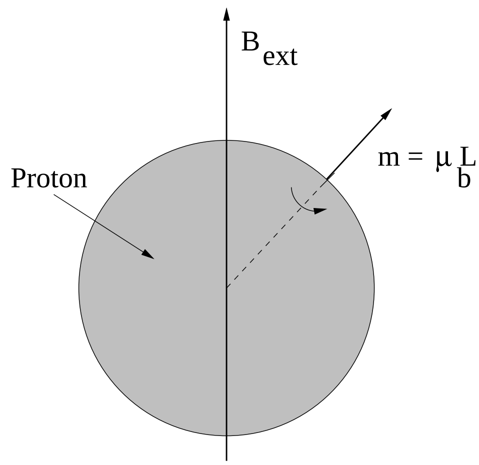
b) Indicate the direction of precession on the figure above (into or out of page, as drawn).

c) Solve the equations of motion to obtain $L_x(t)$ assuming that it is at a positive maximum at $t = 0$. From this and the equations of motion obtained above, find $L_y(t)$ and $L_z(t)$. You now know ***exactly*** what the angular momentum of the proton does in

d) Finally, ***derive*** $\mu_B = Q/2M$. One way you might proceed is to simply derive $\vec{m}$ and $\vec{L}$ separately for the proton, assuming uniform mass and charge distribution and a common angular velocity $\vec{\omega}$. A better way to proceed might be to relate dm_z (along the axis of rotation) to dL_z (ditto) assuming axial symmetry so that $\vec{L}$ is parallel to $\vec{\omega}$.

Problem 139.

problems/magnetic-force-pr-precession-of-proton.tex



A model for a proton (charge $+e$) with mass m_p has an intrinsic angular momentum given by $\vec{L}$ and a magnetic moment given by $\vec{m} = \mu_B \vec{L}$ (where $\mu_B = Q/2M$ for reasons you should completely understand). When the proton is placed in a uniform magnetic field of strength B so that $\vec{L}$ makes an angle of θ with $\vec{B}$, the angular momentum precesses around $\vec{B}$.

Find (*derive*) the angular frequency with which the angular momentum precesses. Indicate the direction of precession on the figure above (into or out of page, as drawn).

Chapter 11

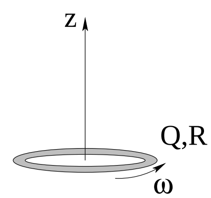
Magnetic Field

11.1 Biot-Savart Law

11.1.1 Multiple Choice

Problem 140.

problems/bfield-mc-magnetic-dipole.tex



A ring of radius R of charge Q is rotating at angular velocity ω about the z-axis as shown. The field at an arbitrary point on the z-axis far from the ring ($z \gg R$) is:

a) $\frac{k_m Q R^2 \omega}{z}$

b) $\frac{k_m Q R^2 \omega}{z^2}$

c) $\frac{k_m Q R^2 \omega}{6z^2}$

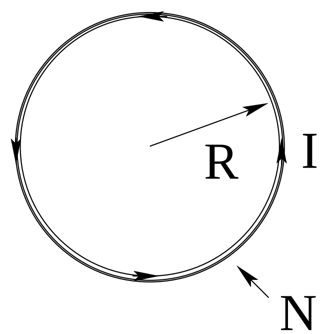
d) $\frac{k_m Q R^2 \omega}{z^3}$

e) $\frac{k_m Q R^2 \omega}{6z^3}$

11.1.2 Ranking/Scaling

Problem 141.

problems/bfield-ra-circular-current-loop-center.tex



A plane circular loop of wire with N turns, radius R, and current in each turn I is shown above and produce a magnetic field with strength B (out of the page) ***at the geometric center of the loop*** j.

Four possible sets of relative values of N, R, I are given below. Rank the magnitudes of B from least to greatest for the four cases where equality might be an answer (so an answer might be, but probably isn't: $a < b < c = d$).

a) $N = N_0$, $R = R_0$, $I = I_0$

b) $N = 2N_0$, $R = 2R_0$, $I = I_0$

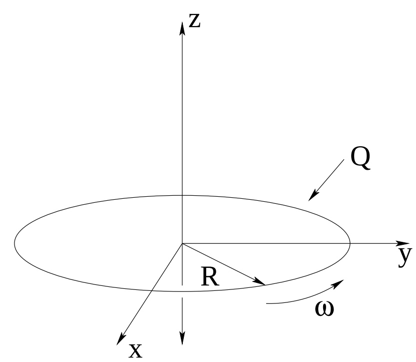
c) $N = 2N_0$, $R = R_0$, $I = I_0/2$

d) $N = 2N_0$, $R = 2R_0$, $I = 2I_0$

11.1.3 Short Answer/Concept

Problem 142.

problems/bfield-sa-spinning-ring-at-center.tex

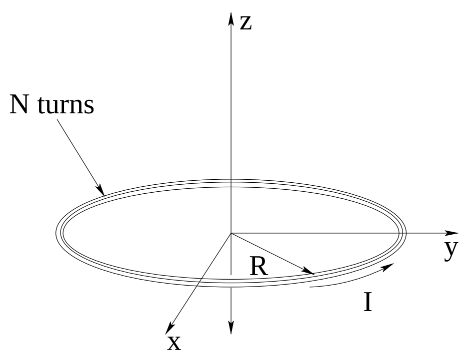


In the figure above a ring of radius R with total charge Q is spinning at angular velocity ω around the z-axis. What is the magnetic field strength at the origin?

11.1.4 Long Problems

Problem 143.

problems/bfield-pr-circular-current-loop-N-turns.tex



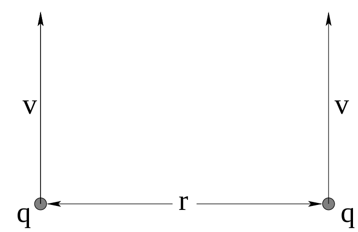
In the figure above a circular current loop of radius R and N turns carries a current I.

a) Find the magnetic field at an arbitrary point on the z-axis.

b) What is the asymptotic form magnetic field in the limit that $z \gg R$, expressed in terms of the magnetic dipole moment of the loop?

Problem 144.

problems/bfield-pr-electrodynamics-two-point-charges.tex



Two identical point charges with charge q are separated by a distance r and have identical velocities $\vec{\boldsymbol{v}}$ perpendicular to $\vec{\boldsymbol{r}}$.

a) Find the magnitude of the electrostatic force between the two charges. Draw it in on the figure as vectors acting on the two charges labelled 'F_e'.

b) Find the magnitude of the magnetic force between the two charges. Draw it in on the figure as vectors acting on the two charges.

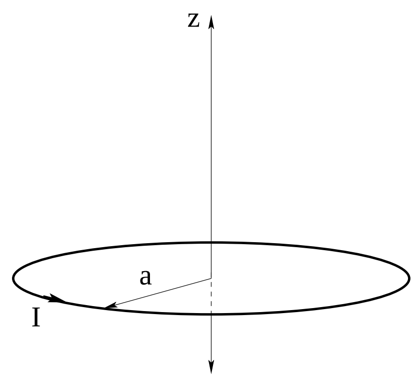
c) The magnetic force increases with the speed v of the particles. Solve for the speed v for which they are equal. You *should* be able to numerically evaluate this speed in meters per second; please do.

The speed you get *should* be ***the speed of light!***

A final meditation: The two charges have this magnetic force in between them only in the inertial frame drawn, where they have identical velocities. But there exists another inertial reference frame, travelling at velocity $\vec{\boldsymbol{v}}$ compared to the one used in the figure, where the velocities of the charges is *zero!* You have just discovered an argument for the *inconsistency of Galilean relativity* with Maxwell's equations, as in this frame there *is no magnetic force at all.* Observers in the two frames in question will thus *disagree* about the magnitudes of the forces and the observed motion of the charges. This makes us sad – until we fix it by inventing relativity theory.

Problem 145.

problems/bfield-pr-ring-on-axis.tex



A circular loop of wire of radius a is carrying a current I counterclockwise (viewed from above) around the z-axis. It is located in the x-y plane, centered on the origin as drawn.

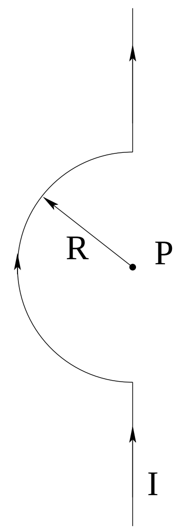
a) Using the Biot-Savart law, find the magnetic field at an arbitrary point on the z-axis. Show all work (don't just write the answer down if you remember it from your homework).

b) What is the magnetic moment of this current loop $\vec{\boldsymbol{m}}$? You don't have to derive this, you can just write it down.

c) Find the field in the (hopefully familiar) limit $z \gg a$ and show that it is the familiar field of a *magnetic* dipole on its axis.

Problem 146.

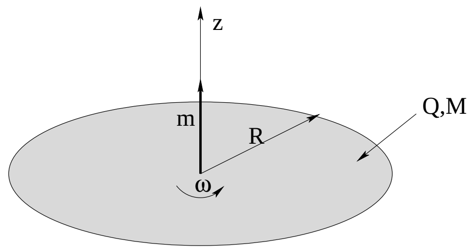
problems/bfield-pr-semicircle-of-current.tex



In the figure above, a long straight wire with a semicircular arc of radius R bent into it carries a current I as shown. Find the magnitude of the magnetic field at the point P. Indicate its direction on the figure.

Problem 147.

problems/bfield-pr-spinning-disk.tex

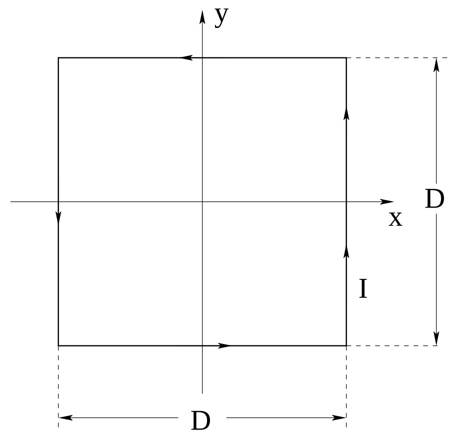


A flat disk of radius R and mass M with uniform surface charge density $\sigma_q = \frac{Q}{\pi R^2}$ is rotating at angular velocity ω about the z-axis as shown.

Find the magnetic field at an arbitrary point on the z-axis of the disk.

Problem 148.

problems/bfield-pr-square-loop-center.tex



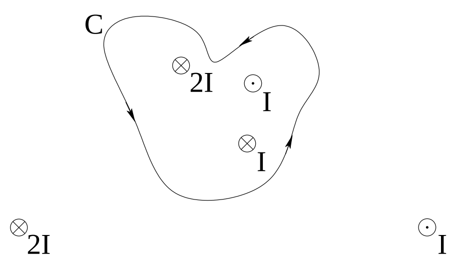
A square loop of wire with side length D carries a current I. What is the magnetic field (vector, so magnitude and direction) at the origin (center of loop as drawn)?

11.2 Ampere's Law

11.2.1 Multiple Choice

Problem 149.

problems/ampere-mc-integral-b-dot-dl.tex



In the figure above, long straight wires carry the currents indicated into or out of the page. The integral of $\vec{B}$ around the indicated curve C:

$$\oint_C \vec{B} \cdot d\vec{\ell}$$

in the indicated direction is:

a) $-1\mu_0 I$

b) $1\mu_0 I$

c) $-2\mu_0 I$

d) $2\mu_0 I$

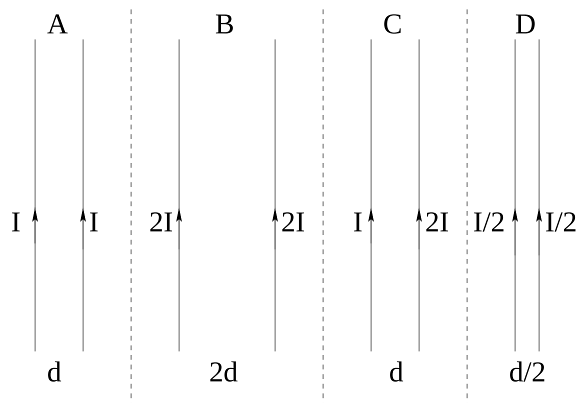
e) $-5\mu_0 I$

f) $5\mu_0 I$

11.2.2 Ranking/Scaling

Problem 150.

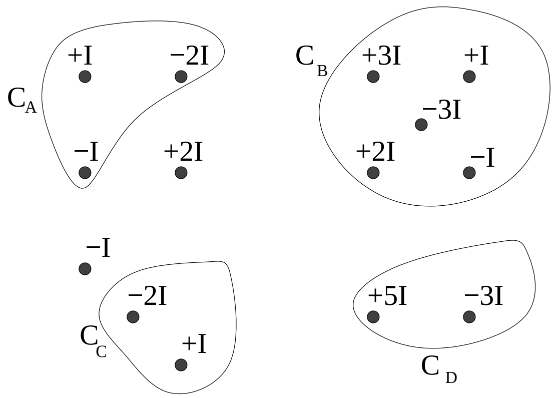
problems/ampere-ra-four-pairs-parallel-wires-force.tex



In the figure above, several arrangements of two long straight parallel wires separated by multiples or fractions of d and carrying currents that are multiples or fractions of I are shown. Rank the *magnitude* of the magnetic force of attraction between the two wires in the figures (separated by the dashed lines), where equality is permitted. That is, a possible answer might be $A > C = B > D$.

Problem 151.

problems/ampere-ra-integral-b-dot-dl.tex



In the figure above are four sets of currents, each with an indicated closed curve $\boldsymbol{C}$. Rank:

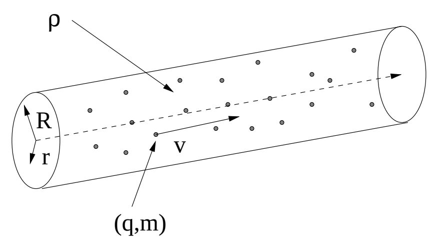
$$\oint_{C_i} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}}$$

for $i = A, B, C, D$. For example, the answer could be (but probably isn't) $A = B < C < D$. Negative values are less than positive values, and positive current is ***out of the page***.

11.2.3 Long Problems

Problem 152.

problems/ampere-pr-particle-beam-electrodynamics.tex



Beam Dynamics: Each part of this problem (a) and b)) will be graded separately. You do not need to get the first part right to do the second part, but obviously you need to get both parts right to get the extra credit from c).

A cylindrical beam of particles each with charge q and mass m has a uniform initial (charge) density ρ and radius R. Each particle in the beam is initially travelling with velocity v parallel to the beam's axis. We will discuss the stability of this beam by examining the forces on a particle travelling in the beam at a distance $r < R$ from the axis (the center of the cylinder).

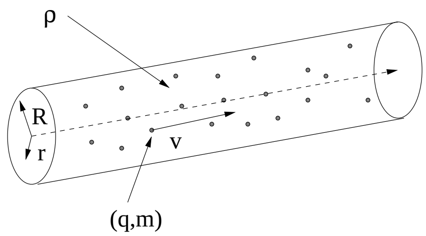
a) Find the approximate force on a particle at radius r caused by the other particles in the beam. You will need to use Gauss's law to calculate the electric field at radius r. Describe your work, and do not skip steps; show that you understand Gauss's law. Make a sketch as needed.

b) Find the magnetic force on a particle at radius r caused by the other particles in the beam. Use Ampere's law to calculate the magnetic field. Describe your work, and do not skip steps; show that you understand Ampere's law. Make a sketch.

c) (5 points extra credit) At what beam velocity do the forces in a) and b) exactly balance? Given the unbalanced electric force *in the rest frame of the particles* from a), offer a hypothesis that can explain both measurements.

Problem 153.

problems/ampere-pr-particle-beam-magnetostatics.tex



Beam Magnetostatics: Not all charged particles travel inside conductors; in the particle beam produced by an accelerator they fly freely together through a vacuum. In the figure above, a cylindrical beam of particles each with charge q and mass m has a uniform initial (charge) density ρ and radius R. Each particle in the beam is initially travelling with velocity v parallel to the beam's axis.

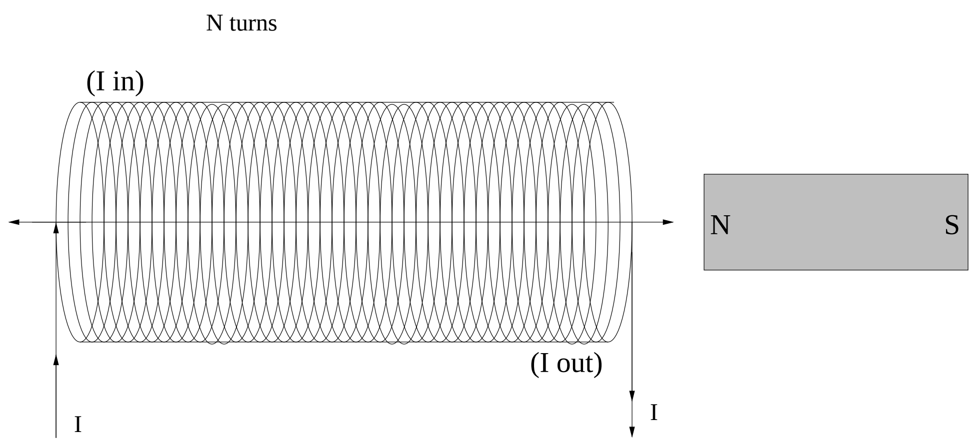
a) Use Ampere's law to calculate the (average) magnetic field at points inside the beam.

b) Find the average magnetic force on one of the particles at radius r caused by the other particles in the beam.

Things to think about: Does this force pull the beam tighter together (compress it) or spread it further apart (disperse it)? Is there another force that might oppose it?

Problem 154.

problems/ampere-pr-solenoid.tex



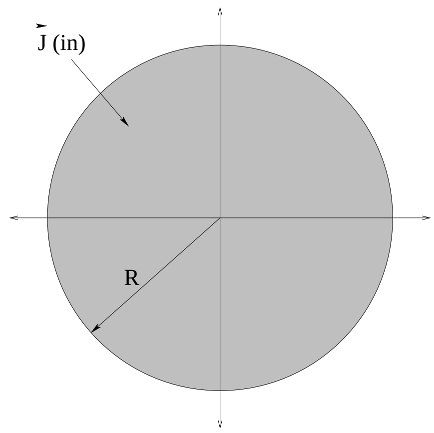
A solenoid with N turns and length L is pictured above. The solenoid is wrapped and connected to a battery (not shown) so that a current I is going into the page at the top of each loop and out of the page at the bottom.

a) Find the magnetic field inside the solenoid using Ampere's Law. Assume that the solenoid is "infinitely long" as usual. Clearly indicate the **direction** of the field in on the figure.

b) If a bar magnet is placed at rest near the right hand end of the solenoid as pictured will it be attracted towards the solenoid or repelled away from the solenoid? (Hint: Think of the "north pole" of a bar magnet as being like a "positive magnetic charge".)

Problem 155.

problems/ampere-pr-thick-wire-uniform-J.tex



A cylindrical long straight wire of radius R carries a current density of magnitude $J = I/\pi R^2$ ***into*** the page as drawn.

a) Find the magnetic field (magnitude and direction) for $r < R$ (inside the wire).

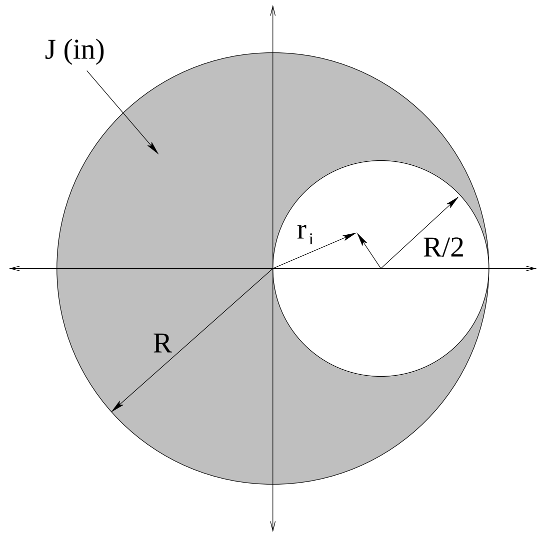
b) Find the magnetic field (magnitude and direction) for $r > R$ (outside the wire).

c) Sketch the magnitude of the magnitude $B(r)$ from $r = 0$ to $r = 2R$. Where is the field maximum and what is its value there in terms of I?

Show all work and ***clearly name and label*** the law or rule used to find the answers.

Problem 156.

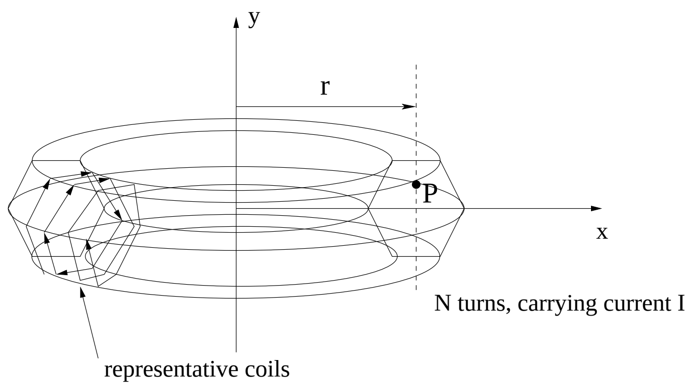
problems/ampere-pr-thick-wire-uniform-J-with-hole.tex



A cylindrical long straight wire of radius R has a cylindrical long straight hole of radius $b = R/2$ and carries a current density of $\vec{\mathbf{J}}$ into the page as drawn. Find the magnetic field at the point $\vec{\boldsymbol{r}}_i$ shown inside of the hole.

Problem 157.

problems/ampere-pr-toroidal-solenoid.tex



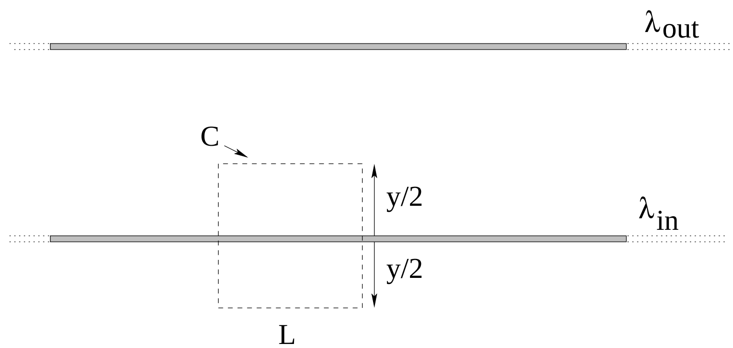
a) State Ampere's Law.

b) *Using Ampere's Law* find the magnetic field at the point labelled P a distance r from the axis and completely inside the toroidal solenoid with N turns and current I in each turn pictured above.

c) **Clearly** indicate the direction of the field on the picture.

Problem 158.

problems/ampere-pr-two-current-sheets.tex



Two infinitely wide and long sheets of current carry charge out of and into the paper as shown. They each carry a current *per unit length* of λ, where the length unit in question is in the plane of the page.

a) From *symmetry* and your knowledge of how the magnetic field depends on the direction of long straight currents, determine the *direction only* of the magnetic field above, in between, and below the sheets of charge.

b) A possible Amperian path C is drawn on the figure as a dashed line. Indicate a direction of integration on this figure for the *lower* sheet only, then use Ampere's Law to find the magnitude of the magnetic field a distance $y/2$ away from the sheet.

c) Use the superposition principle to find the total magnetic field produced by both sheets above, in between and below them. Be sure to give ***both*** magnitude and (from part a) direction.

Chapter 12

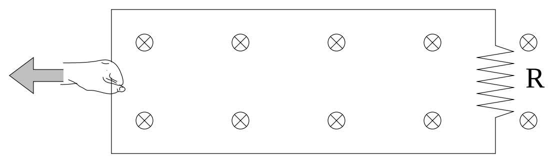
Faraday's Law

12.1 Motional Voltage

12.1.1 Multiple Choice

Problem 159.

problems/motional-voltage-mc-pulling-wire-loop.tex



A conducting loop of wire with a small but finite resistance R sits in and perpendicular to a powerful magnetic field. You grab the loop and try to pull it out of the field. The loop (circle one):

a) Is actively pushed out of the field in the direction you pull.

b) Is actively pushed out of the field in the ***opposite*** direction you pull.

c) Resists your attempt to move it and the resistor heats slightly as you move succeed.

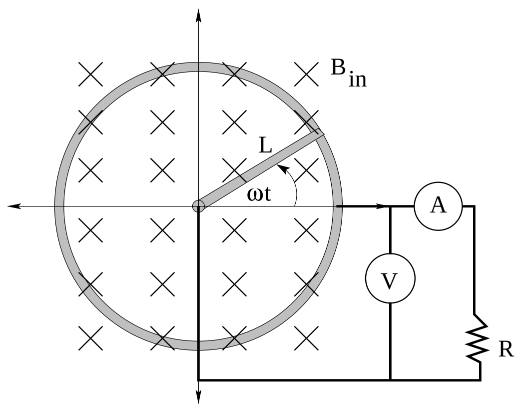
d) Resists your attempt to move it and the resistor does not change temperature as you succeed.

e) Does not move.

12.1.2 Long Problems

Problem 160.

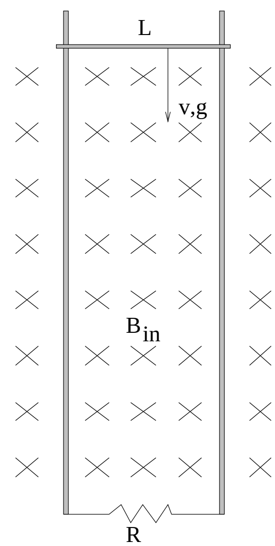
problems/motional-voltage-pr-dc-induction-generator.tex



A rod of length L is pivoted at one end and swings around at an angular frequency ω with its other end sliding along a circular conducting track. A magnetic field B_{in} is oriented perpendicular to the plane of rotation of the rod as shown. The pivot point and the outer ring are connected by (fixed) wires across a resistance R with a voltmeter and ammeter inserted in the circuit as shown. What do the voltmeter and ammeter read?

Problem 161.

problems/motional-voltage-pr-falling-rod.tex



A rod of length L and mass m slides on frictionless conducting guides down vertical rails, connected at the bottom, that enclose a uniform magnetic field of magnitude B as shown, starting at rest at $t = 0$. The loop formed by the rod and rails has a total resistance of R. **Gravity** makes the rod fall. Find:

a) The current $I(v)$ induced in the rod when the speed of the rod is v (down). Indicate the direction on the figure above.

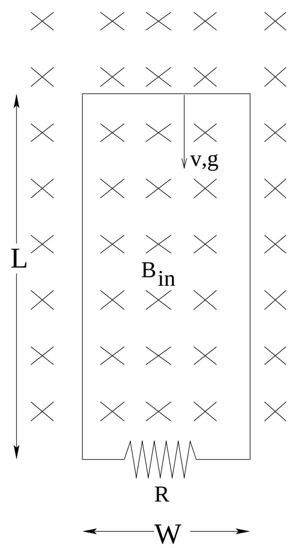
b) The **net** force on the rod as a function of v.

c) The "terminal velocity" of the rod $\vec{v}_t$.

d) For extra credit, explicitly solve the equations of motion and find $v(t)$ for all times, assuming of course that it hasn't yet fallen off of the rails.

Problem 162.

problems/motional-voltage-pr-falling-wire-loop.tex



A rigid rectangular loop of wire of length L, width W, and mass m has a total resistance R and is vertically suspended in a horizontal uniform magnetic field $\vec{B}_{\rm in}$ as shown. At time $t = 0$ it is released and falls under the influence of gravity. Note well that the *bottom* of the loop is *not* in the region of uniform field! Find:

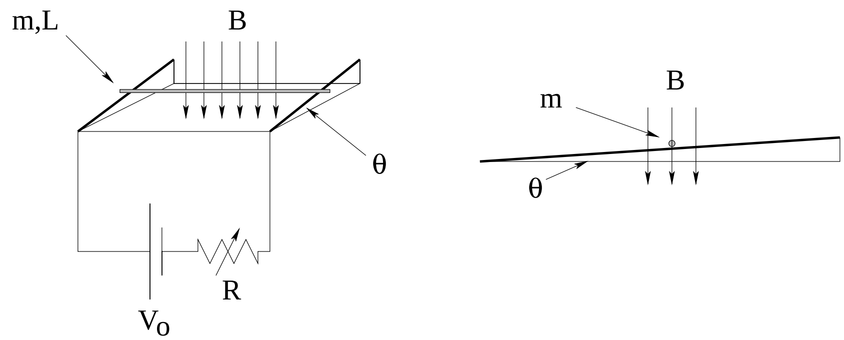
a) The current $I(v)$ induced in the loop when its downward speed is v. Indicate the direction of this current on the figure above.

b) The **net** force on the wire loop as a function of v.

c) The "terminal velocity" of the wire loop $\vec{v}_t$ (this is the velocity it must have when all forces on the wire balance).

Problem 163.

problems/motional-voltage-pr-inclined-plane-rod-on-rails.tex



Two views are shown of a perfectly conducting rod of mass m and length L sitting at rest on frictionless rails elevated at an angle θ with respect to horizontal. The rod is in a vertical magnetic field $\vec{\boldsymbol{B}}$ as shown. A voltage V_0 and *variable* resistance R creates a circuit.

a) Find R such that the rod is in equilibrium and sits at rest.

At $t = 0$, the resistance is suddenly decreased to $R/2$.

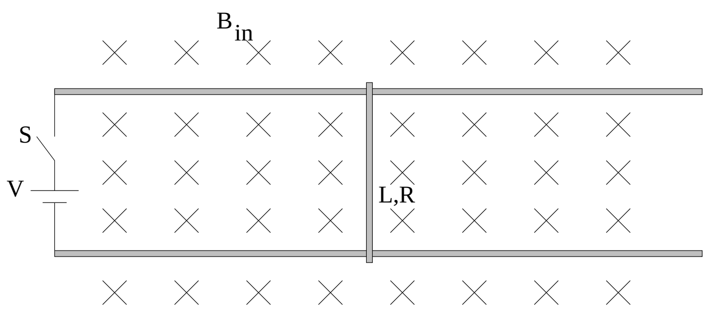
b) Which way does the rod slide, up or down the rails?

c) Derive its equation of motion.

d) For extra credit, determine the terminal velocity of the rod. For a *lot* of extra credit, solve the equation of motion! But don't attempt this until everything else is finished, as it will be a bit messy...

Problem 164.

problems/motional-voltage-pr-rod-on-rails-VR-only-exam.tex



A rod of mass m, resistance R and length L is sitting at rest on frictionless rails in a magnetic field as shown. At $t = 0$, the switch S is closed and a voltage V applied across the rails. Show all work while deriving the following results, clearly indicating the physical law used and reasoning process. Neatness and clarity count.

a) What is the net voltage across the resistance R as a function of $|\vec{v}|$?

b) What is the current I in the loop as a function of $\vec{v}$?

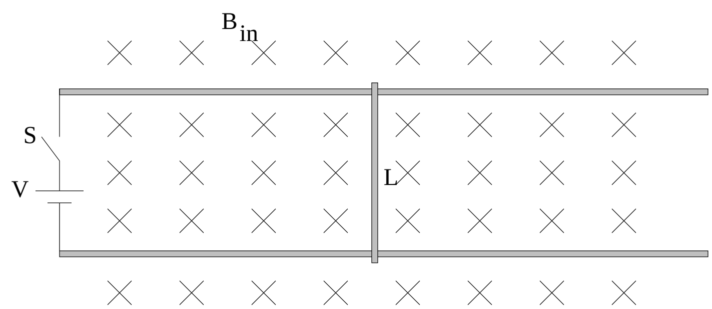
c) What is the force $\vec{F}$ on the rod as a function of $\vec{v}$?

d) What is the terminal velocity of the rod as $t \to \infty$?

10 points of extra Credit: Solve the first order, linear, ordinary, inhomogeneous differential equation and find the velocity of the rod $\vec{v}(t)$ as a function of time. Draw a qualitatively correct curve showing this function and show how it corresponds to your answer to d).

Problem 165.

problems/motional-voltage-pr-rod-on-rails-VR-only.tex



A rod of mass m, resistance R and length L is sitting at rest on frictionless rails in a magnetic field as shown. At $t = 0$, the switch S is closed and a voltage V applied across the rails. Find:

a) The total voltage in the wire as a function of the speed of the rod v.

b) The total current in the wire as a function of v.

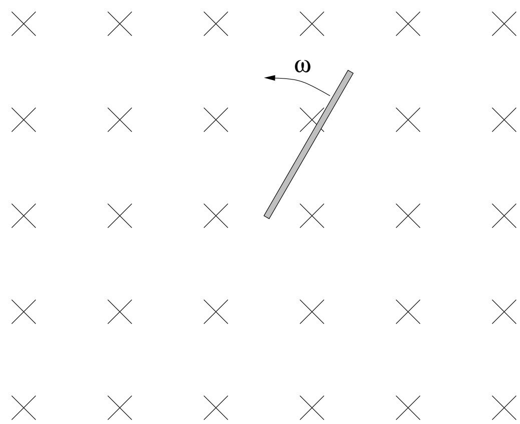
c) The total force on the rod as a function of v.

d) Identify the terminal velocity of the rod v_t.

e) If you can, solve the equation of motion arising from Newton's Law for the rod for $v(t)$.

Problem 166.

problems/motional-voltage-pr-rotating-rod.tex



A conducting bar of length L rotates at an angular frequency ω in a uniform, perpendicular magnetic field as shown.

b) Find the forces acting on a charge $+q$ in the rod (magnitude and direction) at the radius r. What causes these forces and what direction do they point?

a) Find the potential difference developed between the central end of the rod and a point at radius r on the rotating rod.

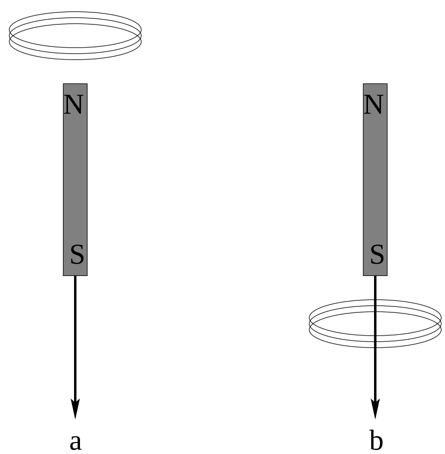
c) Discuss the *qualitative* distribution of charge in the (presumed neutrally charged) rod, assuming that it is in equilibrium (has been rotating for a long time). Draw a qualitative graph of $\rho(r)$, the charge density as a function of r to support your assertions. That is, you don't have to have exactly the right functional form but your curve should have all the right features.

12.2 Faraday's Law

12.2.1 Multiple Choice

Problem 167.

problems/faraday-mc-bar-magnet-and-loop-1.tex



In the figure above, two bar magnets are held near two circular loops of conducting wire with many turns and given a sharp pull downward as shown. The directions of the ***induced magnetic forces*** on the two loops are:

a) Left loop is pulled down, right loop is pushed down.

b) Left loop is pushed up, right loop is pushed down.

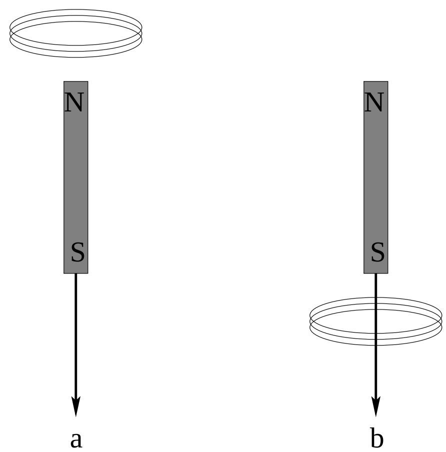
c) Left loop is pulled down, right loop is pulled up.

d) Left loop is pushed up, right loop is pulled up.

e) One cannot tell which direction the forces will act from the information given in the picture.

Problem 168.

problems/faraday-mc-bar-magnet-and-loop-2.tex



In the figure above, two bar magnets are held near two circular loops of conducting wire with many turns and given a sharp pull downward as shown. The directions of the ***induced magnetic forces*** on the two loops are:

a) Left loop is pulled down, right loop is pushed down.

b) Left loop is pushed up, right loop is pushed down.

c) Left loop is pulled down, right loop is pulled up.

d) Left loop is pushed up, right loop is pulled up.

e) One cannot tell which direction the forces will act from the information given in the picture.

Problem 169.

problems/faraday-mc-curie-temperature.tex

What happens at the Curie Temperature of a magnetic material?

a) Frogs float.

b) A diamagnetic material becomes paramagnetic.

c) A paramagnetic material becomes ferromagnetic.

d) The magnetization of an object in a strong field vanishes.

e) A magnetic material becomes a superconductor.

Problem 170.

problems/faraday-mc-diamagnetism.tex

Which of the following characterize a ***diamagnetic*** material? ***Circle all correct answers*** (there can be more than one):

a) When placed in an external magnetic field, they have a ***reduced*** field inside.

b) They can float if placed in a strong enough field.

c) When placed in an external magnetic field, they have a ***stronger*** field inside.

d) They have a magnetic field inside even in zero external field.

e) They attract other magnets.

Problem 171.

problems/faraday-mc-frog-diamagnetic-levitation.tex

In the figure above, a frog is shown that is being levitated by a superpowerful magnetic field. Is the frog being levitated because it is:

a) diamagnetic

b) dielectric

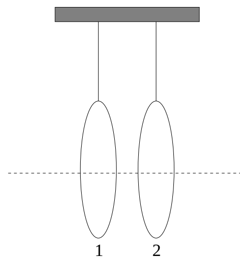
c) paramagnetic

d) ferromagnetic

e) a conductor

Problem 172.

problems/faraday-mc-two-loops-force.tex



Two circular loops of wire are suspended facing each other and carrying no current. A current is suddenly switched on in the first. Does the second loop:

a) move towards the first loop

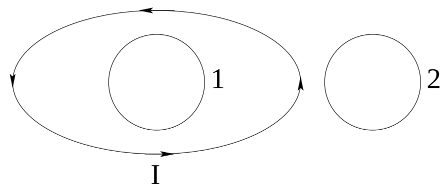
b) move away from the first loop

c) move sideways (staying at the same distance)

d) remain stationary?

Problem 173.

problems/faraday-mc-two-loops-one-current.tex



An ***increasing*** current I flows in an elliptical loop of wire. Two loops of wire are placed as shown. The flow of induced current:

a) is clockwise for loop 1 and counterclockwise for loop 2.

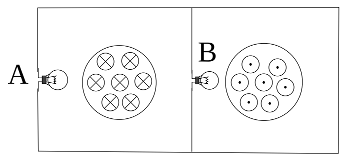
b) is counterclockwise for loop 1 and counterclockwise for loop 2.

c) is clockwise for loop 1 and clockwise for loop 2.

d) is counterclockwise for loop 1 and clockwise for loop 2.

Problem 174.

problems/faraday-mc-two-solenoids-induced-voltage.tex



Two identical long solenoids (shown in cross section) produce a $\vec{B}$-field into the page on the left and out of the page on the right. Both magnetic fields are ***increasing in magnitude with time at the same constant rate***. Assume the field outside the solenoid is negligibly small, so that flux arises ***only from the fields in the solenoid***. A circuit with two identical light bulbs surrounds the solenoids. By considering Faraday's law, you can conclude that:

a) bulb A is dark and bulb B is bright.

b) bulb B is dark and bulb A is bright.

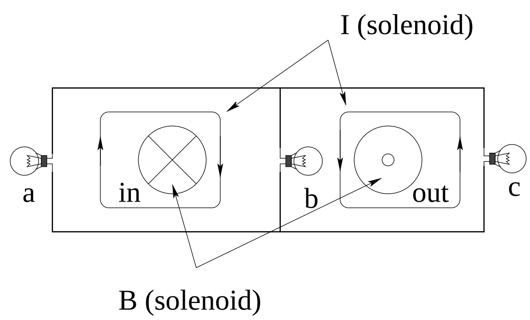
c) both bulbs are bright.

d) both bulbs are dark.

12.2.2 Short Answer/Concept

Problem 175.

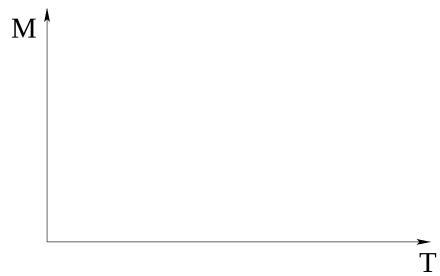
problems/faraday-sa-double-inductance-loop.tex



A toroidal solenoid (seen in cross-section) is surrounded by a double circuit of wires as shown, with bulbs inline that will light up if they carry a current. The current in the solenoid is *increasing*. Which bulb(s) are the brightest?

Problem 176.

problems/faraday-sa-ferromagnetic-transition.tex



On the provided graph above, plot a QUALITATIVE graph of the magnitude of the magnetization of a ***paramagnet in zero external magnetic field*** M as a function of absolute temperature T. Identify and label the Curie Temperature and the paramagnetic and ferromagnetic parts of the graph. The magnetization is, recall, the microscopic "magnetic moment per unit volume" of the material.

Problem 177.

problems/faraday-sa-laminated-transformer-cores.tex

Why are transformer cores generally made of thin slices of laminated iron separated by a thin insulating layer? I'm interested in knowing "why laminated", not "why iron" (although feel free to answer both).

Problem 178.

problems/faraday-sa-limp-loop-responds.tex

A thin, *flexible* loop of wire is carrying a current I (it has some resistance and a built-in small battery) and is sitting in a limp no-particular shape, the shape a loop of string might take if you just dropped it onto a table on edge. A strong, uniform magnetic field is slowly turned on in some direction. It exerts forces on the small segments of wire.

After a short while, the wire loop will be found to have a certain shape and orientation relative to the field. What are they?

Note that this question requires *no actual integration* – consider the forces on all the small segments of wire and what they'll do to the wire. You should, however, give a *qualitative* argument for the form and orientation you decide on, including why it is stable where other possible shapes are not.

Drawing pictures of a proposed initial state, the forces on selected bits of wire, an intermediate state, and the final state showing the balance of forces is highly recommended.

Problem 179.

problems/faraday-sa-magnetic-brake.tex

Many exercycles come with a knob that can be turned one way or another to increase or decrease the "resistance" of the pedals. However, it is not possible to systematically increase the resistance by means of friction (with, for example, a brake shoe) because it would soon wear out. So they use magnets instead. Draw a sketch of a possible magnetic brake below, and indicate the physical principle upon which it would work. Where does the extra work one does pushing the pedals go?

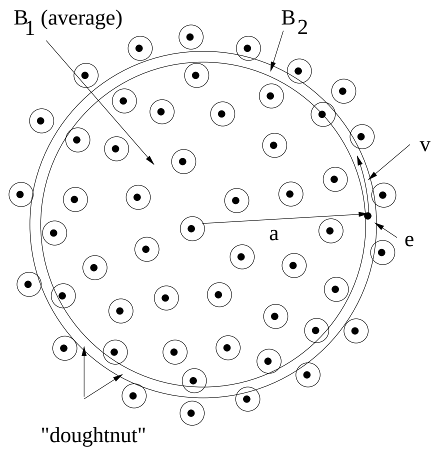
Problem 180.

problems/faraday-sa-magnetic-materials.tex

a) Is the magnetic field inside a paramagnetic material greater than or less than an applied external magnetic field? (Assume that the material is somewhat above the Curie temperature if it has one.)

b) What happens to a paramagnetic material cooled *below* the Curie temperature?

c) Is the magnetic field inside a diamagnetic material greater than or less than an applied external magnetic field?



12.2.3 Long Problems

Problem 181.

problems/faraday-pr-betatron-2.tex

A Betatron is pictured above (with field out of the page). It works by increasing a non-uniform magnetic field $\vec{B}(r)$ in such a way that electrons of charge e and mass m inside the "doughnut" tube are accelerated by the E-field produced by induction (via Faraday's law) from the "average" time-dependent magnetic field $B_1(t)$ inside a, while the magnitude of the magnetic field at the radius a, $B_2(t) = |\vec{B}(a,t)|$, bends those same electrons around in the circle of (constant) radius a.

This problem solves, in simple steps, for the "betatron condition" which relates $B_1(t)$ to $B_2(t)$ such that both things can simultaneously be true.

a) The electrons go around in circles of radius a and are accelerated by an $\vec{E}$ field produced by Faraday's law. We will define the (magnitude of the) average field B_1 by $\phi_m = B_1(\pi a^2) = \int_{(r<a)} \vec{B}(r) \cdot \hat{n} dA$. What is the induced E field (tangent to the circle) in terms of B_1 and a?

(Problem continued on next page!)

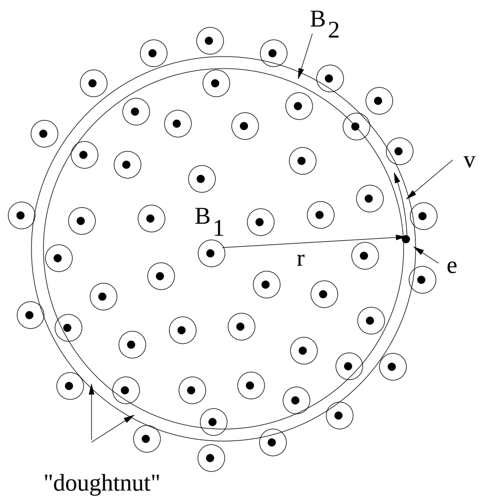
b) The electrons (at their instantaneous speed v tangent to the circle) are bent into the circle of radius a by the field B_2. Relate B_2 to the magnitude of the momentum $p = mv$, the charge e of the electron, and the radius a.

c) The force $\vec{F}$ from the E-field acting on the electron with charge e in the direction of its motion is equal to the time rate of change of the magnitude of its momentum p (if Newton did not live in vain). Substitute, cancel stuff, and solve for $\frac{dB_1}{dt}$ in terms of $\frac{dB_2}{dt}$. If you did things right, the units will make sense and the relationship will only involve dimensionless numbers, not e or m.

Cool! You've just figured out how to build one of the world's cheapest electron accelerators! Or perhaps not....

Problem 182.

problems/faraday-pr-betatron.tex



A Betatron (pictured above with field out of the page) works by increasing a non-uniform magnetic field in such a way that electrons of charge e and mass m inside the "doughnut" tube are accelerated by the E-field produced by induction from the **average** time-dependent magnetic field $B_1(t)$ inside r (via Faraday's law) while the **specific** magnitude of the magnetic field at the radius $B_2(t)$ bends the electrons around in the constant radius circle of radius r.

This problem solves for the "faraday-pr-betatron condition" which relates $B_1(t)$ to $B_2(t)$ such that both things can simultaneously be true.

a) First, assuming that the electrons go around in circles of radius r and are accelerated by an $\vec{E}$ field produced by Faraday's law from the average field B_1 inside that radius, solve for that induced E field in terms of B_1 and r.

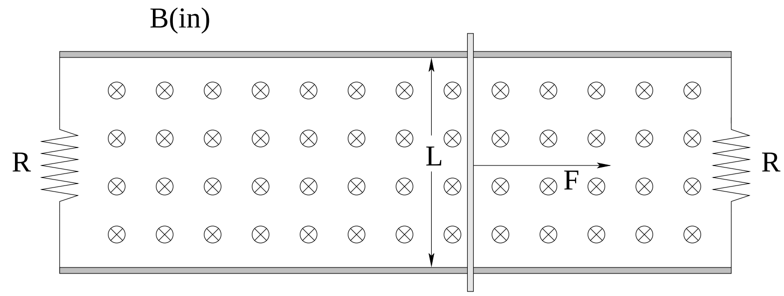
b) Second, assuming that the electrons are bent into a circle of radius r by the specific field at that radius, B_2, relate B_2 to the momentum $p = mv$ and charge e of the electron, and the radius r.

c) Third, noting that the force F from the E-field acting on the electron with charge e in part a) is equal to the time rate of change of p in the result of b) substitute, cancel stuff, and solve for dB_1/dt in terms of dB_2/dt. If you did things right, the units will make sense and the relationship will only involve dimensionless numbers, not e or m.

Cool! By determining how to relate the average field inside r to the actual field at r, you've just figured out how to build one of the world's cheapest electron accelerators!

Problem 183.

problems/faraday-pr-rod-on-rails-two-loops.tex



Two perfectly conducting frictionless rods are connected at the ends with resistors R as shown. It is placed in a uniform magnetic field of magnitude B into the page, and a frictionless perfectly conducting rod with length L (between the rods) and mass M is placed on top. A force $\vec{F}$ is applied such that the rod ***moves with a constant speed*** v_0 ***to the right***.

a) Find the current (magnitude and direction) in each resistor and the bar.

b) Find the magnitude of the force F that keeps the bar moving at constant speed v_0.

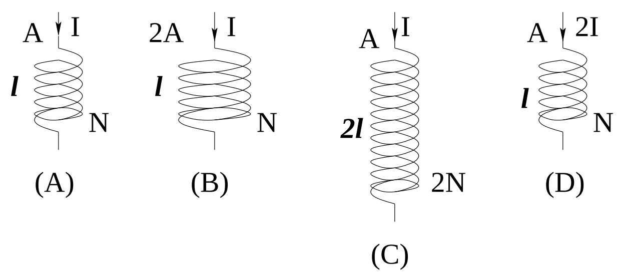
c) Find the total power dissipated in the resistors as the bar moves.

12.3 Inductance and LR Circuits

12.3.1 Ranking/Scaling

Problem 184.

problems/inductance-ra-energy.tex

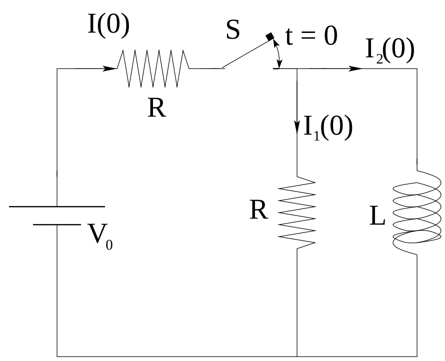


Four inductors are drawn above. Rank the ***magnetic field energy*** stored in each inductor, given the values of I, N, A, ℓ of each. A possible answer might be $A < B < C = D$ (but probably isn't).

12.3.2 Long Problems

Problem 185.

problems/inductance-pr-charging-discharging-LR-circuit.tex



In the circuit above, switch S is closed at $t = 0$. Find:

a) The currents $I(0)$, $I_1(0)$, and $I_2(0)$ at $t = 0$ at the instant after the switch is closed.

b) Find $I(t)$, $I_1(t)$, and $I_2(t)$.

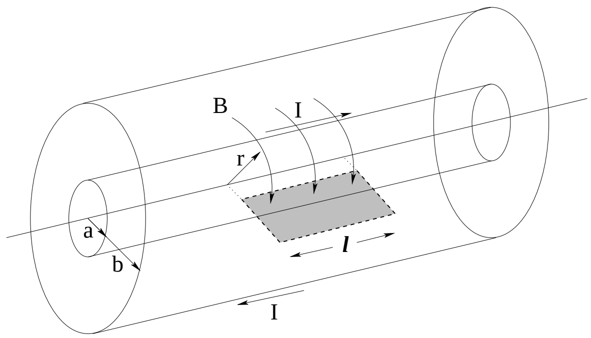
After a very long time we restart our clock, and at $t = 0$ the switch is opened again. Find:

a) The currents $I(0)$, $I_1(0)$, and $I_2(0)$ at $t = 0$ at the instant before the switch is opened.

b) Find $I(t)$, $I_1(t)$, and $I_2(t)$.

Problem 186.

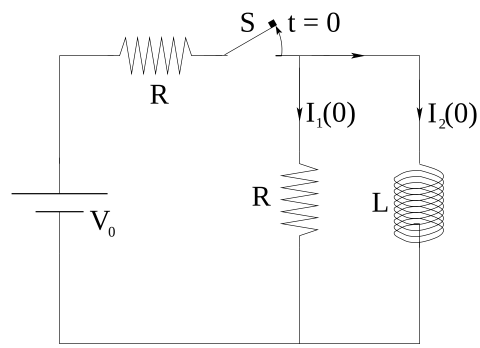
problems/inductance-pr-coaxial-cable.tex



Find the self-inductance per unit length of a coaxial cable consisting of two coaxial cylindrical conducting shells, the inner one with radius a and outer with radius b. I've shaded in a chunk of area for you to use in computing the flux, and have even helped you out by drawing in a cartoon for the field between the shells when the inner one is carrying a current I (and the outer one is returning it).

Problem 187.

problems/inductance-pr-discharging-LR-circuit.tex



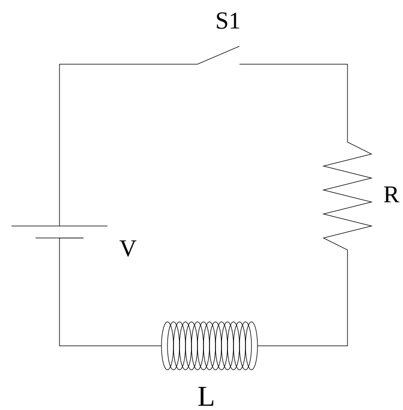
In the circuit above, the switch S has been closed ***a very long time***. At $t = 0$ the switch is opened.

a) What are the currents I_1 and I_2 at the instant before the switch is opened?

b) Find the current in the right hand loop as a function of t after the switch has been opened.

Problem 188.

problems/inductance-pr-LR-circuit.tex



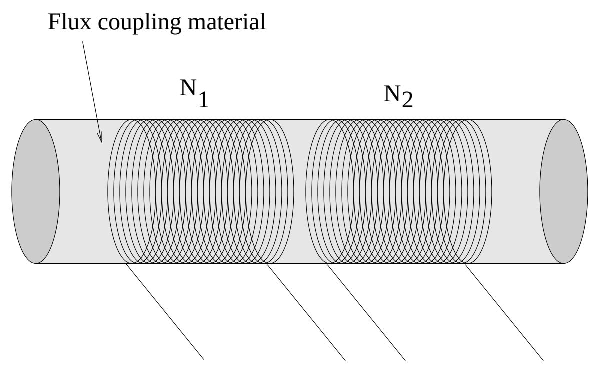
In the circuit above, assume R, L, and V are given.

a) At time $t = 0$, switch S1 is closed. Start by writing Kirchoff's loop rule for the circuit and find the *current* through the inductor as a function of time in terms of the given quantities.

b) Find the power provided by the voltage and delivered to L and R as a function of time and show that $P_V = P_R + P_L$ (so that energy is conserved).

Problem 189.

problems/inductance-pr-mutual-solenoid.tex



Two solenoid coils are wrapped around a paramagnetic core of cross-sectional area A that traps magnetic flux so that the flux (per turn) in both solenoids is always the same. The first coil has N_1 turns, the second coil has N_2 turns.

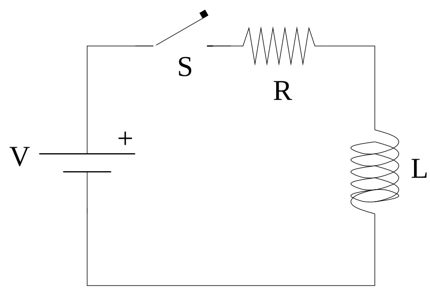
a) If μ_r (the relative permeability) is 10, what is the self-inductance of each solenoid?

b) What is the mutual inductance M_{12} between the two coils?

c) If a current $I_1 \sin(\omega t)$ is running through the first solenoid, what is the ***magnitude*** of the induced voltage V_2 across the second as a function of time?

Problem 190.

problems/inductance-pr-power-in-LR-circuit.tex



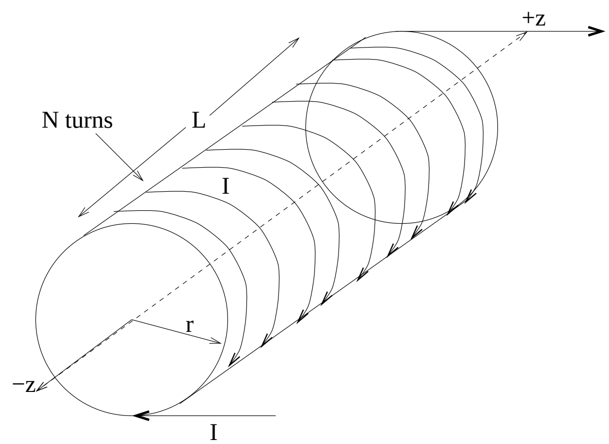
In the figure above, the switch is closed at $t = 0$ and a current $I(t)$ builds up through the inductor. Find (solve for, showing all work):

a) $I(t)$, the current in the wire as a function of time.

b) $P_L(t)$, the power delivered to the inductor as a function of time.

Problem 191.

problems/inductance-pr-solenoid.tex



A solenoid has length L, N turns, and radius r is centered on the z-axis. A current I is driven through the solenoid.

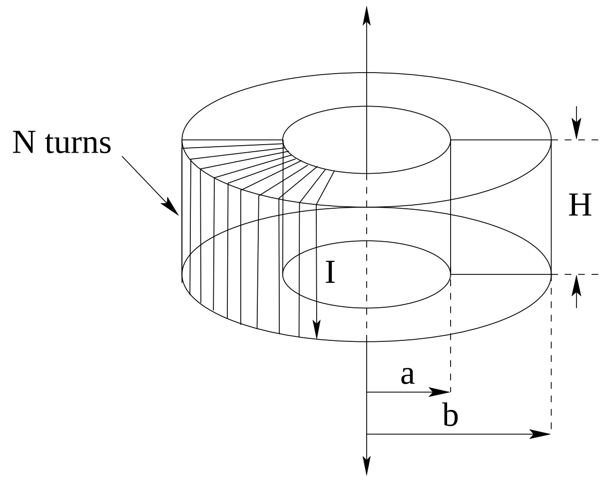
a) *Derive* the magnetic field $\vec{B}$ inside the solenoid, neglecting end effects. Draw the direction of the field lines in on your picture.

b) *Derive* the self-inductance of the solenoid.

In both cases, start from fundamental principles, equations, laws or definitions and clearly state what they are; do not just put down a remembered answer.

Problem 192.

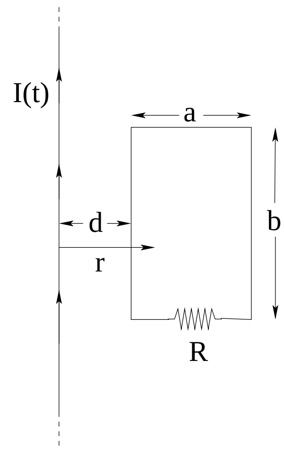
problems/inductance-pr-toroidal-solenoid.tex



Find the self-inductance L of a toroidal solenoid with a rectangular cross-section (height H, inner radius a, outer radius b) and N turns. Presume that the wires are wrapped uniformly all the way around and carry a current of I.

Problem 193.

problems/inductance-pr-wire-rectangular-loop.tex



In the figure above, $I(t) = I_0 \sin(\omega t)$ in the long, straight wire to the left. A rectangular conducting loop of resistance R, width a and length b sits a distance d from and in the plane of the wire. Find:

a) The magnetic field at an arbitrary point a distance r from the wire inside the loop.

b) The total magnetic flux through the loop.

c) The ***mutual inductance M*** between the wire and the loop.

d) The induced voltage in the loop $V_{loop}(t)$.

e) The induced current $I_{loop}(t)$.

f) During the first quarter cycle of $I(t)$ (when the current in the wire is ***increasing***) indicate the ***direction*** of the current in the loop, the direction of its induced magnetic dipole moment $\vec{m}_{loop}$, and the direction of the total force on the loop due to the wire.

Chapter 13

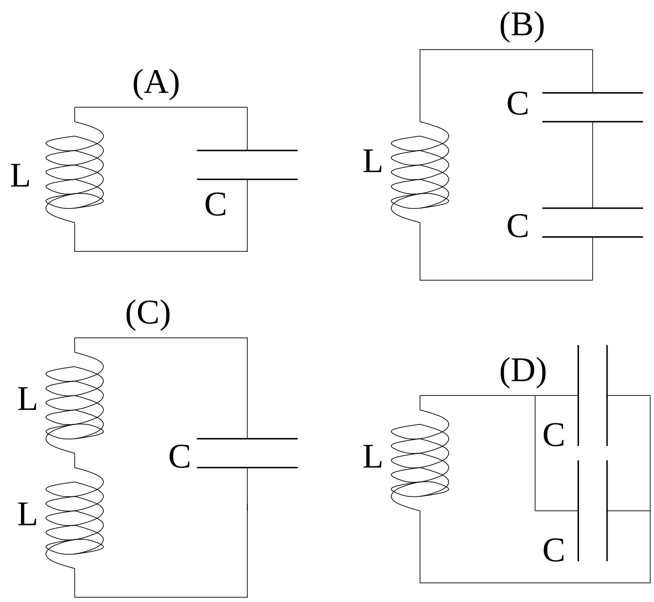
AC Circuits

13.1 Passive AC Circuits

13.1.1 Ranking/Scaling

Problem 194.

problems/ac-passive-ra-resonance-frequency.tex

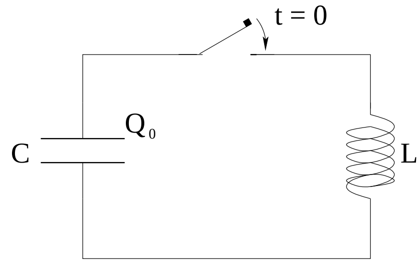


Four LC circuits are drawn above. Rank the resonance frequencies ω_0 of each circuit, with equality a possibility. A possible answer could therefore be $A < B < C = D$ (but probably isn't).

13.1.2 Long Problems

Problem 195.

problems/ac-passive-pr-LC.tex



At time $t = 0$ the capacitor in the LC circuit above has a charge Q_0 and the current in the wire is $I_0 = 0$ (there is no current in the wire).

a) Write Kirchoff's voltage rule for this circuit loop.

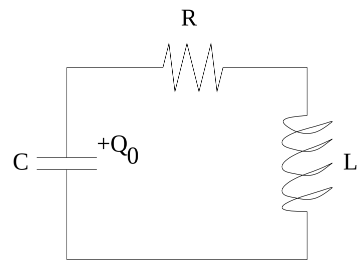
b) Turn it into a "simple harmonic oscillator" differential equation for Q. What is the angular frequency ω of this oscillator?

c) Write down (or derive, if necessary) $Q(t)$.

.

Problem 196.

problems/ac-passive-pr-LRC.tex



At time $t = 0$ the capacitor in the LRC circuit above has a charge Q_0 and the current in the wire is $I_0 = 0$ (there is no current in the wire).

a) Find (or remember) $Q(t)$. Don't forget to define ω', the shifted frequency of this system.

b) Draw a qualitatively correct picture of $Q(t)$ in the case that the oscillation is only weakly damped.

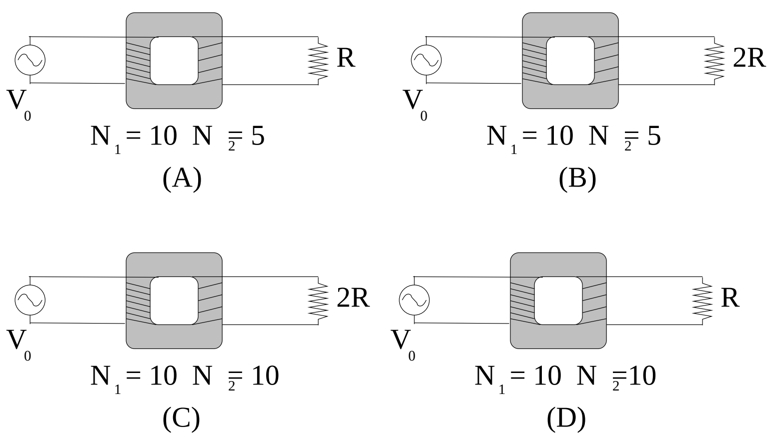
Show all your work. Remember that $Q(t)$ (in the end) is a real quantity, although it may be convenient for you to assume that it is complex while solving the problem.

13.2 Driven AC Circuits

13.2.1 Ranking/Scaling

Problem 197.

problems/ac-driven-ra-transformer.tex



In the figure above, four transformers are drawn with the numbers of turns given on the figures. Rank the magnitude of the currents that appear in the load resistor from lowest to highest with equality an option. That is, a possible answer is $A < B = C < D$ (but probably isn't).

13.2.2 Short Answer/Concept

Problem 198.

problems/ac-driven-sa-build-high-pass-filter.tex

Draw an arrangement of (your choice of) Ls, Rs and Cs that can be used as a "high pass" filter (one that passes high frequencies but blocks low frequencies). Indicate the two points where the output voltage should be sampled.

Note that you do not have to use all of the circuit elements (but can use more than one of any kind if you like) and there is more than one way to do this!

Problem 199.

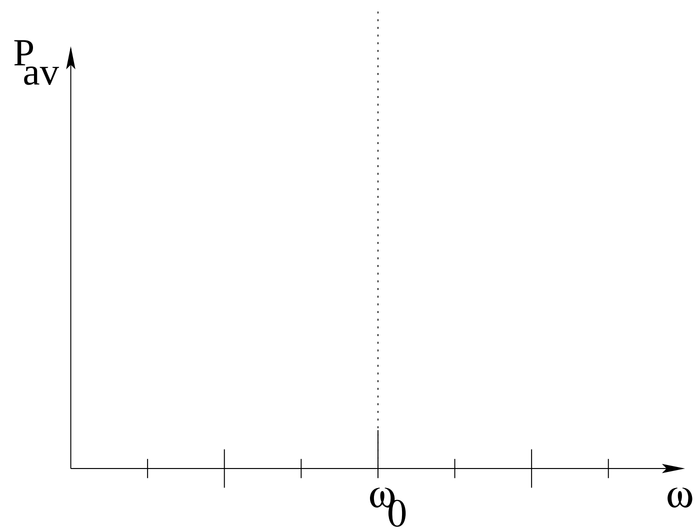
problems/ac-driven-sa-build-low-pass-filter.tex

Draw an arrangement of (your choice of) Ls, Rs and Cs that can be used as a "low pass" filter (one that passes low frequencies but blocks high frequencies). Indicate the two points where the output voltage should be sampled.

Note that you do not have to use all of the circuit elements (but can use more than one of any kind if you like) and there is more than one way to do this!

Problem 200.

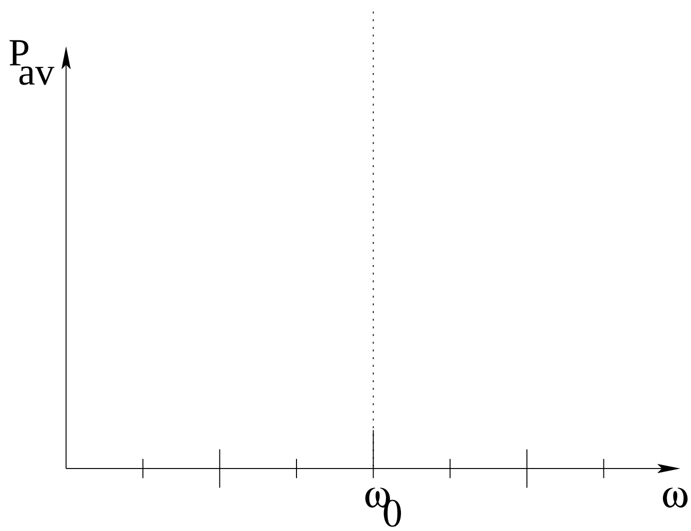
problems/ac-driven-sa-resonance-Q-16.tex



On the axes provided above, draw a qualitatively correct resonance curves for $P_{\text{av}}(\omega)$, the average power delivered to a damped, driven LRC circuit for $Q = 16$. The curve must ***correctly and proportionately exhibit*** $\boldsymbol{\Delta\omega}$, the full width at half max. Be sure to indicate the ***algebraic relation*** between Q and $\Delta\omega$ you use to draw the curves to scale.

Problem 201.

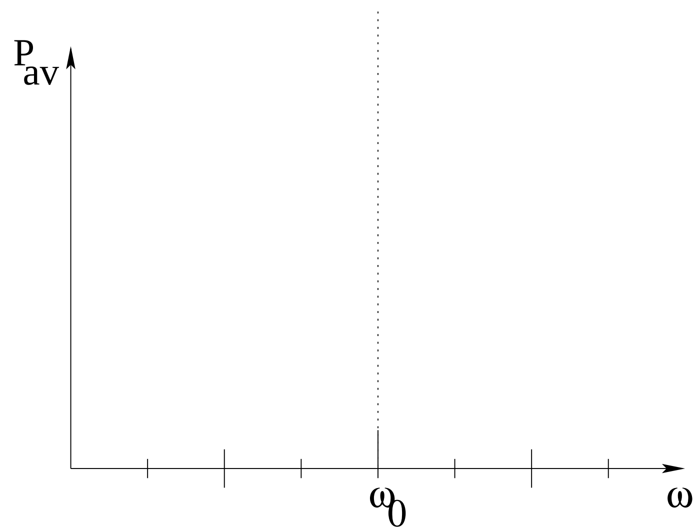
problems/ac-driven-sa-resonance-Q-4-10.tex



On the axes provided above, draw two qualitatively correct resonance curves for $P_{\text{av}}(\omega)$, the average power delivered to a damped, driven LRC circuit: one for $Q = 4$ and one for $Q = 10$. The (labelled!) curves must ***correctly and proportionately exhibit*** $\boldsymbol{\Delta\omega}$, the full width at half max. Be sure to indicate the ***algebraic relation*** between Q and $\Delta\omega$ you use to draw the curves to scale.

Problem 202.

problems/ac-driven-sa-resonance-Q-8.tex

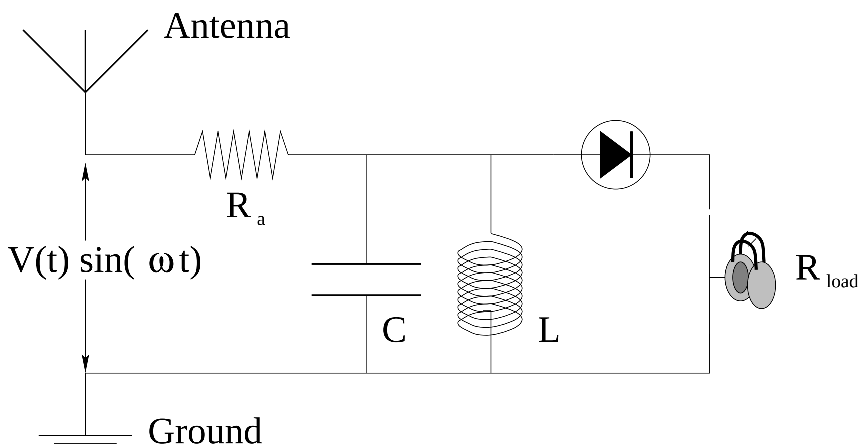


On the axes provided above, draw a qualitatively correct resonance curves for $P_{\rm av}(\omega)$, the average power delivered to a damped, driven LRC circuit for $Q = 8$. The curve must ***correctly and proportionately exhibit*** $\boldsymbol{\Delta\omega}$, the full width at half max. Be sure to indicate the ***algebraic relation*** between Q and $\Delta\omega$ you use to draw the curves to scale.

13.2.3 Long Problems

Problem 203.

problems/ac-driven-pr-AM-radio.tex



Challenge Problem: The simplest crystal radio circuit consists of the ***parallel*** LRC circuit drawn above. The antenna-to-ground connection represents an amplitude-modulated AC voltage source $V = V(t)\sin(\omega t)$. The diode (in series with the earphone of resistance R) is a circuit element that only lets current flow in the direction of the arrow – a one-way gate. The capacitor and/or inductor can be varied to tune the radio. R_a represents the physical and "radiation" resistance of the antenna itself, and $R_{\rm load}$ represents the resistance of e.g. headphones used to listen to the rectified signal.

a) At very low frequencies, what is the reactance of the inductor? What will happen to the current through $R_{\rm load}$?

b) At very high frequencies, what is the reactance of the capacitor? What will happen to the current through $R_{\rm load}$?

c) Therefore, how should one set C with respect to the value of L to tune the radio to deliver the maximum current through the earphones $R_{\rm load}$) and what is that maximum current? Assume that the diode has negligible resistance and capacitance and that the amplitude modulation of $V(t)$ is slow relative to ω^{-1}.

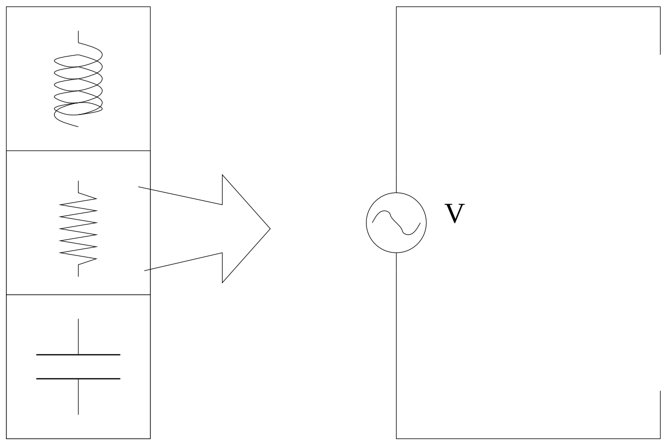
d) Describe qualitatively, with a suitable picture or figure, how the diode allows the amplitude-modulated signal ($V(t)$) to be extracted from the carrier frequency ω.

e) When the radio is correctly tuned, no net current flows through L and C together and the circuit behaves like R_a and $R_{\rm load}$ in series from the antenna to ground. Prove that the ***power*** delivered to $R_{\rm load}$ is maximum when $R_a = R_{\rm load}$.

This last property is called ***impedance matching*** and is actually a general property of even simple series DC circuits – the power delivered to either of a pair of resistors in series is maximum when their resistance is equal.

Problem 204.

problems/ac-driven-pr-build-high-pass-filter.tex

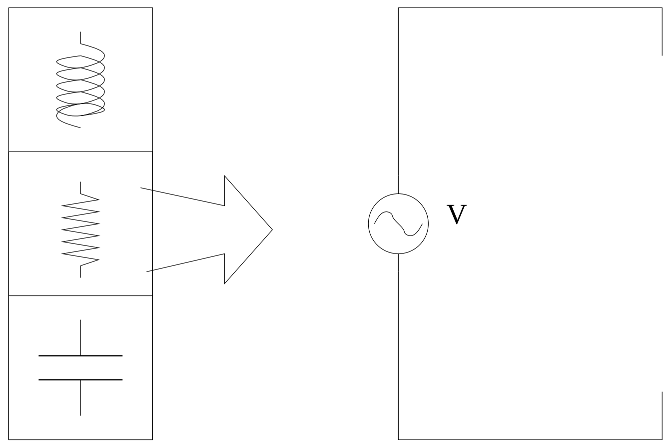


You are given an electronic parts box containing compartments containing 1 ohm, 10 ohm, 100 ohm, ... 1,000,000 ohm resistors. The box also contains compartments of inductors of 1 millihenry, 10 millihenries, ..., 1,000,000 millihenries and capacitors of 1 nanofarad, 10 nanofarads, 100 nanofarads, ... , 1,000,000 nanofarads (basically any power of ten of one ohm, farad, or henry that you like).

Use any parts from this box that you wish to design a *high pass filter* that cuts off frequencies smaller than *approximately* $\omega = 10^4$ radians/second. Draw its schematic and clearly indicate where one should place a resistive load that matches the resistor you choose so that it gets significant current at high frequencies above this cutoff. You may consider the frequency cutoff to occur when $I(\omega = 10^4) = \frac{\sqrt{2}I_0}{2}$ where I_0 is the current at infinite frequency. Prove (algebraically) that your design and the values you select for the components are correct.

Problem 205.

problems/ac-driven-pr-build-low-pass-filter.tex

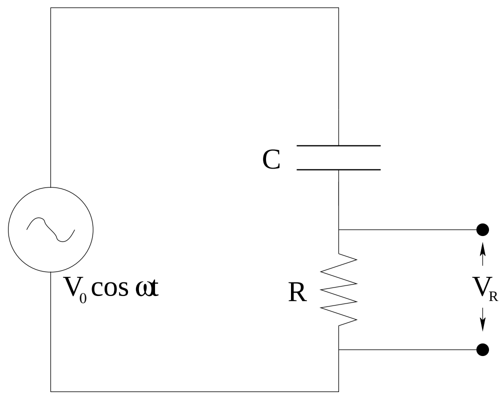


You are given an electronic parts box containing compartments containing 1 ohm, 10 ohm, 100 ohm, ... 1,000,000 ohm resistors. The box also contains compartments of inductors of 1 millihenry, 10 millihenries, ..., 1,000,000 millihenries and capacitors of 1 nanofarad, 10 nanofarads, 100 nanofarads, ... , 1,000,000 nanofarads (basically any power of ten of one ohm, farad, or henry that you like).

Use any parts from this box that you wish to design a *low pass filter* that cuts off frequencies larger than *approximately* $\omega = 10^4$ radians/second. Draw its schematic and clearly indicate where one should place a resistive load that matches the resistor you choose so that it gets significant current at low frequencies. You may consider the frequency cutoff to occur when $I(\omega = 10^4) = \frac{\sqrt{2}I_0}{2}$ where I_0 is the current at zero frequency. Prove (algebraically) that your design and the values you select for the components are correct.

Problem 206.

problems/ac-driven-pr-high-pass-filter.tex



The LC circuit above is connected to an alternating voltage $V_0 \cos(\omega t)$ and the circuit run until it is in a steady state. The following problem steps lead you through an exploration of this circuit so that you end by deducing its purpose and have all quantitative relations in hand to be able to e.g. select C and R to accomplish a given design goal.

a) Write Kirchoff's voltage rule for this circuit loop.

b) Draw the phasor diagram for the voltage, noting that the current must be in phase with the voltage across the resistor.

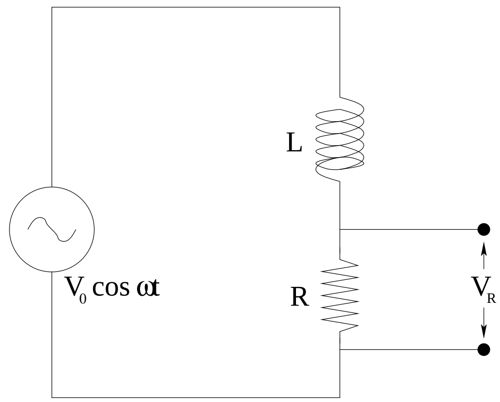
c) From this phasor diagram and the relations between maximum current, reactance or resistance of the circuit elements, and the maximum voltage drop across them, deduce and draw the phasor diagram for the impedance Z of the circuit, clearly labelling the phase angle ϕ.

d) What is the phase angle ϕ and hence the power factor of this circuit (as a function of C, R, and ω)?

e) Find the voltage drops across the resistor V_R and sketch it out *qualitatively* as a function of frequency. If R represents a load of some sort (perhaps the inputs to an amplifier) to which one is sending frequency-encoded information, what kind of filter is this circuit?

Problem 207.

problems/ac-driven-pr-low-pass-filter.tex



The LR circuit above is connected to an alternating voltage $V_0 \cos(\omega t)$ and the circuit run until it is in a steady state. The following problem steps lead you through an exploration of this circuit so that you end by deducing its purpose and have all quantitative relations in hand to be able to e.g. select L and R to accomplish a given design goal.

a) Write Kirchoff's voltage rule for this circuit loop.

b) Draw the phasor diagram for the voltage, noting that the current must be in phase with the voltage across the resistor.

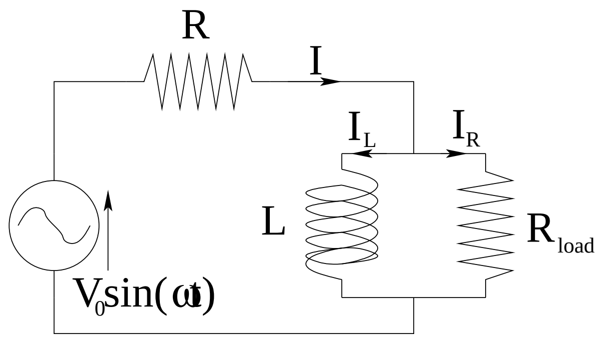
c) From this phasor diagram and the relations between maximum current, reactance or resistance of the circuit elements, and the maximum voltage drop across them, deduce and draw the phasor diagram for the impedance Z of the circuit, clearly labelling the phase angle ϕ.

d) What is the phase angle ϕ and hence the power factor of this circuit (as a function of L, R, and ω)?

e) Find the voltage drops across the resistor V_R and sketch it out *qualitatively* as a function of frequency. If R represents a load of some sort (perhaps the inputs to an amplifier) to which one is sending frequency-encoded information, what kind of filter is this circuit?

Problem 208.

problems/ac-driven-pr-parallel-high-pass.tex



A driven AC circuit is drawn above. Answer the following questions, which ***do not require a complicated algebraic analysis***. V_0, R, R_{load} and L are given, and the frequence ω is variable.

a) What is the ***inductive reactance*** χ_L of the inductor, as a function of the givens and ω?

b) Will the ***current through the inductor*** $\boldsymbol{I_L}$ be maximum at high frequencies or low frequencies?

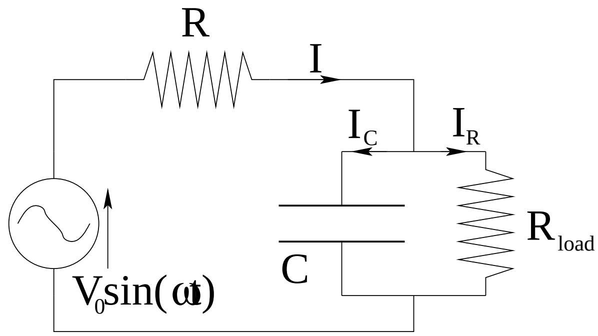
c) Will the ***voltage drop across the load resistor*** $\boldsymbol{V_R}$ be maximum at high frequencies or low frequencies?

d) Based on your analysis above, does this circuit constitute a ***high pass filter*** or a ***low pass filter*** for power delivered to the load resistance R_{load}?

You may give a **short, qualitative** explanation of your reasoning with an equation or two, but do not attempt to solve this as an algebraic system!

Problem 209.

problems/ac-driven-pr-parallel-low-pass.tex



A driven AC circuit is drawn above. Answer the following questions, which ***do not require a complicated algebraic analysis***. $V_0, R, R_{\text{load}}, C$ are given, and the frequence ω is variable.

a) What is the ***capacitive reactance*** χ_C of the capacitor, as a function of the givens and ω?

b) Will the ***current through the capacitor*** $\boldsymbol{I_C}$ be maximum at high frequencies or low frequencies?

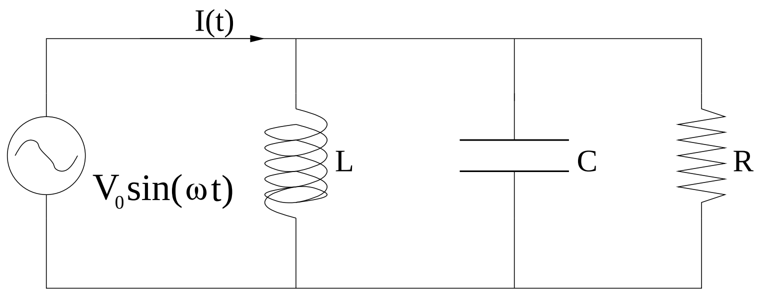
c) Will the ***voltage drop across the load resistor*** $\boldsymbol{V_R}$ be maximum for high frequencies or low frequencies?

d) Based on your analysis above, does this circuit constitute a ***high pass filter*** or a ***low pass filter*** for power delivered to the load resistance R_{load}?

You may give a **short, qualitative** explanation of your reasoning with an equation or two, but do not attempt to solve this as an algebraic system!

Problem 210.

problems/ac-driven-pr-parallel-LRC.tex



A *parallel* LRC circuit connected across a variable AC voltage source $V = V_0 \sin(\omega t)$ is drawn above. Find (in terms of L, R, C, V_0, ω and any quantities you define in terms of these such as χ_L or χ_c):

a) Draw the phasor diagram that represents Kirchhoff's rule for the currents around the loop. What is the form of the total current as a function of time?

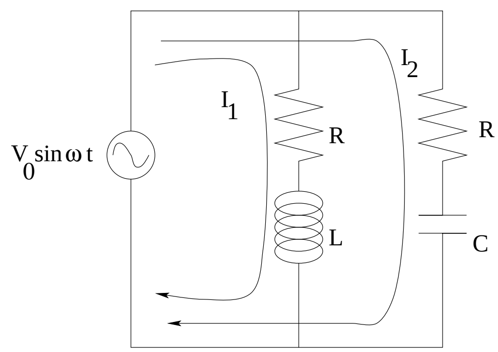
b) Draw the phasor diagram from which the impedance Z can be determined and write down its value in terms of the givens. Also indicate the value of the phase angle δ in terms of the givens.

c) What is the resonant frequency ω_0 for the circuit in terms of the givens?

d) Does the average power delivered to the circuit depend on ω? Why or why not?

Problem 211.

problems/ac-driven-pr-parallel-LR-LC.tex



a) Draw (two) qualitatively correct phasor diagrams that show the voltage drops and gains for each of the two loops shown. Be sure to correctly indicate the phases of the currents I_1 and I_2 relative to the phase of the applied voltage and the voltage drop across each element.

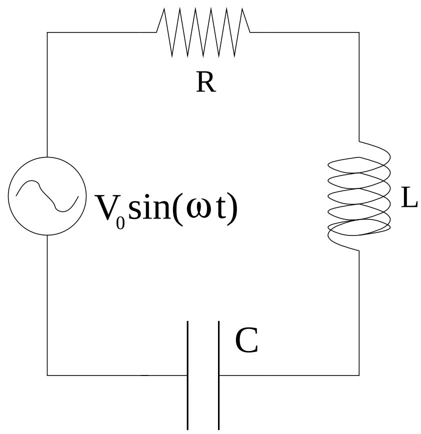
b) Write the Kirchoff's Law (Voltage) for each of the two loops shown that corresponds to your phasor diagram.

c) From a) and b), find the impedance of each loop Z_1 and Z_2, the current phase of each loop δ_1 and δ_2, and write down an expression for $I_1(t)$ and $I_2(t)$. Try to work neatly enough that I can grade this.

d) For extra credit, use Kirchoff's Law (current) to find the total impedance of the circuit, the total current provided by the voltage, and the total power provided by the voltage.

Problem 212.

problems/ac-driven-pr-series-LRC.tex



The LRC circuit above is connected to an alternating voltage $V_0 \sin(\omega t)$ and the circuit is run until it is in a steady state. Assume that the current in this circuit is given by $I_0 \sin(\omega t - \delta)$.

a) Write Kirchoff's voltage rule for this circuit loop.

b) Draw the phasor diagram for the voltages in the loop, noting that the current must be in phase with the voltage across the *resistor*. Clearly label e.g. δ.

c) Draw the related figure for the impedance Z for the circuit. Write an algebraic expression for Z in terms of R, L, C and ω.

d) Write an algebraic expression for δ in terms of R, L, C and ω.

Chapter 14

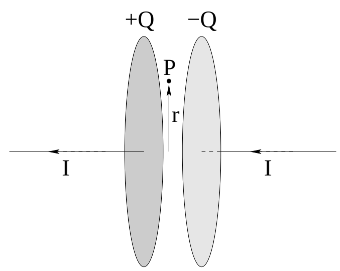
Maxwell's Equations

14.1 Maxwell's Equations

14.1.1 Multiple Choice

Problem 213.

problems/maxwell-mc-discharging-capacitor-poynting-vector.tex



An air-spaced capacitor with circular plates that is charged up with a total charge Q is ***discharging*** with a current I. At a point P between the capacitor plates a distance r from the axis of the plates, the Poynting vector for the combined electric and magnetic field points:

a) outward away from the center.

b) inward toward the center.

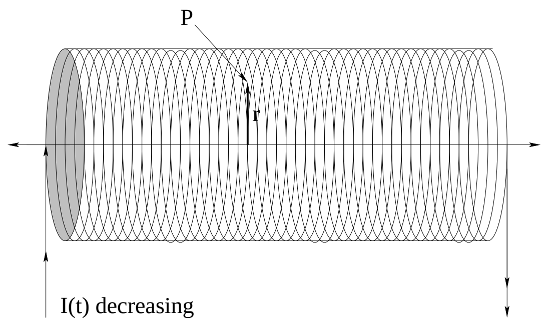
c) to the right.

d) to the left.

e) impossible to tell.

Problem 214.

problems/maxwell-mc-discharging-inductor-poynting-vector.tex



An (ideal) inductor has a current $I(t)$ running through it. That current is ***decreasing***. At a point P inside the inductor and a distance r from its axis, the Poynting vector for the combined electric and magnetic field points:

a) outward away from the center.

b) inward toward the center.

c) to the right.

d) to the left.

e) impossible to tell.

14.1.2 Short Answer/Concept

Problem 215.

problems/maxwell-sa-benefit-of-transformers.tex

Transformers are ubiquitous in our society (on poles outside of our houses) for a very important reason. What is it? I don't need a long essay here, just the bottom line and an indication that you know how it works.

Problem 216.

problems/maxwell-sa-intensity-of-em-wave.tex

Suppose you are given an electromagnetic field whose equations are:

$$\begin{aligned}\vec{\boldsymbol{E}}(z,t) &= \hat{\boldsymbol{y}}E_0\sin(kz+\omega t)\\ \vec{\boldsymbol{B}}(z,t) &= \hat{\boldsymbol{x}}B_0\sin(kz+\omega t)\end{aligned}$$

a) What is the ***time-averaged intensity*** of this field in terms of E_0?

b) Does this wave propagate to the ***right*** ($+z$ direction) or the ***left*** ($-z$ direction)?

You may use any or all of ϵ_0, μ_0, c in your answer, and there is more than one correct way to write the answer.

Problem 217.

problems/maxwell-sa-maxwells-equations-complete.tex

What are Maxwell's Equations? To get full credit for them, they need to be written *exactly* the way I write them in class, with all the little vector arrows, hats, loopy thingies in the middle of integral signs and so forth.

In addition, explain the "meaning" of Gauss's Law for Magnetism. Circle and label Lenz's Law. Label Maxwell's Displacement Current.

a) (GLE)

b) (GLM)

c) (AL+MDC)

d) (FL)

Problem 218.

problems/maxwell-sa-maxwells-equations.tex

What are Maxwell's Equations (as you have learned them so far)? To get full credit for them, they need to be written *exactly* the way I write them in class, with all the little vector arrows, hats, loopy thingies in the middle of integral signs and so forth. An illustrative figure should accompany each one. Circle Lenz's Law.

a) (GLE)

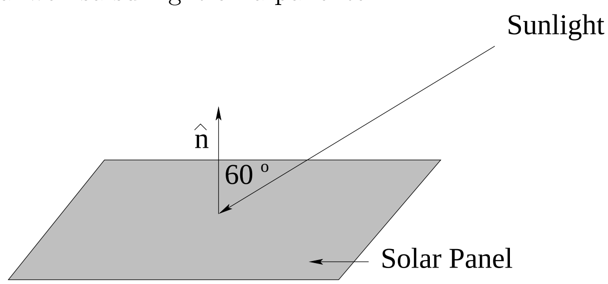
b) (GLM)

c) (AL)

d) (FL)

Problem 219.

problems/maxwell-sa-sunlight-on-a-panel.tex

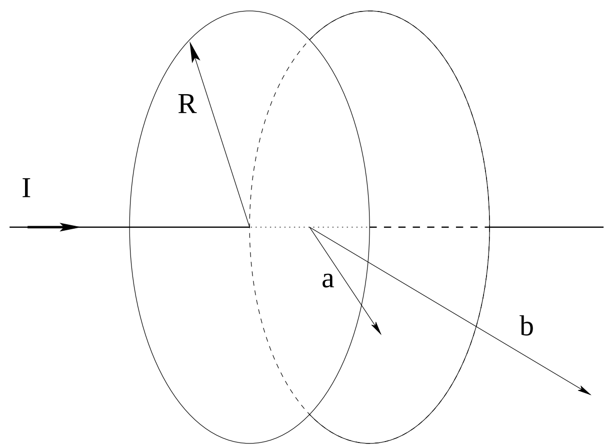


Suppose that the intensity of incoming sunlight is 1000 Watts/meter^2 at the surface of the earth. At a certain time of day, the angle of incidence (relative to a unit vector *normal* to the surface) is 60°. The efficiency of the solar panel is 10%. How much *power* can one collect from five square meters of solar panels at this time of day and angle of incidence?

14.1.3 Long Problems

Problem 220.

problems/maxwell-pr-bfield-in-capacitor.tex



In the picture above, a circular capacitor is being charged by a current I. Using Ampere's Law and the Maxwell Displacement Current, derive a formula for the magnitude of the magnetic field at the two points shown (one at radius $a < R$ from the axis of the capacitor in between the plates, one at radius $b > R$ from the axis of the capacitor outside of the plates).

Problem 221.

problems/maxwell-pr-light-sail.tex

In some science fiction stories, spaceships get their propulsion from lasers, that is, directly from light pressure.

So, put on your sunglasses (really, really dark ones) and explore the design of such a spaceship to see if this is at all feasible. Assume that the ship to be lifted has a mass of 10^4 kg (ten metric tons) – that's about two and a half times the mass of my Ford Excursion and hence not very big. Assume that you get propulsion from a panel of 10^6 lasers with a cross-sectional area of 1 cm each (or about 100 square meters – 10 meters square – of lasers).

a) What would the power of each those lasers need to be, in watts, in order to barely lift the ship (generate a force on the ship equal to that of gravity, assuming $g = 10$ m/sec^2)?

b) Let's assume instead that the lasers have the not totally unreasonable (but still *very* large) intensity of one watt apiece, so that the bank delivers 1 megawatt in 100 m^2. Now what force would be exerted on the spaceship, and what (roughly) would be its acceleration. Can you lift such a ship off of the earth?

So, what do you report to NASA about your massless "photon drive"? Good idea or bad? Don't forget those sunglasses...

Problem 222.

problems/maxwell-pr-light-vs-mass-propulsion.tex

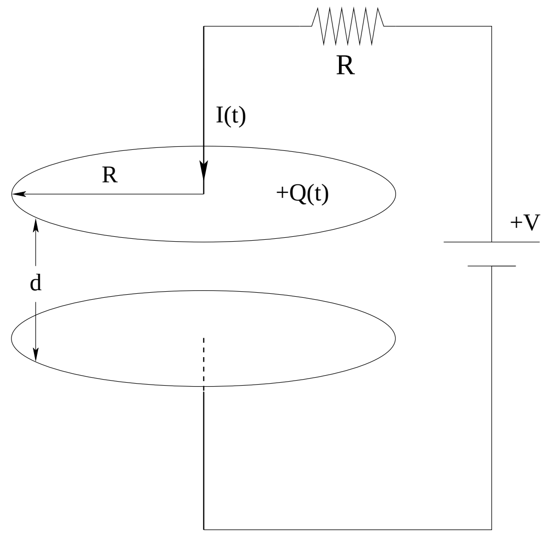
Invent and compare spaceships (draw them in the blank space above) that are driven according to the following (ideal) criteria. The actual source of power is e.g. a small fusion plant onboard the spaceship.

a) Suppose a spaceship is powered by a laser that emits 1000 Watts in a beam 1 cm^2 in cross-sectional area. What is the recoil force (per KW) exerted by the laser?

b) Suppose instead the spaceship is powered by throwing mass. If it throws 1000 small beads per second, each with mass $m = 1$ gram and with a kinetic energy of 1 Joule per bead (so the power required to operate it is still 1000 Watts), what is the average force (per KW) exerted by the mass-driver?

Problem 223.

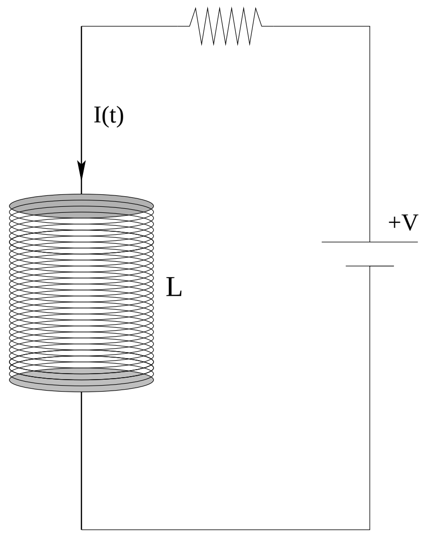
problems/maxwell-pr-poynting-vector-capacitor.tex



Our archetypical model for a capacitor of capacitance $C = \frac{\epsilon_0 \pi R^2}{d}$ is drawn above: two circular "perfectly conducting" plates with radius R, separated at a distance d by a vacuum. Assuming that the current is flowing as shown to charge the capacitor at rate $I = +dQ/dt$, show that the flux of the Poynting vector into the volume between the plates is equal to the rate energy is being stored in the capacitor.

Problem 224.

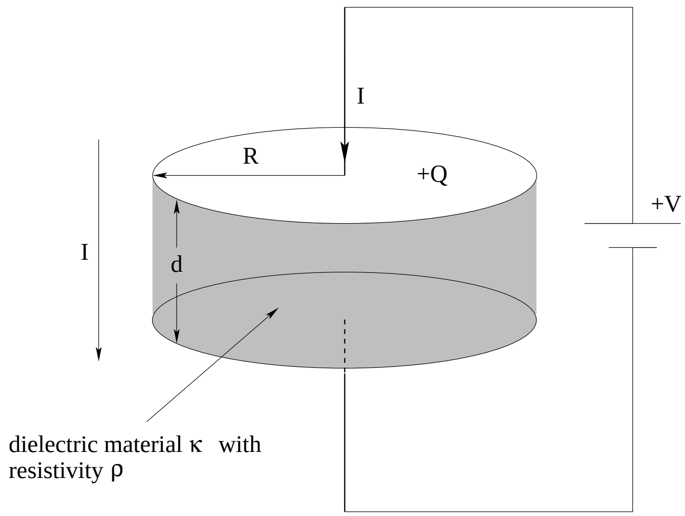
problems/maxwell-pr-poynting-vector-inductor.tex



Our archetypical model for an inductor is drawn above: N circular turns of wire forming a solenoid of length L and radius R, carrying a current $I(t)$. Show that the flux of the Poynting vector for this inductor into its interior volume equals the power flowing into it evaluated as $P(t) = V_L(t)I(t)$ where $V_L(t)$ is the voltage drop across the inductor.

Problem 225.

problems/maxwell-pr-poynting-vector-resistor.tex



Our archetypical model for a resistor is drawn above: two circular "perfectly conducting" plates (metal contacts) with radius R, separated at a distance d by a material with resistivity ρ.

a) In a steady state situation where a DC voltage V is applied as shown, find the field $\vec{E}$ inside the resistive material.

b) Find the current density $\vec{J}$ inside the resistive material.

c) From Ampere's law, find the magnetic field as a function of r in the region between the plates.

d) From your answers to a) and c), find the Poynting vector $\vec{S}$ (magnitude and direction) as a function of r in the region in between the plates.

e) NOW show that:

$$\oint_A \vec{S} \cdot \hat{n} dA = -I^2 R$$

where A is the outer surface of the resistor and $\hat{n}$ is its outward-directed normal unit vector.

Thus the heat that appears in the resistor *can* be thought of as the electromagnetic field energy that flows in through its outer surface!

Problem 226.

problems/maxwell-pr-properties-of-em-field.tex

Suppose you are given an electromagnetic field whose equations are:

$$\begin{aligned}\vec{\boldsymbol{E}} &= \hat{\boldsymbol{x}} E_0 \sin(kz - \omega t)\\ \vec{\boldsymbol{B}} &= \hat{\boldsymbol{y}} B_0 \sin(kz - \omega t)\end{aligned}$$

Answer the following questions about it. You may use any or all of ϵ_0, μ_0, c in your answers in addition to the actual parameters of the fields.

a) What is the magnitude B_0 in terms of E_0?

b) What is the direction of propagation of the electromagnetic field?

c) What is the speed of the wave *in terms of the wavenumber* k *and the angular frequency* ω?

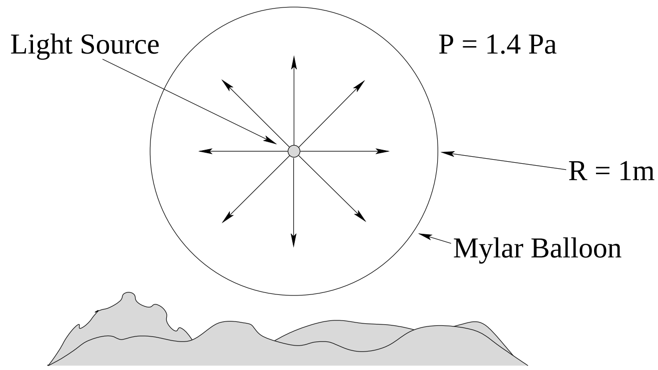
d) What is the instantaneous Poynting Vector for this electromagnetic field?

e) What is the *time average intensity* of this field in terms of E_0?

f) What *radiation force* would be exerted by this field on a *reflective* screen of area A held perpendicular to the direction of propagation?

Problem 227.

problems/maxwell-pr-radiation-inflates-balloon.tex



The pressure on Triton, a moon of Neptune, is roughly 1.4 pascals. Suppose you have a weather balloon that is made of reflective mylar that, if fully inflated, is 1 meter in radius. You would like to inflate it on Triton using only a very bright light source to exert radiation pressure on the inside that exceeds Triton's atmospheric pressure on the outside. The arrangement to accomplish this is pictured above.

a) The balloon will inflate nicely if you make the radiation pressure inside equal to 2 pascals. What must the ***intensity*** of the light be at the inner surface of the balloon in order to exert this pressure, assuming that the light is *absorbed* (not reflected)? You may answer with an algebraic expression, but the arithmetic is pretty easy to do in your head.

b) What must the ***power*** of the light source at the center be to achieve this intensity? Again, an algebraic expression is sufficient, but at least estimating the arithmetic isn't that difficult if you remember that $4\pi \approx 12.5$.

Given the power requirement (compared to, say, just pumping up or warming some of the atmosphere) is this a reasonable way to inflate the balloon, even against an atmosphere so tenuous that a reasonable person would call it "a vacuum"?

Problem 228.

problems/maxwell-pr-radiation-pressure-spherical-dust.tex

A spherical grain of dust of radius r is a distance $R \gg r$ from Mr. Sun. Mr. Sun has a mass $M_{\rm sun} = 2 \times 10^{30}$ kg and produces power (all radiated away in the form of light) at a rate $P_{\rm sun} = 3.83 \times 10^{26}$ watts. The mass density of the grain of dust is ρ.

a) Draw a figure schematically representing what's going on to help you solve the rest of the problem.

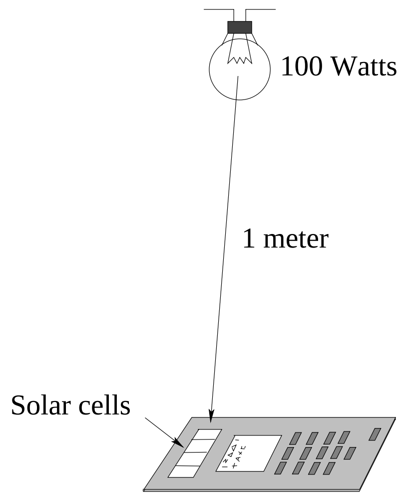
b) Assuming that light is reflected from the grain in such a way that (on average) all the momentum in the light that hits the grain at all is transferred to the grain (per unit time), find an **algebraic expression** for the radius r of the particle that will cause it to hang precisely balanced between gravitation and light pressure. Note well, do not use the numbers above yet.

c) Does your answer depend on R?

d) Evalute this radius, assuming a density $\rho = 1000$ kg/m^3 (the density of water – most condensed matter has a density between this and about ten times this).

Problem 229.

problems/maxwell-pr-solar-cell-from-light-bulb.tex



Many pocket calculators today run on solar cells instead of batteries, using ambient light (such as a nearby light bulb) for their power source.

You have a pocket calculator has a solar cell with a collection area of $A = 4$ cm^2. It works fine using the light from an ordinary $P_0 = 100$ Watt light bulb located $R = 1$ meter away. Make a reasonable (upper bound) estimate for the power used by the calculator. Assume that the efficiency of the solar cell is $\eta = 0.1$ (10% efficient).

You may answer this problem algebraically if you wish (or don't have a solar powered calculator handy to help with the arithmetic). If you do have a calculator (or do the arithmetic by hand; it isn't difficult) and get the right numerical answer as well as the right algebraic answer, you may have 2 extra points.

Problem 230.

problems/maxwell-pr-solar-panel-analysis.tex

You need to decide whether or not to buy a solar panel for your house. Use the facts below and your understanding of light energy to *estimate* whether or not you will invest in a panel based energy system.

- The sun produces 4×10^{26} Watts of power and is 1.5×10^{11} meters away from the earth.
- Only about 70% of this makes it down through the atmosphere on a clear day at the equator.
- On average, at our latitude the sun will strike your panel at an angle of 45°.
- The solar panel converts sunlight into electrical energy and stores it into a battery to be recovered later at an overall efficiency of about 10%.
- Electricity can be purchased from a power company at a cost of $0.10 kW-hour (what is that in joules?).
- A 1 m^2 solar panel and associated battery storage system cost approximately $1000, with additional panels (that will feed the same battery) costing around $250 each.

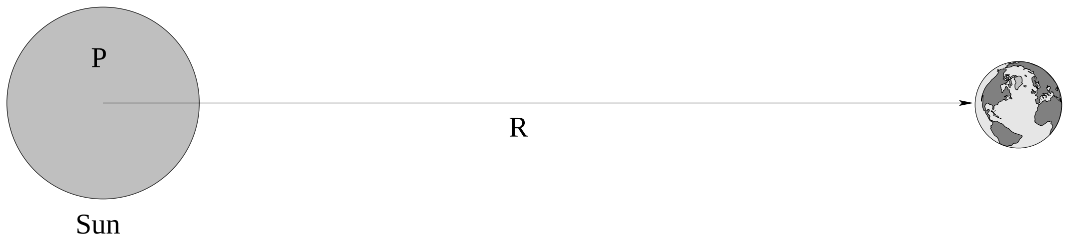
If you collect an average of six hours of sunlight a day, roughly how long would it take to recover the cost of a single panel? Show *all* of your reasoning, supporting it with figures and diagrams as needed. Note that to be *completely* fair, you'd need to add in the cost of borrowing the money for the panel in which case the answer might well be "never", but let's go with the easy answer first.

Next, assume that you double your investment and spend $2000 for five panels. How long will it take to recover your investment now?

You may make "reasonable" simplifying assumptions to make your arithmetic easier as you proceed as long as you are very clear as to what they are, e.g. – six hours is 0.25 days, $0.25 * 365 \approx 100$ days...

Problem 231.

problems/maxwell-pr-sunlight-at-the-Earth.tex



The power output of the Sun is roughly $P = 4 \times 10^{26}$ Watts. The Sun is $R = 1.5 \times 10^{11}$ meters away from Earth. Answer the problems below ***algebraically***, but feel free to plug the numbers into a calculator if you have one (for one extra point per item below) as the numerical answers are instructive.

a) What is the intensity of sunlight at the top of the atmosphere?

b) What is the approximate *force* exerted by the Sun on a sheet of shiny aluminum foil 1 meter square placed out in the Sun on a clear day at midday? Assume that the atmosphere absorbs roughly half of the available energy before the light reaches the ground. (Use 1500 Watts/m^2 at the top of the atmosphere if you couldn't do better in your answer to a).)

c) Suppose one builds a solar power plant that has 1 kilometer squared worth of collectors, each of them 5% efficient at converting light energy into electricity. What is the expected power output of this plant under optimal conditions?

Chapter 15

Light

15.1 Snell's Law

15.1.1 Multiple Choice

Problem 232.

problems/snells-law-mc-critical-angle-diamond.tex

Diamond has an index of refraction of around $n_d = 2.4$, air has an index of refraction around $n_a = 1$. What is the ***critical angle*** at which total internal reflection will occur when white light is incident on the surface of a diamond facet from the inside?

a) $\theta_c = \cos^{-1}(2.4)$

b) $\theta_c = \cos^{-1}(1/2.4)$

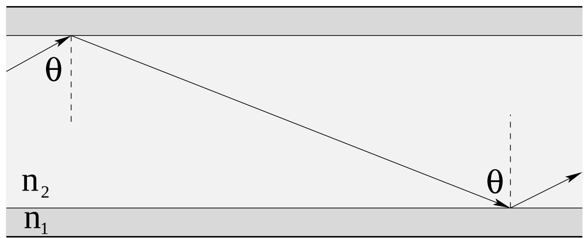
c) $\theta_c = \sin^{-1}(1/2.4)$

d) $\theta_c = \tan^{-1}(1/2.4)$

e) $\pi/2.4$

Problem 233.

problems/snells-law-mc-fiber-optic-cable.tex



A fiber optic cable is made with a glass core (index of refraction n_2) and an outer sheath (index of refraction n_1). To achieve efficient transmission of the light along the fiber, one should choose:

a) $n_1 < n_2$ and sufficiently large θ.

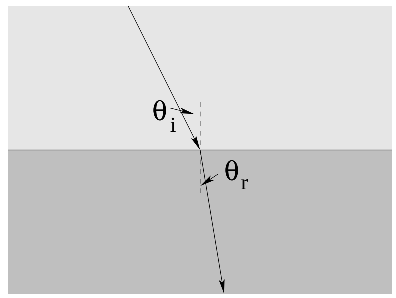
b) $n_1 > n_2$ and sufficiently large θ.

c) $n_1 < n_2$ and sufficiently small θ.

d) $n_1 > n_2$ and sufficiently small θ.

Problem 234.

problems/snells-law-mc-water-to-glass.tex



A beam of light is incident on the interface between water ($n_w = 4/3$) and glass ($n_g = 3/2$) at an angle of incidence of $\theta_i = 30°$. Which statement below is true:

a) Water is over glass, $\theta_r = \sin^{-1}(2/9)$.

b) Water is over glass, $\theta_r = \sin^{-1}(4/9)$.

c) Glass is over water, $\theta_r = \sin^{-1}(2/3)$.

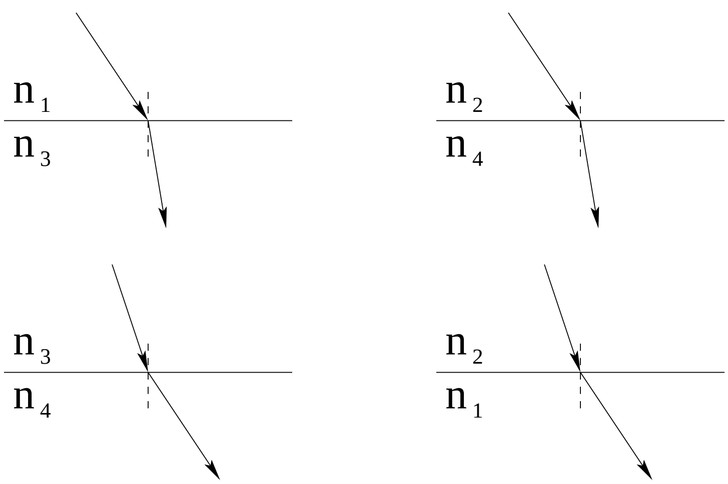
d) Glass is over water, $\theta_r = \sin^{-1}(9/2)$.

e) This picture could not represent a light beam going from water into glass.

15.1.2 Ranking/Scaling

Problem 235.

problems/snells-law-ra-index-of-refraction.tex



The figures above represent data indicating the way light rays refract when passing between four media with indices of refraction n_1, n_2, n_3, and n_4. Rank the indices of refraction from ***smallest to largest***. Note well: You will have to solve a (fairly easy) logic puzzle to answer this question!

15.1.3 Short Answer/Concept

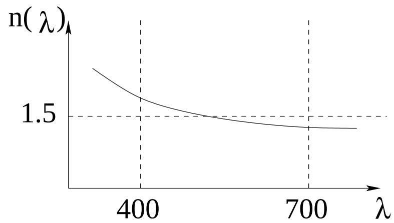
Problem 236.

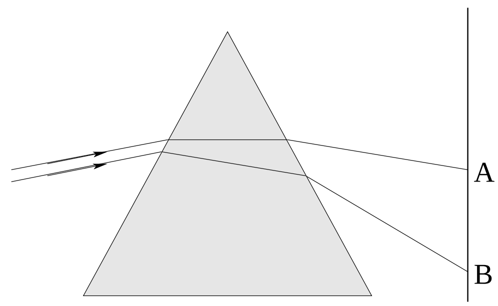
problems/snells-law-sa-critical-angle-light-fiber.tex

Light propagates down a light fiber “without loss” even as it goes around (gentle) curves by reflecting off the interface between the fiber and its surroundings. Assuming that a fiber has an index of refraction of $n_f = 3/2$ and it is submerged in water (with $n_w = 4/3$), what is the critical angle of incidence such that light will remain trapped in the fiber?

Problem 237.

problems/snells-law-sa-dispersion-prism.tex





The graph above shows the **dispersion** (index of refraction as a function of wavelength) for a kind of glass around the mean value for visible light of 3/2. The wavelengths of violet light (λ = 400 nanometers) and red light (λ = 700 nanometers) are indicated with vertical dashed lines. Two rays of light are shown entering a prism on the right. *Give a short argument based on explicit principles of physics* that explains which ray is which color and fill in the blanks below:

A:

B:

15.1.4 Long Problems

Problem 238.

problems/snells-law-pr-critical-angle-diamond.tex

Unpolarized light is incident on the surface of a large diamond (n = 2.4). Some of the light is reflected from the diamond; the rest penetrates the diamond surface and is refracted.

a) Find the angle at which the reflected light is *completely* polarized and indicate the direction of polarization on a suitable figure.

b) Diamond is interesting for another reason. It "traps light" and reflects it internally many times as it bounces from facet to facet. Explain how a diamond (with n = 2.4) traps more light more than an identically shaped piece of glass (n = 1.5). Your answer should be at least partly quantitative.

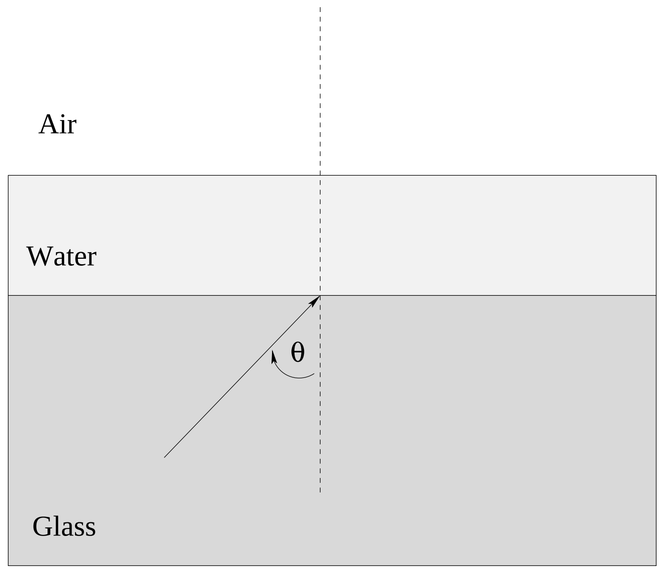
Problem 239.

problems/snells-law-pr-derive.tex

Derive Snell's Law. You may use either the wave picture (that I gave in class) or the Fermat principle (which was on your homework). For a bit of extra credit, do it both ways. Be sure to give the definition of index of refraction.

Problem 240.

problems/snells-law-pr-double-critical-angle.tex



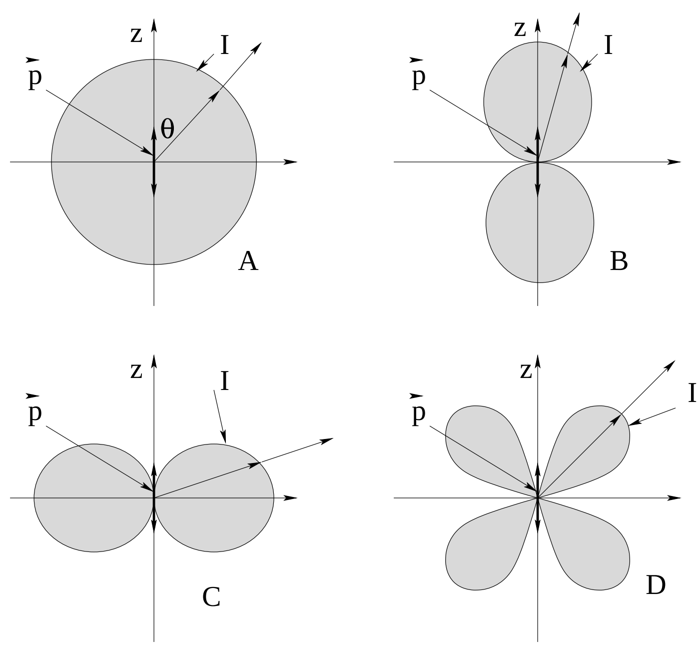
Suppose that you have a sheet of glass with a thin layer of water on top, in air, as shown (where $n_a = 1 < n_w = 4/3 < n_g = 3/2$). Prove that the critical angle in the glass (where total internal reflection occurs for rays coming from the glass through the water into the air) is not changed by the presence of the water.

15.2 Polarization

15.2.1 Multiple Choice

Problem 241.

problems/polarization-mc-dipole-antenna.tex



Four graphs are presented above of the distribution of outgoing radiated intensity from an oscillating charged dipole (antenna) aligned with the z-axis. The polar angle θ is shown in the first graph. Which one is correct?

a) A

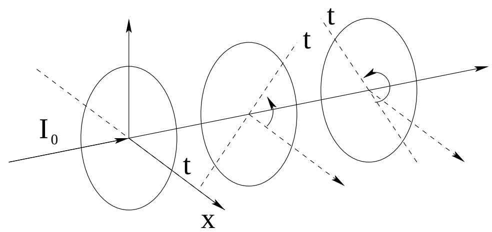
b) B

c) C

d) D

Problem 242.

problems/polarization-mc-malus-3-filters.tex



Unpolarized light of intensity I_0 is normally incident along the z-axis onto three successive polaroid filters with transmission axes at $\theta = 0, 60°, 120°$ with respect to the x-axis. What is the intensity of light that makes it through the last filter?

a) $I = 0$

b) $I = \frac{I_0}{4}$

c) $I = \frac{I_0}{8}$

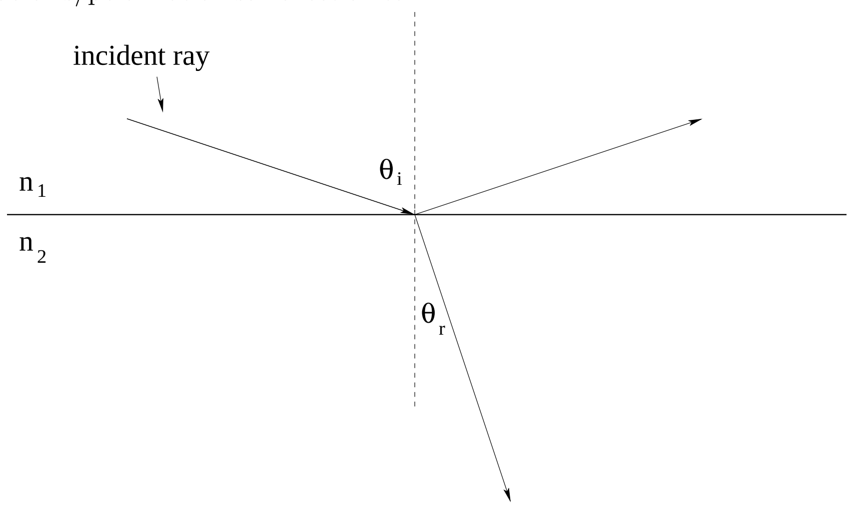
d) $I = \frac{I_0}{16}$

e) $I = \frac{I_0}{32}$

15.2.2 Short Answer/Concept

Problem 243.

problems/polarization-sa-reflection.tex



Unpolarized radiation is incident upon a reflecting surface between two different media with indices of refraction as shown. Draw E-field vectors onto the figure that illustrating the polarization of the transmitted and reflected rays. Write down the Brewster formula for the angle of *incidence* at which the reflected ray is completely polarized in terms of n_1 and n_2.

Problem 244.

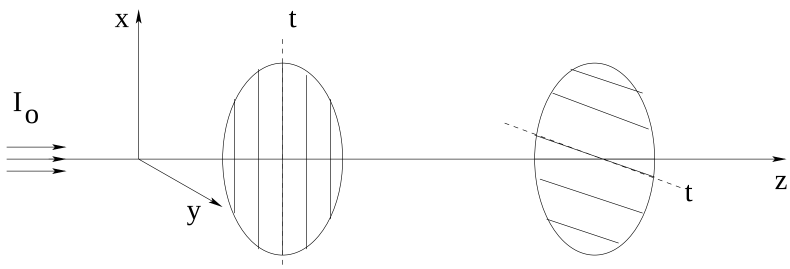
problems/polarization-sa-sunglasses-scattering.tex

You are out fishing and your polarized sunglasses do great job of reducing reflected glare off the water in the late afternoon. Do they *also* reduce the *scattered* glare from the sky just above the horizon at noon? No good just answering yes or no, have to draw a picture to indicate why to get credit.

15.2.3 Long Problems

Problem 245.

problems/polarization-pr-maximize-3-filter-transmission.tex



Unpolarized light of intensity I_0 is normally incident along the z-axis onto two successive polaroid filters with transmission axes at $\theta_1 = 0°$ and $\theta_2 = 90°$ with respect to the x-axis as drawn above. No light is transmitted through these two filters in this orientation.

You are given a third filter.

a) Where (along the z-axis should you place it to ***maximize*** the intensity of transmitted light?

b) What should its angle θ be relative to the x-axis?

c) What is the (maximized) transmitted intensity relative to I_0?

Derive or verify/justify your answers.

Problem 246.

problems/polarization-pr-reflection-absorption-scattering.tex

Indicate, with pictures and/or a short descriptions, how light is polarized by:

a) **Absorption:** Derive and explain (with a figure) Malus's law, which quantitatively describes how much light polarized in one direction passes through a filter whose transmission axis is rotated through an angle θ with respect to that direction.

b) **Reflection:** Derive and explain (with a figure) the formula for the Brewster angle (telling us what the Brewster angle is).

c) **Scattering:** Explain with a figure, and indicate the rule that leads to the result.

Problem 247.

problems/polarization-pr-scattering-and-sunglasses.tex

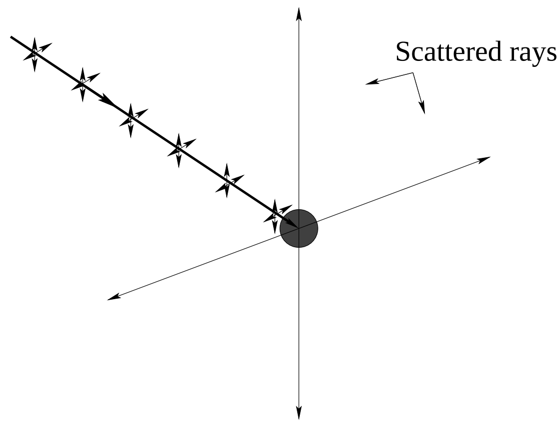
Polaroid sunglasses are lovely because they reduce reflected glare in the morning and evening when the sun is low AND because they darken the blue sky near the horizon in the middle of the day when the sun is directly overhead.

How does all this work? To answer this, I need a description of polarization by absorption (the sunglasses) including the transmission axis used in the glasses, and the "standard pictures" that indicate QUALITATIVELY how the reflected glare is polarized (so that the sunglasses will block it) and how the scattered light from the sky is polarized (so that the sunglasses will block it).

Clearly good pictures are essential to your answer.

Problem 248.

problems/polarization-pr-scattering.tex



Unpolarized radiation is incident upon a molecule and is scattered at right angles as shown. State the rule we use to determine the polarization of each of the light rays and draw the polarization into the figure in the usual way.

Chapter 16

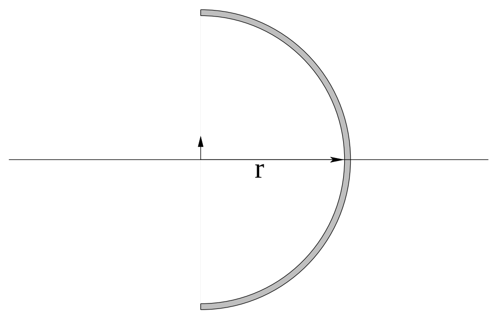
Lenses and Mirrors

16.1 Mirrors

16.1.1 Multiple Choice

Problem 249.

problems/mirrors-mc-concave-object-at-r.tex



A concave mirror is shown above with an object located at its center of curvature, $s = r$. The image is at:

a) $s' = -r$, virtual, erect, and the same size as the object.

b) $s' = r$, real, inverted, and larger than the object.

c) $s' = r/2$, virtual, inverted, and smaller than the object.

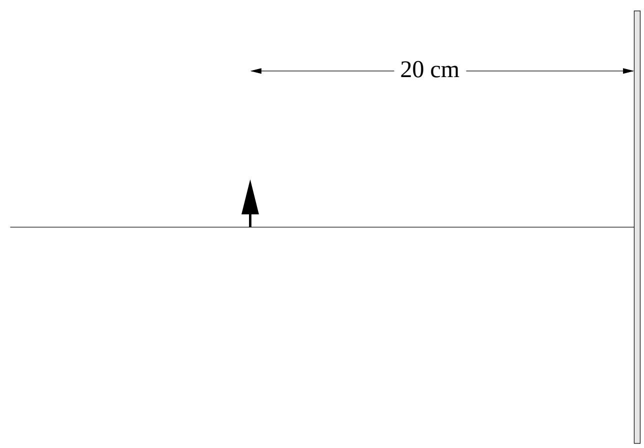
d) $s' = r$, real, inverted, and the same size as the object.

e) $s' = -\infty$, virtual, inverted, and larger than the object.

16.1.2 Long Problems

Problem 250.

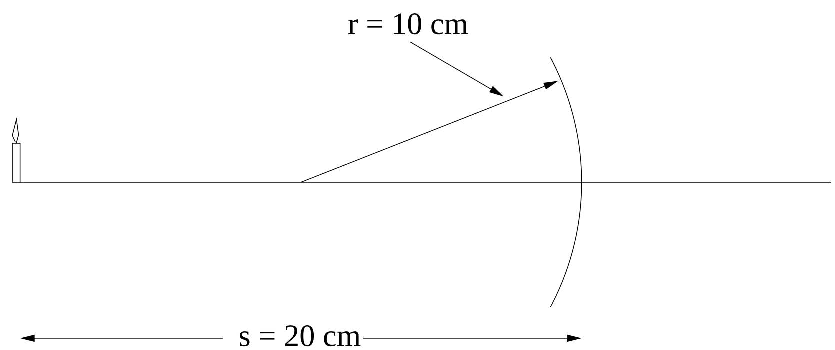
problems/mirrors-pr-real-image-mag3.tex



There is an object 20 cm away from a screen. Using a concave mirror, I would like to throw an image of this object upon the screen that is three times larger than the object itself. Find the location of the mirror (with respect to object and screen and the focal length of the mirror necessary to accomplish this. Draw the corresponding ray diagram.

Problem 251.

problems/mirrors-pr-s20-f10-converging.tex



The mirror above has a radius of curvature $r = 10$ cm. A candle is placed at $s = 20$ cm as shown. Find:

- The focal length of the mirror (draw the focal point in on the diagram above).
- The location s' of the image in centimeters.
- The magnification of the image.
- State whether the image is real or virtual, erect or inverted.
- Draw the ray diagram for this arrangement using the three "named" rays used for both lenses and mirrors as shown in class. Obviously it should validate your answers to the above.

Problem 252.

problems/mirrors-pr-s=2f-converging.tex

This problem will be solved algebraically in terms of the positive length $f > 0$. If it pleases you to make this length definite, say 10 cm, feel free, but it is not necessary.

A small object is placed $2f$ in front of a *diverging* (convex) mirror with focal length $-f$ – negative because it is diverging. Determine (in terms of f where appropriate):

a) The image distance s'.

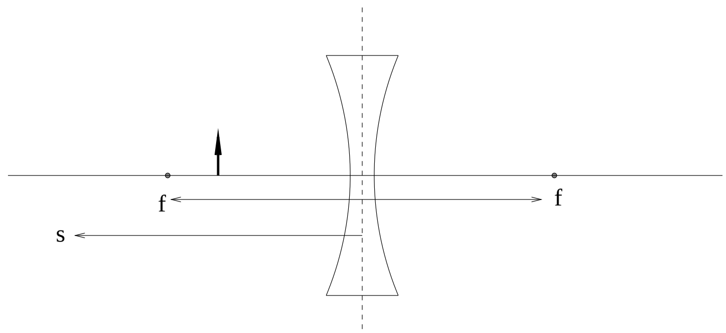
b) The magnification m.

c) The kind of image (erect/inverted, real/virtual).

Draw a **neat ray diagram** for the arrangement using (and labelling!) the three standard rays covered in class to locate the image. It should at least approximately correspond to your numerical results above. It's a good idea to use a straightedge of some sort, and try to make the size of the diagram reasonable so it clearly illustrates the problem.

Problem 253.

problems/mirrors-pr-s60-f-20-diverging.tex



A candle 20 cm high is placed 60 cm in front of the center of a thin lens. The lens has a focal length of -80 cm.

a) Find the location s' of the image, its magnification, and indicate whether the image is real or virtual.

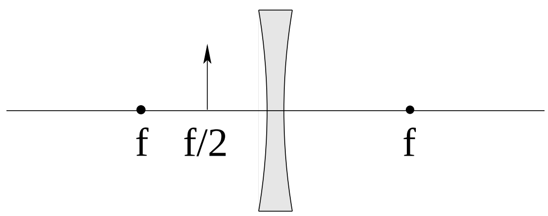
b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include 3 rays that uniquely specify the image location.

16.2 Lenses

16.2.1 Multiple Choice

Problem 254.

problems/lenses-mc-diverging-image.tex



An object (arrow) is placed at an object distance $s = f/2$ in front of a ***diverging*** lens with a ***negative*** focal length as shown. Note that the dots indicate the location, but not the sign, of f in the figure above. The image formed is:

a) real and larger than the object.

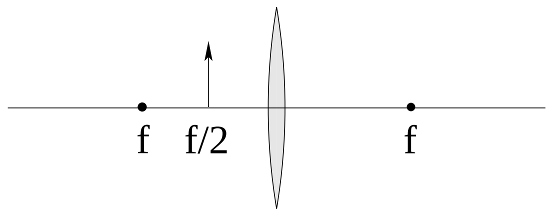
b) real and smaller than the object.

c) virtual and larger than the object.

d) virtual and smaller than the object.

Problem 255.

problems/lenses-mc-virtual-image-m2.tex



An object (arrow) is placed at a distance $f/2$ from a lens with a ***positive*** focal length f as shown. The image is:

a) real and twice as large as the object.

b) real and half as large as the object.

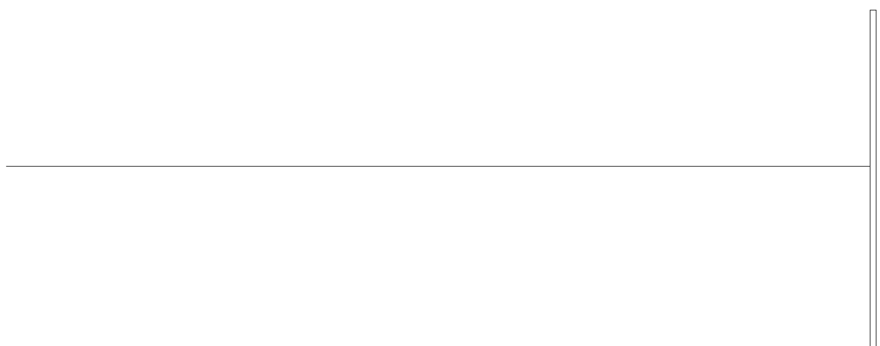
c) virtual and twice as large as the object.

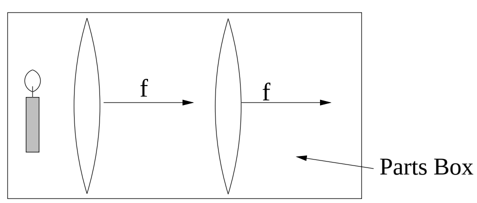
d) virtual and half as large as the object.

16.2.2 Long Problems

Problem 256.

problems/lenses-pr-candle-and-screen.tex



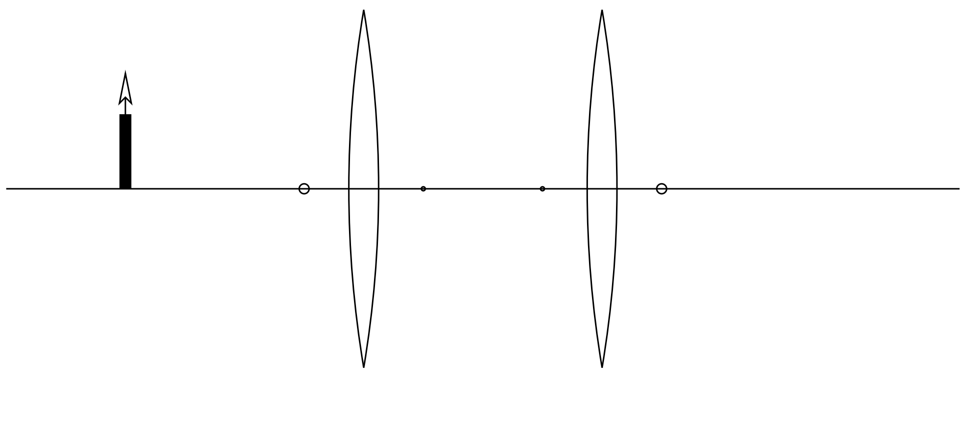


You have a candle and two lenses with a focal length of 10 cm (each). You wish to cast a real image of the candle *right side up* upon a screen. You want the image size (magnitude) to be exactly two times the size of the actual candle.

a) Determine the set of locations of the object, the lenses, and the image/virtual objects such that this condition is satisfied *and* so that the absolute value of the magnification of the second lens is $|m_2| = 1$.

b) Carefully place the components on the figure above and draw a ray diagram to locate the image, to scale, in agreement with your answers to a. Be sure to include the 3 rays that uniquely specify the image location, for each lens.

Don't burn yourself on the candle.



Problem 257.

problems/lenses-pr-candle-and-two-lenses.tex

A candle 20 cm high is placed 40 cm in front of the center of a thin lens. This lens has a focal length of 10 cm. A second thin lens, also with a focal length of 10 cm, is placed 40 cm from the first. Find:

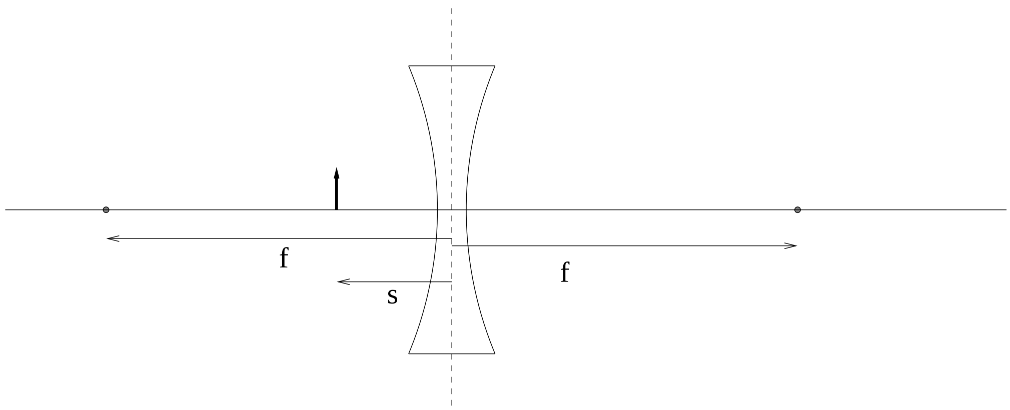
a) The location s' of the image due to the **first** lens and its magnification. Indicate whether the image is real or virtual.

b) The location s'' of the image (of the image of the first lens) of the **second** lens. Find the overall magnification, and indicate if the final image is real or virtual.

c) Draw a ray diagram to locate the image in agreement with your answers to a and b. Be sure to include and label 3 rays that uniquely specify the image locations.

Problem 258.

problems/lenses-pr-candle-diverging-virtual-image.tex



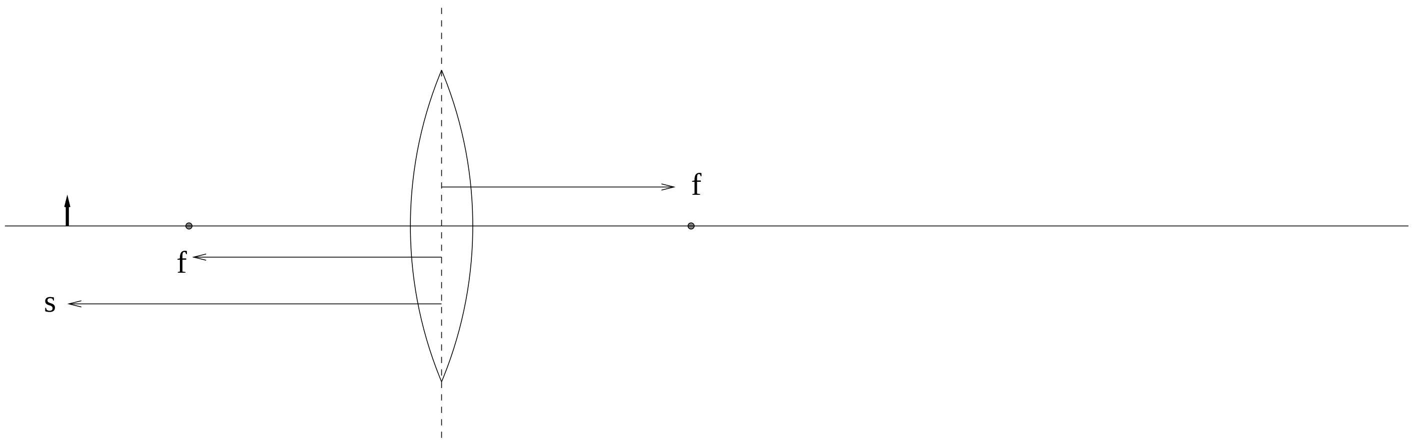
A candle $y = 5$ cm high is placed $s = 20$ cm in front of the center of a thin diverging lens. The lens has a focal length of $f = -60$ cm.

a) Find the location s' of the image, its magnification, and indicate whether the image is real or virtual.

b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include 3 rays that uniquely specify the image location. Use a straightedge (folded piece of paper) or ruler to draw the rays if at all possible, and be neat.

Problem 259.

problems/lenses-pr-candle-real-image.tex



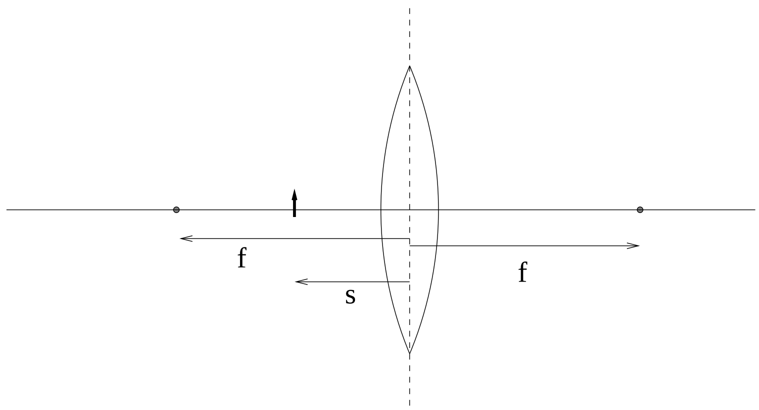
A candle 10 cm high is placed 75 cm in front of the center of a thin lens. The lens has a focal length of 50 cm.

a) Find the location s' of the image, its magnification, and indicate whether the image is real or virtual.

b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include the 3 rays that uniquely specify the image location.

Problem 260.

problems/lenses-pr-candle-virtual-image.tex



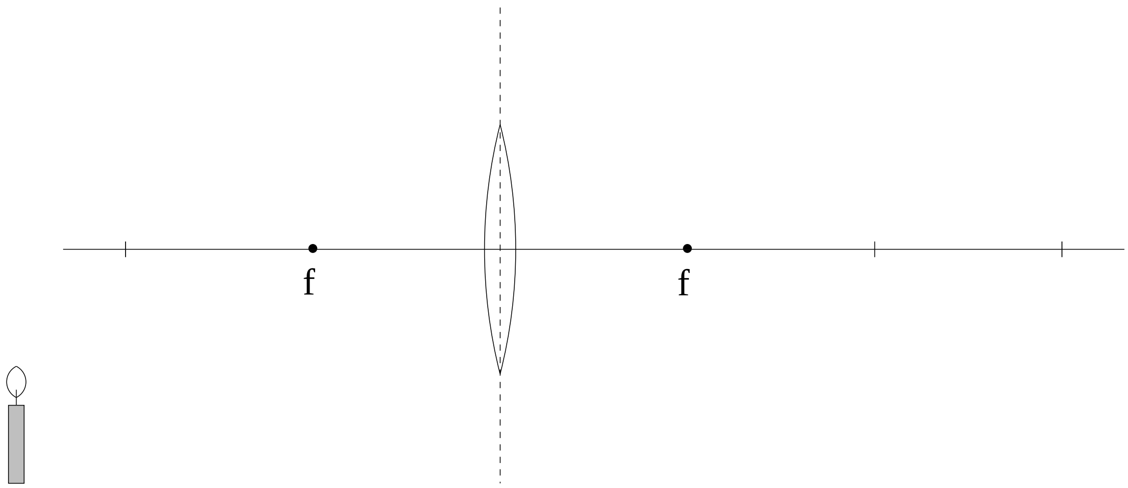
A candle 5 cm high is placed 20 cm in front of the center of a thin lens. The lens has a focal length of +40 cm.

a) Find the location s' of the image, its magnification, and indicate whether the image is real or virtual.

b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include 3 rays that uniquely specify the image location.

Problem 261.

problems/lenses-pr-f10-real-m2.tex



You have a candle and a lens with a focal length $f = +10$ cm. You wish to view a ***real, inverted*** image of a candle that is exactly ***two times the size of the actual candle***.

a) Find s and s' such that this kind of image is formed.

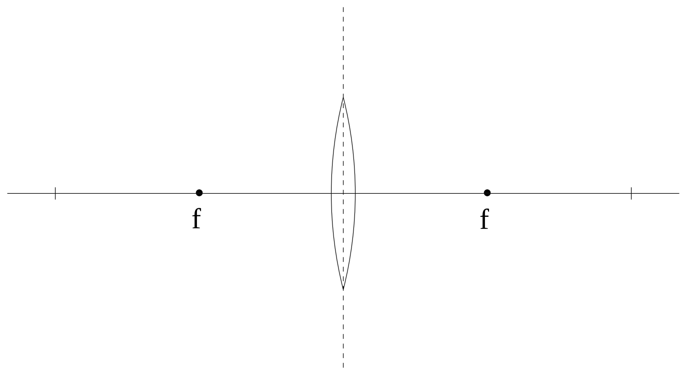
b) Carefully 'place' the candle into the figure above at the position you determine and draw a ray diagram to locate the image, to scale, in agreement with your answers to a). Be sure to include the 3 rays that uniquely specify the image location.

c) If you view this image with the naked eye, is the apparent size of the image larger or smaller than it would be if you used the lens as a simple magnifier to view the image?

Don't burn yourself on the candle.

Problem 262.

problems/lenses-pr-f10-virtual-m2.tex



You have a candle and a lens with a focal length of +10 cm. You wish to view a virtual, erect image of a candle that is exactly two times the size of the actual candle.

a) Find s and s' such that this kind of image is formed.

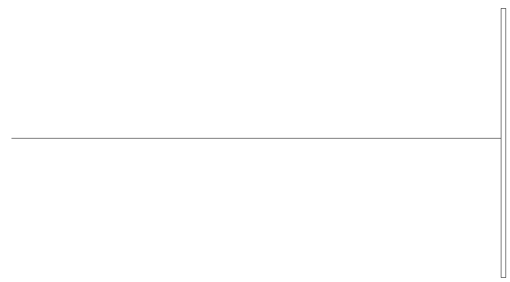
b) Carefully 'place' the candle into the figure above at the position you determine and draw a ray diagram to locate the image, to scale, in agreement with your answers to a). Be sure to include the 3 rays that uniquely specify the image location.

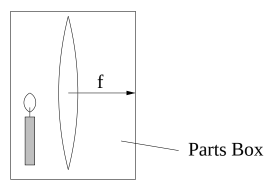
c) If you view this image with the naked eye, is the apparent size of the image larger or smaller than it would be if you used the lens as a simple magnifier to view the image?

Don't burn yourself on the candle.

Problem 263.

problems/lenses-pr-f15-real-m2.tex





You have a candle and a lens with a focal length of 15 cm. You wish to cast a real image of the candle upon a screen. You want the image size (magnitude) to be exactly two times the size of the actual candle.

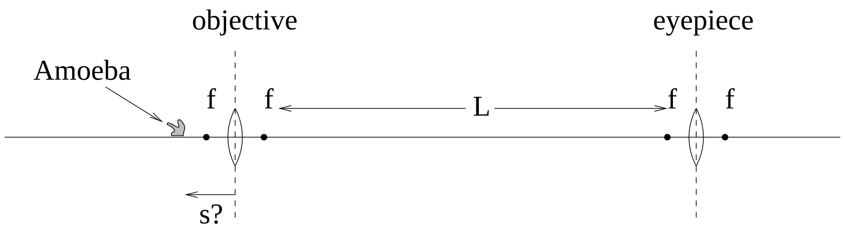
a) Find s and s' such that this kind of image can be formed.

b) Carefully place the components on the figure above and draw a ray diagram to locate the image, to scale, in agreement with your answers to a. Be sure to include the 3 rays that uniquely specify the image location.

Don't burn yourself on the candle.

Problem 264.

problems/lenses-pr-inverting-microscope.tex



The arrangement of lenses that makes up a compound microscope is pictured above. The focal lengths of the objective and eyepiece lenses are $f_o = f_e = f = 1$ cm. The tube length is $L = 9$ cm.

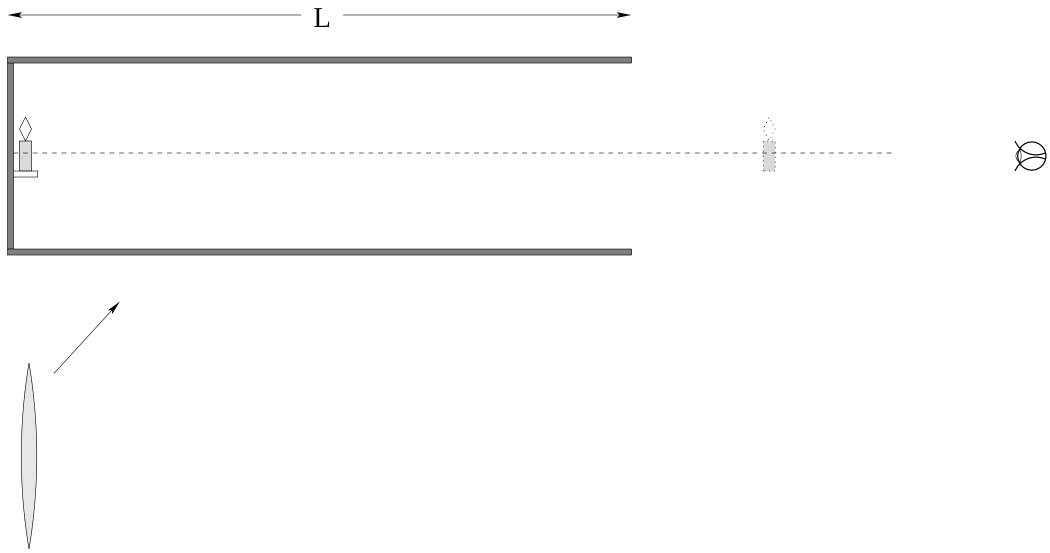
a) Find s (the object distance from the objective lens) such that the final image viewed by the eye is in focus (at infinity, as imaged by the eyepiece).

b) Draw the ray diagram from which you can find the overall magnification (try to use a straight edge to do this).

c) From this diagram and your knowledge of the separate purposes of the two lenses, find the overall magnification. Explain each part (that is, what are the separate roles of the objective and eyepiece).

Problem 265.

problems/lenses-pr-physics-demo-m1.tex



You are given the job of designing a display for the physics department lobby that will demonstrate real images. You are given a box of large, high quality, thin lenses with focal lengths of $\pm 10, 20, 30, 50$ cm (several of each size) along with other construction materials such as a small electric candle, plywood, black spray paint, and construction adhesive to use to make a hooded box (to hide/house the candle) and lens mounts.

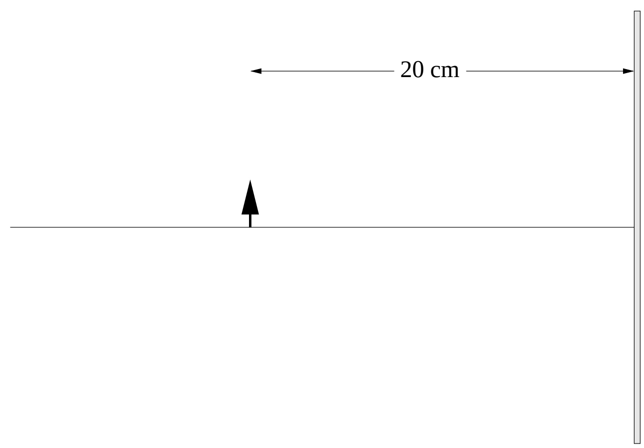
The plan is for you to construct an arrangement of lenses that creates a real image of the candle with overall magnification of $+1$ at the front of a short platform, so that a "real candle" appears to be attached there if one looks through the lenses from in front. Of course the candle is just the ghost of a candle and cannot be grasped by the hand if a student tries.

Select one or more lenses from the box and draw a plan for an arrangement that will do the trick. Your plan should include the location (to scale) of the lens(es), its(their) focal length(s) on the diagram, a ray diagram that shows how it will work, proof that the total magnification is $+1$, and an estimate of the overall length of the box L (accurate within a few cm).

Note that there are an infinite number of choices and arrangements that will "work", but bear in mind that it would be desirable for the overall box and platform to to fit on a moveable cart or small table, so maintaining a total length of $L < 1.75$ meters would be a good idea...

Problem 266.

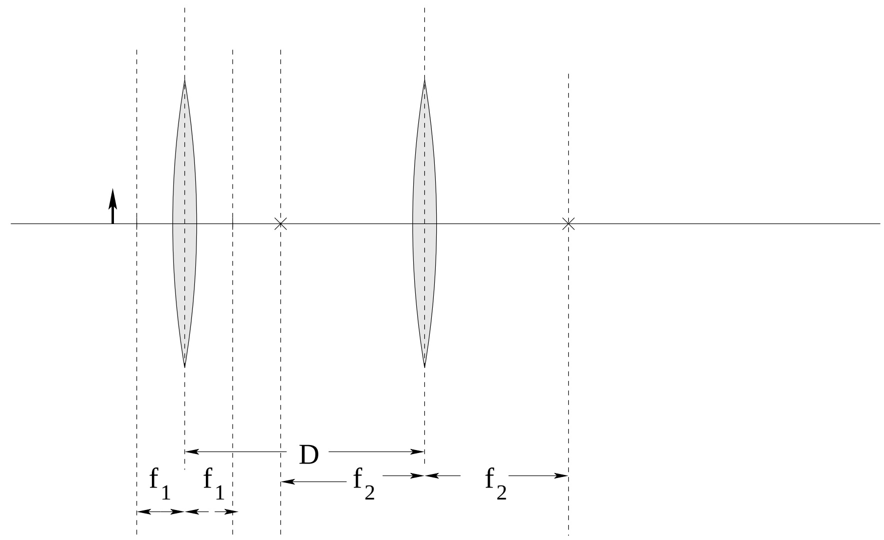
problems/lenses-pr-real-image-mag3.tex



There is an object 20 cm away from a screen. Using a converging (thin) lens, I would like to throw an image of this object upon the screen that is three times larger (in magnitude) than the object itself. Find the location of the lens (with respect to object and screen and the focal length of the lens necessary to accomplish this. Draw the corresponding ray diagram. [Hints: Remember the central ray! Is the image required real or virtual?]

Problem 267.

problems/lenses-pr-two-converging.tex



Two converging lenses are shown above. The first has a focal length of $f_1 = 1$ cm. The second has a focal length of $f_2 = 3$ cm and is placed a distance of $D = 5$ cm from the first lens.

An object is placed a distance $s_1 = 3/2$ cm in front of the first lens. Dashed lines are drawn for your convenience into the figure at the focal points and down the center plane of the lenses.

a) Draw (using a straightedge if possible) the ray diagrams for *both* lenses, using the image from the first lens as the virtual object for the second one. Locate and circle the final image and indicate whether the final image is erect or inverted, real or virtual.

b) Solve for the magnification of this final image. Show all work! Just because there are numbers, don't go all crazy. You shouldn't need a calculator for these numbers, but you are welcome to use one *after* setting up the arithmetic required on the paper.

16.3 The Eye

16.3.1 Multiple Choice

Problem 268.

problems/eye-mc-vision-underwater.tex

If you dive into a clear freshwater lake or pool and open your eyes, everything is a blur. This is because:

a) The muscles that control accommodation spasm when the eye is in contact with water, making you effectively nearsighted. This is why people wear swim masks.

b) Contact with the water makes the lens swell, making you effectively nearsighted. This is why people wear swim masks.

c) Your eyes' lenses have become farsighted underwater beyond your ability to accommodate by altering the bending of light at their surface.

d) Your eyes' lenses have become nearsighted underwater beyond your ability to accommodate by altering the bending of light at their surface.

e) Water isn't really transparent – it naturally makes things look blurry.

16.3.2 Short Answer/Concept

Problem 269.

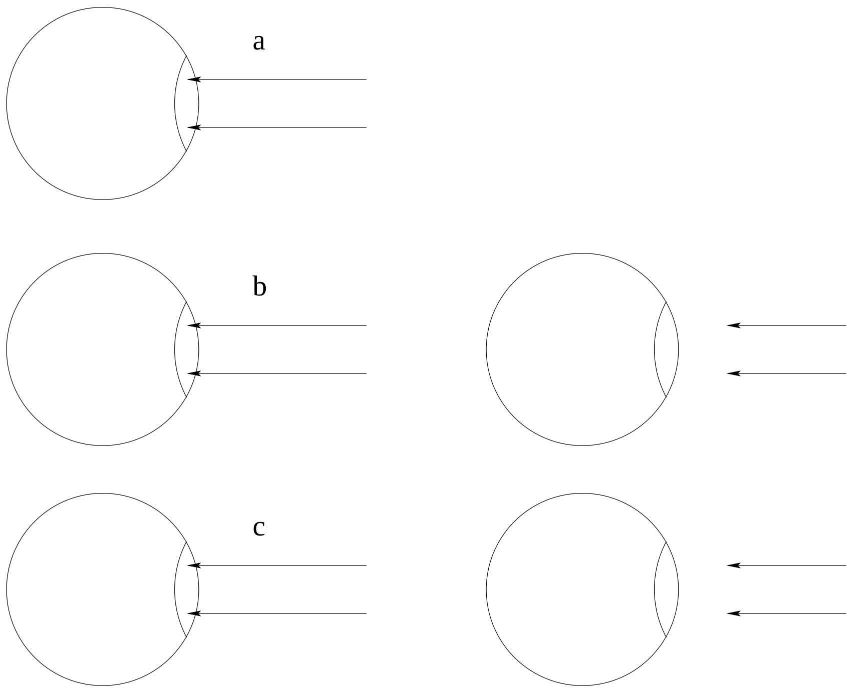
problems/eye-sa-bull-shark-in-fresh-water.tex

Bull sharks are known to swim from the ocean into freshwater rivers and lakes and are sometimes found a thousand miles or more away from the sea. Freshwater is considerably less dense than salt water, and the index of refraction of saltwater is correspondingly *greater* than that of fresh water.

If a bull shark has perfectly normal vision in the ocean, is it shortsighted or farsighted in fresh water? In particular, can it still accommodate to a clear vision of distant prey or is everything a bit of a blur to it in fresh water?

Problem 270.

problems/eye-sa-corrections.tex



On the figure above there are three "eyeballs" schematically represented. On each figure draw:

a) The correct location of the focal point of the relaxed eye of each kind (in front of, on, or behind the retina).

b) The lenses required to correct nearsightedness and farsightedness, drawn in front of each eye.

c) Complete the two rays given (arising from a very distant object "at infinity"), tracing their path through the lenses needed (if any) to the retina, assuming the objects are in focus.

Problem 271.

problems/eye-sa-fish-out-of-water.tex

You catch a fish that has normal vision – in the water! In air you appear all blurry to it. Is the fish nearsighted or farsighted in air? Should you outfit the fish with converging or diverging lenses so that it can see you clearly?

Problem 272.

problems/eye-sa-nearsighted.tex

Draw a representation of a nearsighted eye, indicating the focal length of the relaxed lens. Then draw the eye with the appropriate corrective lense in front and indicate with rays how it "fixes" the problem.

Problem 273.

problems/eye-sa-scuba-mask.tex

Explain in very simple terms why you can see clearly underwater wearing a diving mask but see everything as a blur when your eyes are in direct contact with the water. (Pictures would certainly help your explanation.)

16.4 Optical Instruments

16.4.1 Short Answer/Concept

Problem 274.

problems/optical-instruments-sa-draw-reflector-telescope.tex

One can make a telescope by using a *mirror* instead of a *lens* for the primary (objective) stage of magnification. Using what you know about how telescopes work as inspiration, draw a schematic for such a (Newtonian) reflecting telescope. The diagram should indicate how the focal lengths of primary mirror and eyepiece lens are placed so that one can view distant objects, magnified, with a relaxed normal eye. You do not have to draw an actual ray diagram, and you may or may not choose to use a small flat mirror in the barrel of the telescope to allow you to put the eyepiece on the side.

Problem 275.

problems/optical-instruments-sa-three-telescope-question.tex

Answer the three short questions below with a word, phrase or picture. For example, an answer to the first question might be "To avoid sinusoidal wiggle" (but probably isn't).

a) Why would one make a parabolic lens or mirror?

b) Why are big telescopes almost invariably built with a primary mirror instead of a lens?

c) Why are the optics of good binoculars "coated" with a thin film?

16.4.2 Long Problems

Problem 276.

problems/optical-instruments-pr-build-microscope-m500.tex

A physics professor hands you a box that contains the following material: (mounted) lens A with $f_A = 10$ cm, lens B with $f_B = 1$ cm, lens C with $f_C = 5$ mm and lens D with $f_D = -2$ mm. There is also a piece of tubing 15 cm long that fits the lens mounts exactly and can be cut to any length you like with the enclosed hacksaw, a focus gear (that can be used to move the objective lens mount small distances along its axis in the tube), glue, screws, a slide/tube mounting bracket, and things like that.

a) Create a rough design in the space above for a simple microscope with a magnification of $M = -500$ using this material and equipment. ***Clearly indicate the lenses you use and their arrangement in the tube.***

b) Draw the rays needed to prove that the magnification of your design is correct and do so.

Problem 277.

problems/optical-instruments-pr-build-reflecting-telescope-m200.tex

A physics professor hands you a box that contains the following material: a converging mirror A with $f_A = 100$ cm, a converging mirror B with $f_B = 200$ cm, a diverging lens C with $f_C = -2$ mm, a converging lens D with $f_D = 5$ mm and diverging lens E with $f_E = -5$ mm. The box also contains a small, round flat mirror centered on an axle so that it can be rotated to any angle, 4 meter sections of PVC pipe that fit each lens or mirror and that can be cut with a handy hacksaw, sleeves that can nest PVC pipe sections together, some glue, focus gears (that can be used to move the eyepiece lens small distances along its axis), and things like that.

a) Create a rough design in the space above for a ***reflecting*** telescope with an angular magnification of ***magnitude*** $M = 200$, made using this material and equipment. Clearly indicate the lenses you use and their arrangement in the tube(s). Note that there might be more than one way to do this.

b) Draw the rays needed to prove that the magnification of your design is correct and do so. It need not be precisely to scale, but should have all of the correct features.

Problem 278.

problems/optical-instruments-pr-build-refracting-telescope-m200.tex

A physics professor hands you a box that contains the following material: lens A with $f_A = 100$ cm, lens B with $f_B = 200$ cm, lens C with $f_C = -2$ mm, lens D with $f_D = 5$ mm and lens E with $f_E = -5$ mm. There are also 4 meter sections of PVC pipe that fit each lens and that can be cut with a handy hacksaw, sleeves that nest the PVC pipe sections together, some glue, focus gears (that can be used to move the eyepiece lens small distances along its axis), and things like that.

a) Create a rough design in the space above for a ***refracting*** telescope with an angular magnification of ***magnitude*** $M = 200$, made using this material and equipment. Clearly indicate the lenses you use and their arrangement in the tube(s). Note that there might be more than one way to do this.

b) Draw the rays needed to prove that the magnification of your design is correct and do so. It need not be precisely to scale, but should have all of the correct features.

Problem 279.

problems/optical-instruments-pr-design-microscope-m250.tex

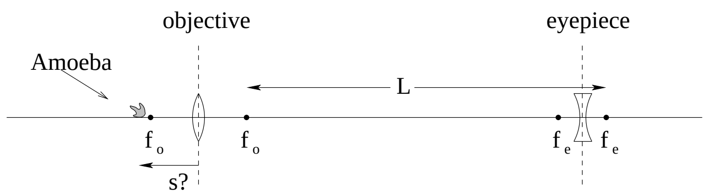
a) Design a microscope with a tube length $\ell = 10$ cm and a magnification of 250. Draw it below to scale. You may pick f_o and f_e to have any "sensible" values, and can make the microscope invert the image you see or not as you wish.

b) Draw the rays needed to prove that it has the correct magnification and do so.

c) Determine where (that is, the actual object distance s in cm) one has to place the object in front of the objective lens in order for the relaxed, normal eye to view its image at infinity through the eyepiece. Note that this answer will depend (obviously) on your choice of f_o and other parameters, so the number answer is less important than the algebra (which is what will be checked).

Problem 280.

problems/optical-instruments-pr-galilean-microscope.tex



The arrangement of lenses that makes up a "Galilean" compound microscope is pictured above. The focal lengths of the objective and eyepiece lenses are $f_o = 2$ cm and $f_e = -1$ cm. The tube length is $L = 20$ cm.

- Find s (the object distance from the objective lens) such that the final image viewed by the eye is in focus (at infinity, as imaged by the eyepiece).
- Draw the ray diagram from which you can find the overall magnification. NOTE WELL the tube length goes to the second (negative) focal point of the eyepiece. Why?
- From this diagram, find the overall magnification. Explain each part (that is, what are the separate roles of the objective and eyepiece).
- What is the advantage of this kind of microscope compared to one with two converging lenses?

Problem 281.

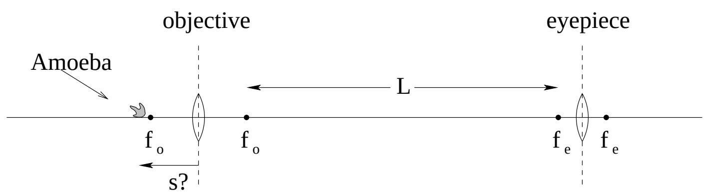
problems/optical-instruments-pr-galilean-telescope-design.tex

Draw below a Gallilean telescope (one built with a converging primary lens and a *diverging* eyepiece lens). Draw it to scale so that the overall angular magnification is $M = 10$. Derive (with a figure and the correct rays and triangles and angles) its magnification in terms of f_p, f_e, and any other parameters you think necessary. Remember, f_e is *negative* for a Gallilean telescope – be sure to specify whether the brain perceives the final image right side up or upside down so that there is no ambiguity.

Note that the rays used to derive the magnfication are tricky for a diverging eyepiece, so be careful.

Problem 282.

problems/optical-instruments-pr-regular-microscope.tex



The arrangement of lenses that makes up a compound microscope is pictured above. The focal lengths of the objective and eyepiece lenses are $f_o = 2$ cm and $f_e = 1$ cm. The tube length is $L = 20$ cm.

a) Find s (the object distance from the objective lens) such that the final image viewed by the eye is in focus (at infinity, as imaged by the eyepiece).

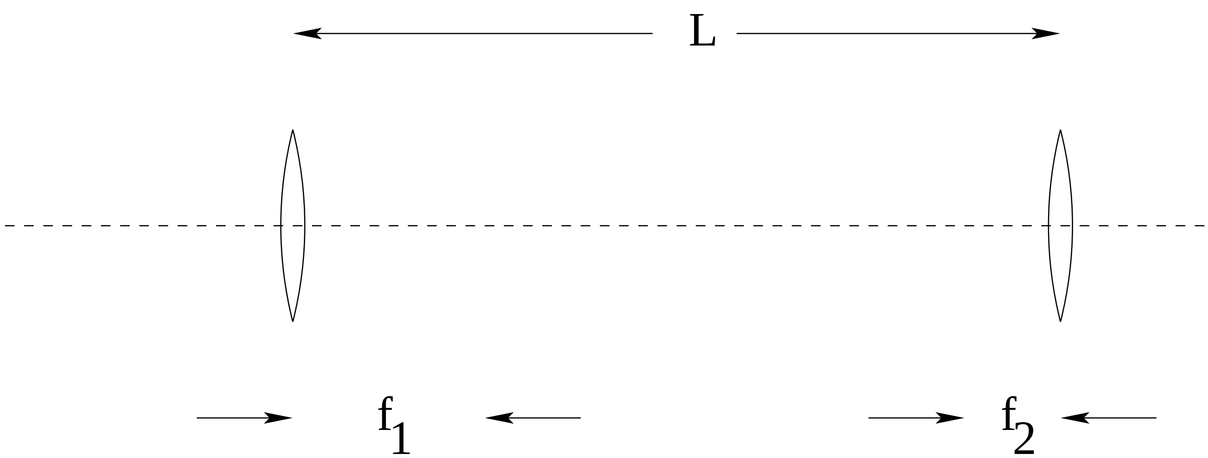
b) Draw the ray diagram from which you can find the overall magnification.

c) From this diagram, find the overall magnification. Explain what each part contributes to the overall magnification (that is, what are the separate roles of the objective and eyepiece in allowing you to see a significantly magnified image at infinity).

d) Is the final image you see inverted or erect compared to the way you would see the object with your naked eye (if you *could* see the object with your naked eye)?

Problem 283.

problems/optical-instruments-pr-small-microscope.tex



An arrangement of two lenses is drawn above. $f_1 = 2$ cm and $f_2 = 1$ cm. The *lenses themselves* are separated by a distance $L = 8$ cm. Then:

a) What is this particular arrangement of lenses called?

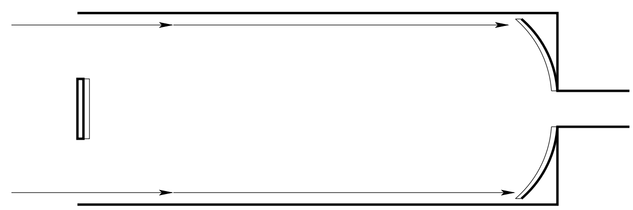
b) Find where (s) to put an object to be viewed relative to the first lens (on the left) so that the final image seen through the second lens (on the right) can be viewed with the relaxed, normal eye.

c) The ***angular*** magnification of the final image compared to what it would be when seen by the relaxed, normal eye without any lenses.

d) Draw the ray diagram for this arrangement using the object distance determined above that can be used to locate the final image and (with some rules) find the magnification. You may or may not have to redraw the arrangement to have room.

Problem 284.

problems/optical-instruments-pr-telescope-cassegrain-reflector.tex



This problem asks you to analyze the Cassegrain reflecting telescope design, drawn above. The two incoming rays drawn from an object at infinity in the center of its visual field are reflected from the parabolic primary mirror and then re-reflected by the *flat* secondary mirror so that they converge in the vicinity of the eyepiece tube. The focal length of the primary mirror is 100 cm. You have two eyepiece lenses, one converging and one diverging, each with a focal length of five cm.

a) Pick either of these eyepieces and locate it in the eyepiece tube in such a way that the telescope permits you to observe this distant object with a relaxed normal eye. Clearly indicate where its focal point has to be relative to the doubly reflected focal point of the primary mirror. Show the two distances that must add up to 100 cm.

b) Draw the rest of the ray diagram for the two incoming rays that show how they emerge from the eyepiece lens to enter your eye so that the condition required in a) is true. What is the expected magnification of this telescope?

c) This telescope has the reflecting secondary mirror situated right in the middle of the telescope mouth. Is there a corresponding hole in your visual field?

d) What happens to the image you observe as you vary the diameter of this secondary mirror relative to the diameter of the primary reflecting mirror?

Problem 285.

problems/optical-instruments-pr-telescope-galilean.tex

Consider an Galilean refracting telescope (one built with a ***converging*** primary/objective lens with focal length f_o and a ***diverging*** eyepiece lens with focal length $-f_e$). Then:

a) Draw it to scale (below) so that the overall angular magnification is $|M| = 4$. (Note: You have to determine the correct sign of M below, so I only give you its magnitude here.)

b) Derive (with a figure and the correct rays and triangles and angles) its magnification in terms of f_o, f_e, and any other parameters you think necessary.

c) Does this telescope invert its image (as seen by the eye) or not?

Problem 286.

problems/optical-instruments-pr-telescope-near-object.tex

You have a telescope constructed with a primary lens with a focal length $f_o = 1$ meter, and an eyepiece lens with a focal length of $f_e = 0.1$ meter (ten centimeters).

a) What is the magnification of this telescope when used to view truly distant objects, i.e. the moon? Does it invert the image or not?

b) Suppose one wishes to use the telescope to view a bird ten meters (or $10 f_o$) in front of the primary lens. Where is the (real) image formed (that is, find s' in terms of f_o)?

c) Where must one locate the eyepiece lens (via the "focus" knob) in this case in order to view the bird through the telescope with a relaxed, normal eye?

d) Draw the arrangement of lenses and a representation of the image of the bird inside the telescope, approximately to scale horizontally, labelling all distances. You do not need to draw the bird/object itself – it is too far away to easily fit on your page if the telescope is clearly drawn. Do, however, draw the rays one might use to estimate the effective magnification of the telescope in this case.

Problem 287.

problems/optical-instruments-pr-telescope.tex

Consider an ordinary refracting telescope (one built with a ***converging*** primary/objective lens with focal length f_o and a ***converging*** eyepiece lens with focal length f_e). Then:

a) Draw it to scale (below) so that the overall angular magnification is $|M| = 4$. (Note: You have to determine the correct sign of M below, so I only give you its magnitude here.)

b) Derive (with a figure and the correct rays and triangles and angles) its magnification in terms of f_o, f_e, and any other parameters you think necessary.

c) Does this telescope invert its image (as seen by the eye) or not?

Chapter 17

Interference and Diffraction

17.1 Interference

17.1.1 Short Answer/Concept

Problem 288.

problems/interference-sa-4-slit-phasors.tex

Draw the phasor diagrams that correspond to ***interference minima*** for the ***four slit*** problem – four slits that are much narrower than the wavelength of monochromatic light illuminating them, separated by a distance d, a long way away from a screen. Write down the *first* set of angles $0 \leq \delta = kd\sin\theta \leq 2\pi$ (***Note well:*** δ in the range $[0, 2\pi]$) for which these minima occur.

Problem 289.

problems/interference-sa-5-slit-phasors.tex

Draw the phasor diagrams that correspond to ***interference minima*** for the ***five slit*** problem – five slits that are much narrower than the wavelength of monochromatic light illuminating them, separated by a distance d, a long way away from a screen. Write down the *first* set of angles $0 \leq \delta = kd \sin\theta \leq 2\pi$ (***Note well:*** δ in the range $[0, 2\pi]$) for which these minima occur.

Problem 290.

problems/interference-sa-6-slit-phasors.tex

Draw the phasor diagrams that correspond to ***interference minima*** for the ***six slit*** problem – six slits that are much narrower than the wavelength of monochromatic light illuminating them, separated by a distance d, a long way away from a screen. Write down the *first* set of angles $0 \le \delta = kd\sin\theta \le 2\pi$ (***Note well:*** δ in the range $[0, 2\pi]$) for which these minima occur.

Problem 291.

problems/interference-sa-four-slit-minima.tex

What are the four values of $\delta = kd \sin\theta$ for the first four *interference minima* produced by five slits. Should be able to read them right off the phasor pictures...

Problem 292.

problems/interference-sa-resolution-grating-sodium.tex

Light from a sodium lamp has a double line in the yellow part of the spectrum with wavelengths $\lambda_1 = 589.6$ nm and $\lambda_2 = 589.0$ nm ($\Delta\lambda \approx 0.6$ nm correct to the number of significant digits displayed). What is the minimum number of slits that must be illuminated within the coherence length of the light so that a diffraction grating that can resolve the lines:

a) In first order (for $m = 1$)?

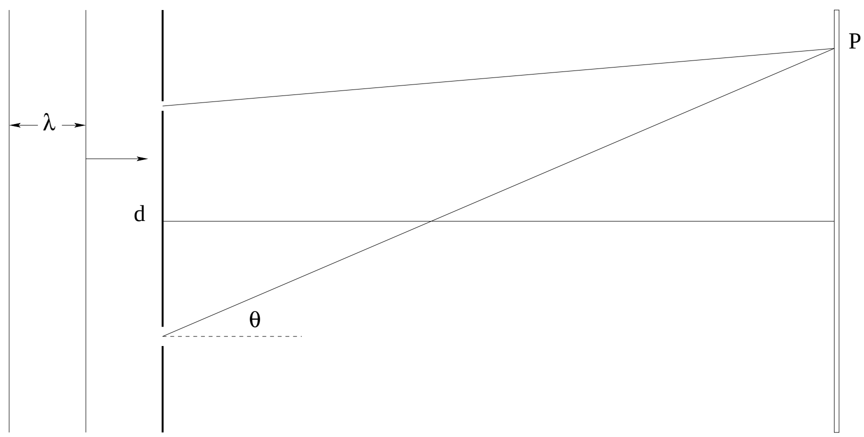
b) In second order (for $m = 2$)?

You may approximate freely, e.g. you can assume that $\lambda_1 \approx \lambda_2 \approx 600$ nm to get your answer without a calculator...

17.1.2 Long Problems

Problem 293.

problems/interference-pr-2-asymmetric-slits.tex



Light with wavelength λ is incident on a barrier with two narrow (width much smaller than a wavelength) slits separated by a distance $d = 3\lambda$ cut in it. The lower slit is *twice as wide* as the upper one. The light passes through the two slits and falls upon a distant screen at a point P that is at an angle θ above both slits, where the figure is not horizontally to scale.

a) Write an expression for the total electric field you expect to get at P from both slits, in terms of the field strength E_0 at P from the upper slit only.

b) Draw a phasor diagram that you *could* use to solve for the total field amplitude for arbitrary $\delta = kd\sin(\theta)$. Do not attempt to solve it at this time.

c) Draw (small) phasor diagrams that schematically indicate the phase angles δ where you expect to get maximum and minimum intensity on the screen. What are the magnitudes you expect for maximum and minimum intensity on the basis of these diagrams.

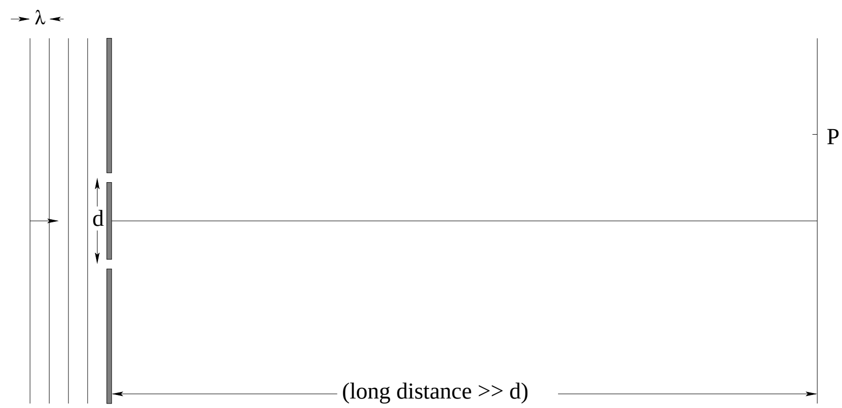
d) Find the angles θ where the maxima and minima occur and sketch the intensity as a function of θ, approximately to scale.

Only when this is done and checked should you then attempt:

e) For five points of extra credit, find an explicit expression for $E_{\rm tot}$, and use it to express the intensity $I(\delta)$ in terms of the intensity of the upper slit by itself, I_0. Hint: remember the law of cosines.

Problem 294.

problems/interference-pr-2-narrow-slits-1.tex



Light with wavelength $\lambda = 600$ nm passes through two extremely narrow slits. The slits are spaced a distance $d = 2400$ nm apart. The light then travels a long distance and falls on a screen. The intensity of light reaching the midpoint of the screen from any *single* slit (with the other one covered) is I_0 and corresponds to a field strength E_0.

a) Using the superposition principle, write down an expression for the total electric field evaluated at P resulting from the waves that pass through the two slits.

b) What is δ (the phase difference between the waves from the upper and lower slits) in terms of d, λ, and θ?

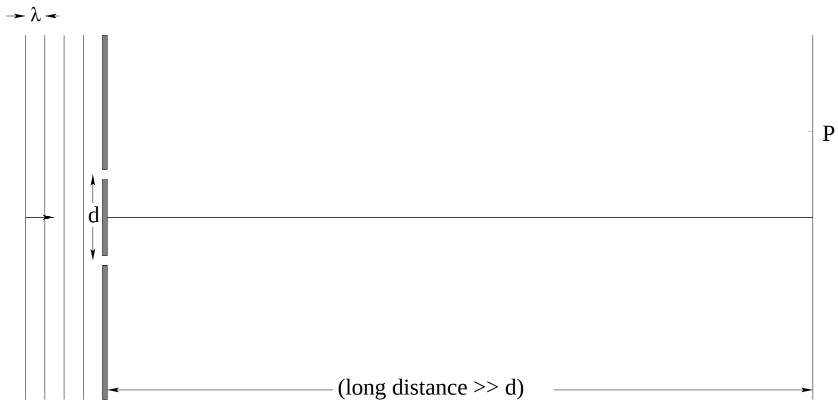
c) ***Draw the phasor diagram*** and use it to evaluate the total field amplitude.

d) Make a short table of all of the angles where interference maxima and minima occur.

e) Draw a semi-quantitatively correct graph of the intensity as a function of θ between 0 and $\pi/2$. Correctly label the peak intensity expressed in terms of I_0.

Problem 295.

problems/interference-pr-2-narrow-slits.tex



Light with wavelength $\lambda = 500$ nm passes through two extremely narrow slits. The slits are spaced a distance $d = 2000$ nm apart. The light then travels a long distance and falls on a screen. The intensity of light reaching the midpoint of the screen from any *single* slit (with the other two covered) is I_0 (and corresponds to a field strength E_0).

a) Draw onto the figure above lines and coordinates that will help you determine the intensity of the interference pattern produced by the slits. Use θ for the angle to an arbitrary point P on the screen relative to the midline drawn from the central slit (as done in class).

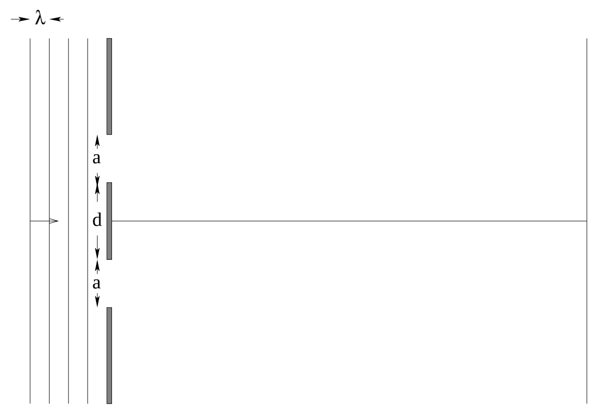
b) Using the superposition principle, write down the sum of the two waves from the slits at P, using δ as the phase angle introduced by the path difference between them.

c) *Draw the phasor diagram* that allows you to find the total field amplitude as a function of an arbitrary δ (choose and draw a convenient one as I did in class), and evaluate the total field amplitude. Write an expression for the total intensity as a function of δ and I_0. What is the peak intensity in terms of I_0?

d) Write an expression for the angles θ where a *principle* interference maximum occurs. How many (non-negative) angles are there? Draw a qualitatively correct graph of the intensity as a function of θ between 0 and $\pi/2$.

Problem 296.

problems/interference-pr-2-wide-slits-1.tex



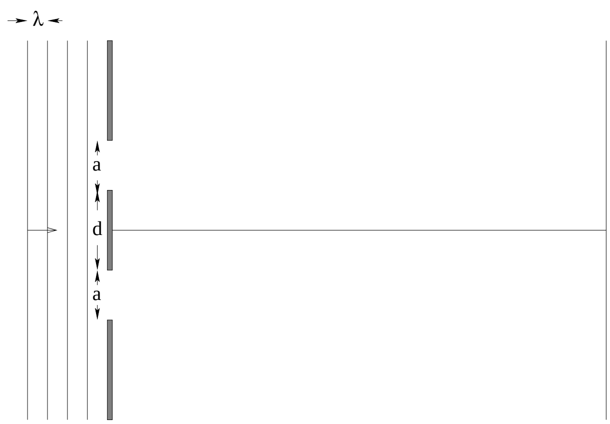
Light with wavelength $\lambda = 700$ nm passes through two slits of a width $a = 1400$ nm. The centerpoints of these two slits are separated by a distance of $d = 3500$ nm. The light then travels a long distance and falls on a screen. It is *not* necessary (for once) to derive or justify the equation(s) you use below, but if you do you will get partial credit even if your numerical answers are wrong.

a) Write down (or derive) the algebraic formula for the intensity of the combined interference-diffraction pattern for this arrangement.

b) Write down (or derive) the formula from which the angles at which diffraction minima occur can be found, and apply it to find *all* these angles (put them in a table).

c) Write down (or derive) the formulas from which the angles at which interference maxima and minima occur, and apply them to find **the first three of each** (only) (put them in a table).

d) Draw a qualitatively correct picture of the expected diffraction/integration pattern $I(\theta)$.



Problem 297.

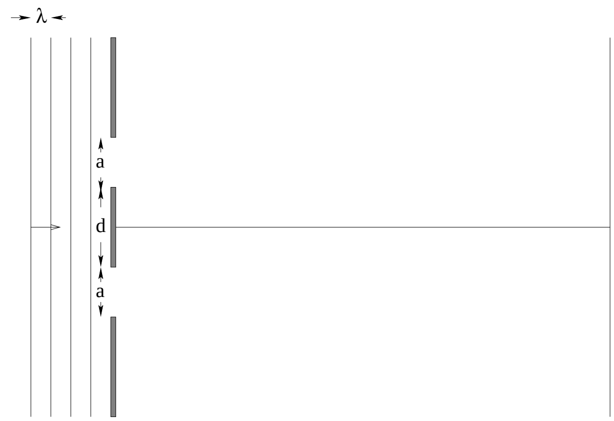
problems/interference-pr-2-wide-slits-2.tex

Light with wavelength $\lambda = 300$ nm passes through two slits of a width $a = 900$ nm. The centerpoints of these two slits are separated by a distance of $d = 2700$ nm. The light then travels a long distance and falls on a screen. It is *not* necessary (for once) to derive or justify the equation(s) you use below, but if you do you will get partial credit even if your answers are wrong.

a) Write down (or derive) the formula from which the angles at which diffraction minima occur can be found, and apply it to find *all* these angles (put them in a table).

b) Write down (or derive) the formulas from which the angles at which interference maxima and minima occur, and apply them to find the first three of each (put them in a table).

c) Draw a qualitatively correct picture of the expected diffraction/integration pattern $I(\theta)$.



Problem 298.

problems/interference-pr-2-wide-slits-3.tex

All angles in the parts a-c may be expressed by means of tables of inverse trigonometric functions of simple fractions, e.g. $\cos^{-1}(1/2)$, $sin^{-1}(2/7)$, etc.

Two vertical slits of width $a = 1200$ nanometers (nm) are separated (center to center) by a distance of $d = 3000$ nm and illuminated by light of wavelength $\lambda = 600$ nm. The light which passes through is then projected on a distant screen. Find:

a) The location (angles θ) of all **diffraction minima**.

b) The location of all **interference minima**.

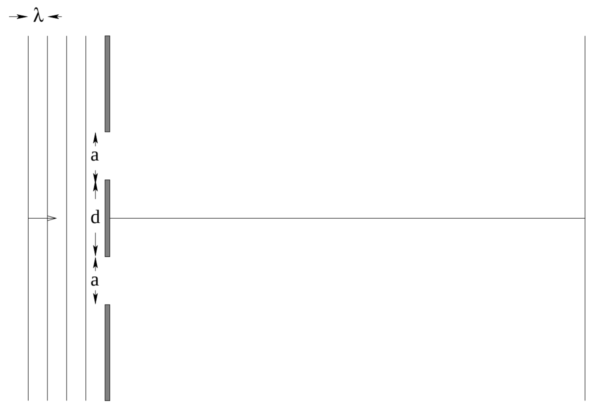
c) The location of all **interference maxima**.

d) Finally, draw a properly proportional figure of the resulting interference pattern between 0 and $\pi/2$ (on either side), indicating the maximum intensity in terms of the central maximum intensity that would result from a single slit.

e) For five points of extra credit, write down the algebraic expression for $I(\theta)$ in terms of I_0 (the central intensity of a single slit), defining all variables used (like ϕ and δ) in terms of a, d, λ and θ.

Problem 299.

problems/interference-pr-2-wide-slits-4.tex



Two vertical slits of width 1500 nanometers (nm) are separated (center to center) by a distance of 2500 nm and illuminated by light of wavelength 500 nm. The light which passes through is then projected on a **distant screen**. Find θ (or $\sin(\theta)$) for:

a) The location of all **diffraction minima**.

b) The location of all **interference minima**.

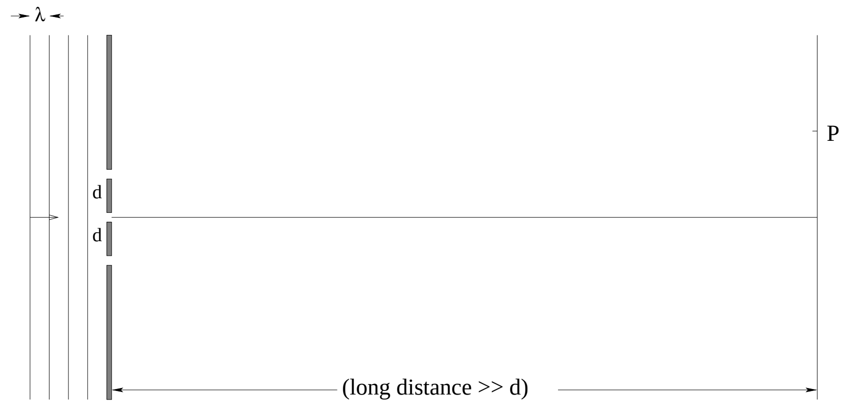
c) The location of all **interference maxima**.

d) Finally, draw a properly proportional graph of $I(\theta)$ (or $I(\sin(\theta))$) between $-\pi/2$ and $\pi/2$ (or -1 and 1) indicating the maximum intensity in terms of the central maximum intensity that would result from a single slit.

Note well that you may answer in terms of $\sin(theta)$ *or* θ as you prefer, but without a calculator $\sin(\theta)$ is usually somewhat simpler.

Problem 300.

problems/interference-pr-3-narrow-slits.tex



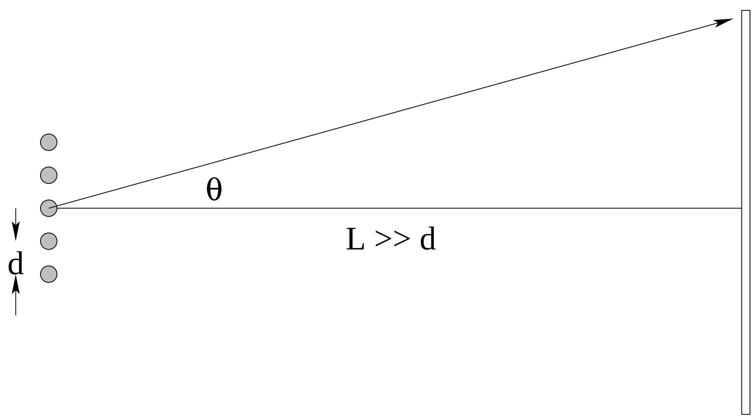
Light with wavelength $\lambda = 500$ nm passes through three extremely narrow slits. The slits are spaced a distance $d = 2000$ nm apart. The light then travels a long distance and falls on a screen. The intensity of light reaching the midpoint of the screen from any *single* slit (with the other two covered) is I_0 (and corresponds to a field strength E_0).

a) Draw onto the figure above lines and coordinates that will help you determine the intensity of the interference pattern produced by the slits. Use θ for the angle to an arbitrary point P on the screen relative to the midline drawn from the central slit (as done in class).

b) Using the superposition principle, write down the sum of the three waves from the slits at P, using δ as the phase angle introduced by the path difference between them.

c) *Draw the phasor diagram* that allows you to find the total field amplitude as a function of an arbitrary δ (choose and draw a convenient one as I did in class), and evaluate the total field amplitude. Write an expression for the total intensity as a function of δ and I_0. What is the peak intensity in terms of I_0?

d) Write an expression for the angles θ where a *principle* interference maximum occurs. How many (non-negative) angles are there? Draw a qualitatively correct graph of the intensity as a function of θ between 0 and $\pi/2$.



Problem 301.

problems/interference-pr-5-slits.tex

Suppose you have five point sources of monochromatic, coherent light with wavelength λ and a common phase lined up and separated by a distance d as shown above. The light from all five sources falls upon a *distant* ($L \gg d$) screen as shown.

Derive expressions for the angles at which maxima and minima occur. Your derivation should include the *phasor diagrams* for the minima and the maxima and should relate $\phi = kd\sin(\theta)$ to suitable fractional multiples of π. End by drawing a representative cycle or two (primary maximum to primary maximum) of the resulting interference pattern, showing the correct relative intensity of primary and secondary peaks and the right number of minima and maxima per cycle.

Note that in this case "derive" pretty much means draw the correct five-sided phasors for each of the max and min cases and read off what ϕ must be for each, and arrange it in a pretty (simple) pattern.

17.2 Thin Films

17.2.1 Short Answer/Concept

Problem 302.

problems/thin-film-sa-oil-on-water-sheen.tex

When a a layer of oil spreads out on top of water and gets much thinner than the wavelength of visible lignt, it becomes shiny and bright, reflecting all wavelengths. Why? (A diagram with the reason marked and circled is fine.)

Problem 303.

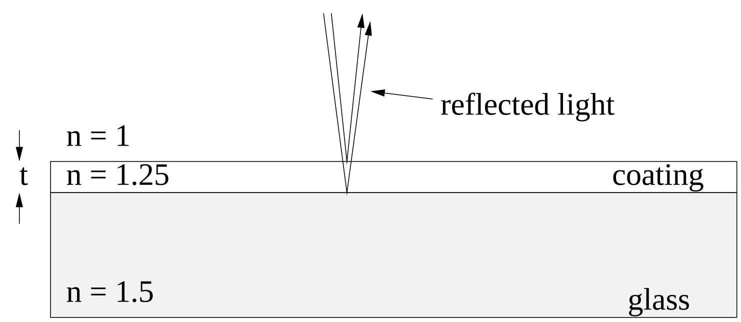
problems/thin-film-sa-oil-on-water.tex

A drop of oil ($n_o = 1.25$) floats on top of water ($n_w = 4/3$) creating a very think film as it spreads out. At first you see a riot of vaguely toxic rainbow colors in the reflection of white overhead light, but then, as its thickness gets to be much less than any wavelength in visible light, it either turns bright (reflecting all colors like a mirror) or dark (transmitting all colors and reflecting none of them.

Which? Justify your answer with a picture and a few words explaining.

Problem 304.

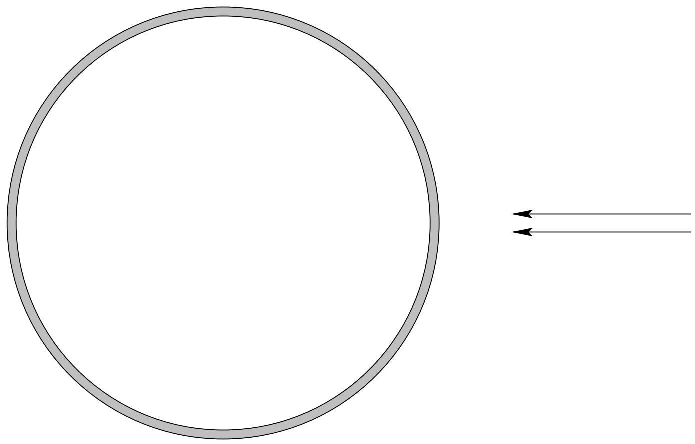
problems/thin-film-sa-optical-coating.tex



You would like to eliminate the reflected light from a flat glass pane for perpendicularly incident light of wavelength 550 nm. The index of refraction of the glass is $n_g = 1.5$, and the index of refraction of the coating material to be used is $n_c = 1.25$. What minimum thickness t of the coating material will have the desired effect? (Try to show your reasoning, and don't forget "details".)

Problem 305.

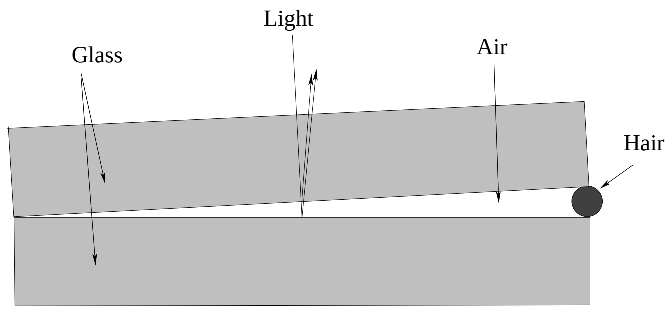
problems/thin-film-sa-soap-bubble-transparency.tex



When a soap bubble like the one drawn above gets much thinner than the wavelength of visible light, it becomes transparent. Why? (A diagram with the reason marked and circled on the figure above, plus a sentence or two of explanation is fine.)

Problem 306.

problems/thin-film-sa-wedge-caliper.tex

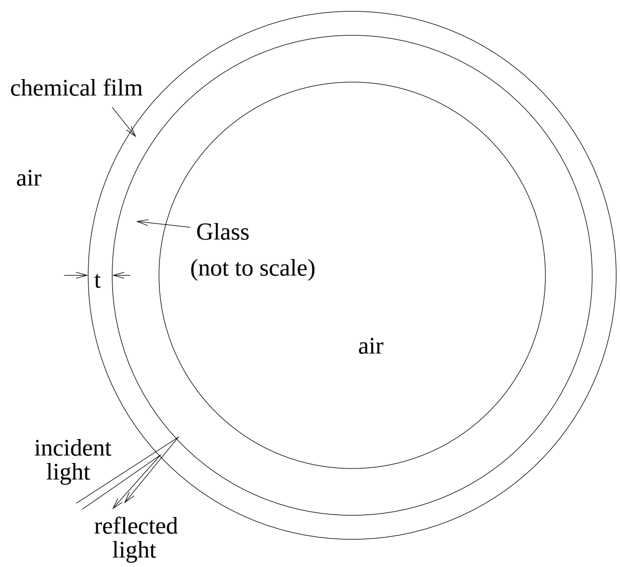


Two pieces of very flat glass are used in the arrangement above to measure the thickness of a human hair. When viewed with light of 600 nm from above, 30 dark fringes are observed in the light reflected from the wedge of air. How thick (approximately) is the hair? (Note well: Derive/explain your answer and show all work).

17.2.2 Long Problems

Problem 307.

problems/thin-film-pr-ornament-1.tex



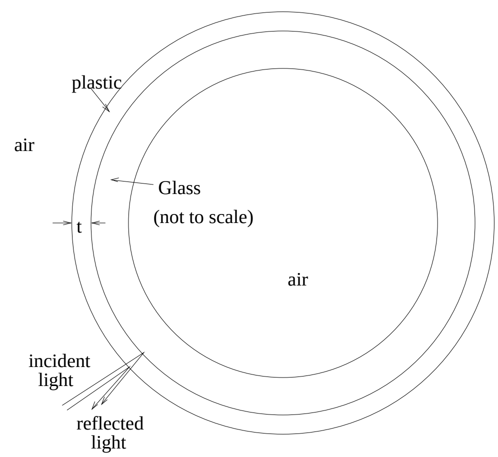
A Christmas tree ornament is constructed by vapor-depositing a chemical film (with n = 1.7) on a “thick” ($\sim$ 2 mm) spherical glass (n = 1.5) bubble as drawn schematically above. The thin chemical film is not uniform in thickness, and its variation in the range 0-2 microns (micrometers) produces brilliant streaks of color in the reflected light.

a) What is the smallest (nontrivial) mean thickness t of the film such that reflected light to has a constructive interference *maximum* in the center of the visible spectrum (λ = 400-700 nm in free space where $n = 1$).

b) When the film first starts to deposit on the glass (and has a thickness t of only a few nanometers) does the film on the bulb turn shiny (constructively reflecting all wavelengths) or transparent (destructively reflecting all wavelengths)? Explain.

Problem 308.

problems/thin-film-pr-ornament-2.tex



A Christmas tree ornament is constructed by vapor-depositing a thin, transparent film (with $n = 1.25$) on a "thick" (~ 2 mm) spherical glass ($n = 1.5$) bubble as drawn schematically above. The thin plastic film is not quite uniform in thickness, and this variation produces brilliant streaks of color in the reflected light.

a) What does the light reflected from the ornament look like where $t \sim 0$ (or $t \ll \lambda$, at any rate). Explain the physics behind your answer with a single sentence and/or diagram.

b) At what thickness $t \sim \lambda > 0$ of the film will the reflected light first have a constructive interference *maximum* at $\lambda = 550$ nm (where λ, recall, is the wavelength in free space where $n = 1$)?

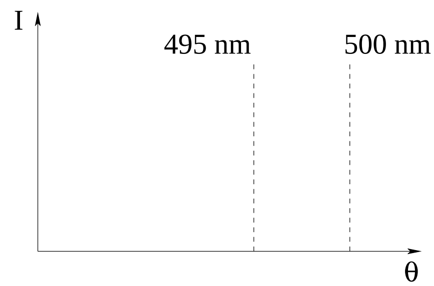
c) At that thickness, will any other visible wavelengths have an interference maximum or minimum? Justify your answer – just 'yes' or 'no' (even if correct) are incorrect.

17.3 Diffraction and Resolution

17.3.1 Multiple Choice

Problem 309.

problems/diffraction-mc-grating-resolution.tex



One wishes to resolve two spectral lines at $\lambda_1 = 495$ nm and $\lambda_2 = 500$ nm, respectively at ***second order*** with a diffraction grating. What is the ***minimum*** number of slits that must be illuminated (within the coherence length) to accomplish this?

a) $N = 50$

b) $N = 100$

c) $N = 250$

d) $N = 500$

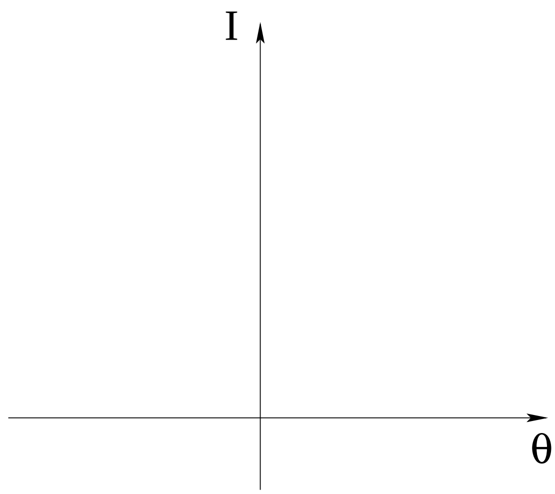
e) Answer not on the list above.

Draw the ***minimally resolved*** lines in on the figure above (where the horizontal axis is not to scale!) for a point of partial or extra credit. It is also probably wise to indicate the formula or reasoning process you are using.

17.3.2 Short Answer/Concept

Problem 310.

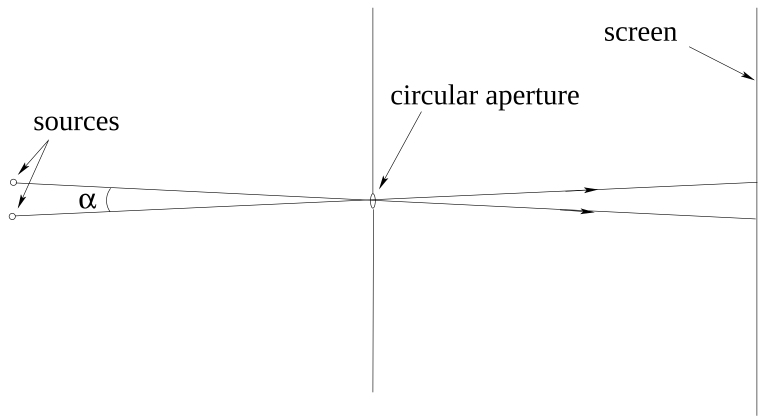
problems/diffraction-sa-minimal-resolution-graph-only.tex



Draw a qualitatively correct graph of the intensity that schematically represents two minimally resolved diffraction patterns (from e.g. a single slit) according to the ***Rayleigh Criterion of Resolution***. Carefully mark the location of the maximum and first minimum of the patterns on your figure corresponding to the criterion.

Problem 311.

problems/diffraction-sa-rayleigh-circular-aperture.tex



What is the smallest angle α for which light from two point-like sources will be minimally resolved (according to Rayleigh' criterion for resolution) if the light from the objects has wavelength λ and passes through a circular aperture of diameter $D \gg \lambda$ (so the small angle approximation will hold)?

17.3.3 Long Problems

Problem 312.

problems/diffraction-pr-1-wide-slit-1.tex

Light with wavelength $\lambda = 700$ nm passes through a single slit with a width $a = 2100$ nm. The light then travels a long distance (compared to a and λ) and falls on a screen. It is ***not necessary*** (for once) to derive or justify the equation(s) you use below, but if you do you might get partial credit even if your numerical answers are wrong.

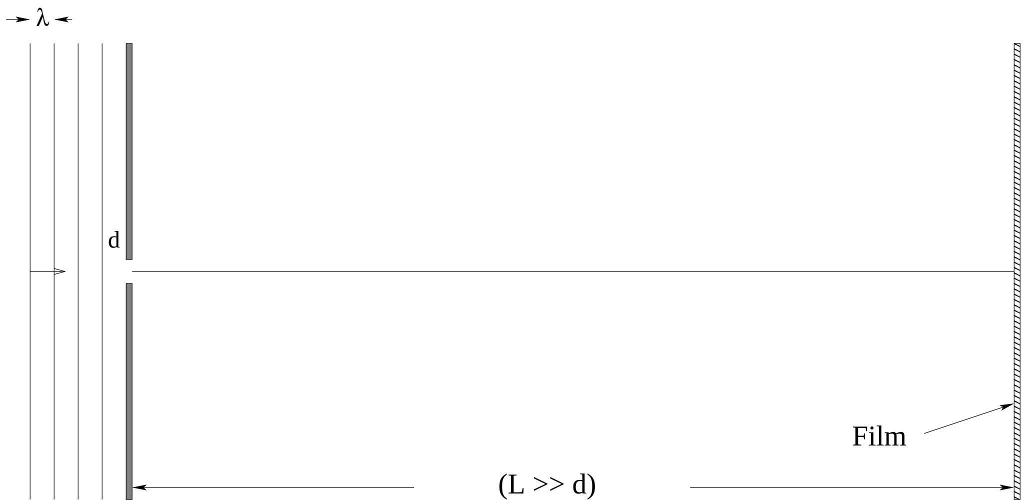
a) Write down (or derive) the algebraic formula for the intensity of the diffraction pattern for this arrangement, $I(\theta)$ where θ is measured from the midline of the slit to a line directed at the point of observation on the screen.

b) Write down (or derive) the formula from which the angles at which diffraction minima occur can be found, and apply it to find ***all*** these angles (put them in a table).

c) Draw a qualitatively correct picture of the expected diffraction pattern $I(\theta)$.

Problem 313.

problems/diffraction-pr-pinhole-camera-resolution.tex

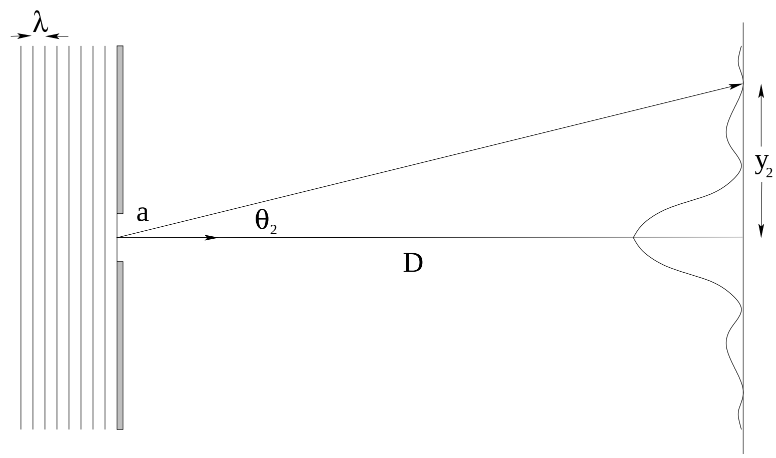


The pinhole (with diameter d) in a pinhole camera functions like a high-resolution universal-focus "lens" by permitting only the "central ray" through to form an image on a piece of film placed a distance $L = 50$ cm behind it as shown. A point source a distance $s \gg L$ in front of the pinhole creates a dot on the film the size of the aperture in the geometric optic approximation (where wavelength does not matter and light travels in straight lines). To get the best possible resolution we would then try to use the smallest possible dot (and wait longer for enough light to pass through to activate the film) with no lower bound in size. Light with wavelength $\lambda = 500$ nm passing through the aperture, on the other hand, casts a diffraction pattern onto the film that for small enough d will be wider than the geometric dot.

From these two competing limits, determine the diffraction-limited optimum minimum size for the pinhold diameter d that will give you the smallest possible image of the point source on the screen in terms of the givens. I gave you some (simple) numbers because the actual geometry matters a bit and the number is a good/reasonable one to know, but feel free to use algebra first to answer the question.

Problem 314.

problems/diffraction-pr-width-of-slit.tex



Light of wavelength $\lambda = 500$ nanometers passes through a slit of width a and falls on a screen a distance $D = 1$ meter away. The ***second*** minimum of the resulting diffraction pattern is observed at a position $y_2 = 2$ cm above the central maximum on the screen.

a) Find the width a of the slit.

b) Now the slit is illuminated with light with a wavelength of $\lambda = 400$ nanometers. What is y_1, the location of its ***first*** minimum on the screen?

As always, you can answer the questions algebraically first, but please try to do the arithmetic. You may use the small angle approximation to avoid having to use a calculator.